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RECHERCHES SUR LES COMÈTES *b* ET *c* 1881.

PAR TH. BREDICHIN.

1. *Comète b* 1881.

Les observations des comètes *b* et *c* employées dans ce mémoire sont faites par MM. Th. Bruhns, candidat de notre université (Simféropol en Crimée, $2^h\ 16^m$ à l'Est de Greenwich); H. J. H. Groneman et J. C. Kapteyn, professeurs à Groningue (Hollande; 26^m à l'Est de Greenwich); G. Tempel, astronome d'Arcetri, et par moi-même (à Nicolajeff).

M. Bruhns a soigneusement dessiné les contours des queues des comètes *b* et *c* sur les cartes de l'atlas de Heis et il m'a envoyé une copie fidèle de ces dessins, dont j'ai tiré les positions des points des queues.

Les positions des queues décrites dans la lettre de M. Groneman, que je publie ci-dessous, ont été portées par moi sur les cartes de Heis, pour en déduire les co-ordonnées des queues.

Les observations de M. Tempel sont représentées par des excellents dessins dont je reproduis ici celui de la comète *b*.

Que ces messieurs veuillent bien recevoir ici mes plus vifs et plus sincères remerciments de leurs communications.

Mes observations sont aussi portées sur les cartes de Heis.

Les positions des queues sont d'abord représentées par les co-ordonnées rapportées à l'équinoxe de 1855·0; dans les calculs ces positions sont réduites à l'équinoxe de 1881·5. Les numeros des étoiles sont ceux de catalogue Heis. Le temps est rapporté au méridien de Greenwich.

Observations de M. Bruhns:—

Juin 25, $11^h{\cdot}7$. Pour le bord gauche 82 Camelopard. 85° 13′, + 57° 55′; pour le bord droit 82° 48′, + 57° 55′.

Juin 26, $11^h{\cdot}7$. Pour l'axe de la queue 85° 36′, + 61° 54′; la divergence des bords 18°.

Juin 28, $9^h{\cdot}7$. Camelop. 91, mais l'ascension droite doit être diminuée de 5°, 89° 30′, + 78° 7′; divergence des bords 14°.

Juin 29, $9^h{\cdot}0$. Camelop. 91, l'ascension droite diminuée de 3°·6, 90° 54′, + 78° 7′.

Juin 30, $10^h{\cdot}5$. Camelop. 91, ascension droite diminuée de 2°·9, 91° 36′, + 78° 7′; à la longueur angulaire calculée *s* de la queue il faut ajouter 1°; divergence 10°.

Juillet 1, $8^h{\cdot}7$. Camelop. $\frac{1}{2}(91 + 93)$, 94° 56′, + 78° 55′; à *s* il faut ajouter 2°; divergence 10°.

Juillet 2, 11^{h}·7. Camelop. $\frac{1}{2}$(93 + 97), 97° 32′, + 78° 26′.
Juillet 3, 9^{h}·7. Camelop. $\frac{1}{2}$(102 + 103), 104° 45′, + 82° 5′ ; divergence 7°.
Juillet 4, 13^{h}·0. Camelop. 102, 104° 25′, + 81° 30′.
Juillet 6, 13^{h}·0. Camelop. 116, 119° 0′, + 82° 52′.
Juillet 15, 11^{h}·2. Camelop. 131, 192° 1′, + 84° 12′.
Juillet 19, 10^{h}·2. Camelop. 135, 206° 40′, + 83° 29′.

Le bord droit de la queue (antérieur par rapport au mouvement dans l'espace) était toujours plus clair, que le bord postérieur.

Observations de MM. Groneman et Kapteyn :—Diese Beobachtungen wurden in der Weise gemacht, dass erstens die Richtung der Tangente an dem Kernpunkte der Axe, zweitens diejenige einer Geraden, die von dem Kerne nach einem Punkte P gezogen wurde, welches in der Axe in einer Entfernung αβ Ursæ majoris gedacht wurde (la longueur de cet s est égale à 5°·5), während drittens die von P aus fast geradlinige Schweifaxe durch ihre verlängerte Richtung bestimmt wurde. Die Mehrzahl dieser Bestimmungen sind Mittelwerthe aus unabhängig von Prof. Kapteyn und mir mit blossem Auge erhaltenen Resultaten, was ich mit (K.) und (G.) andeute. Die übrigen, nur mit (G.) bezeichneten, sind von mir allein gemacht worden. Weiter sind noch einige wenige Bemerkungen anderer Art in Betreff dieses Kometen hinzugefügt.

Juni 24, 11^{h}·1. Länge des Schweifes mit blossem Auge = αδ Ursæ majoris. (G.) Die gerade Linie Kern — P weist nach einem Punkte zwischen Polaris und γ Cassiopeiæ auf $\frac{1}{4}$ dieser Linie, von Polaris an gerechnet (Le point de la queue est déterminé par l'étoile 55 Aurigæ, mais l'ascension droite doit être augmentée de 1°, 83° 24′, + 53° 25′ ; s = 5°·5). In einem Merz'schen Refractor von beiläufig 8 c.m. Oeffnung und 80-facher Vergrösserung zeigten sich aus dem Kerne verschiedene Ausströmungen : (1) Im umgekehrten Bilde nach rechts eine sectorähnliche, fast bis zum ziemlich gut begrenzten Rande des Kopfes reichende, (2) eine viel kleinere weiter links und (3) eine schwache und fast linienförmige, geradlinige, in der Richtung der Axe des Schweifes austretende und von der Sonne abgewandte.* (G. und K.) Ausser diesen drei Ausströmungen war eine enganschliessende neblige Umhüllung des Kernes, so wie ein ziemlich deutlich begrenzter Kopf mit äusserer parabolischer Begrenzung sichtbar, die von einer aus dem Kerne ausgehenden, verwaschenen Doppelparabole, dem Zeichen des Widders nicht unähnlich, von der uebrigen Nebulosität des Schweifes getrennt war (G.).

Juni 23, 11^{h}·1. Die oben angedeutete Tangente am Kernpunkte der Axe ist nach δ Draconis gerichtet. Die gerade Linie Kern — P weist nach Polaris hin, während die Schweifaxe von P aus nach ε Ursæ minoris gerichtet ist. (Pour la direction 86 Camelop., 89° 34′, + 65° 43′; s = 5°·5.)

Der Schweif ist nicht scharf begrenzt, ist aber an der rechten oder vorangehenden

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* Simplement l'effet de la perspective.—Br.

Seite, dem Scheine nach, etwas schärfer. Die Schweiflänge war mit blossem Auge um $10^{\mathrm{h}}\cdot5 = \alpha\delta$ Ursæ majoris. Um $11^{\mathrm{h}}\cdot5$ reichte der Schweif aber bis zur Mitte des Bogens Kern—Polaris.

Juni 29, $11^{\mathrm{h}}\cdot6$. Die Linie Kern – P weist nach der Mitte des Bogens $\delta - \chi$ Draconis hin. Von P an ist die Schweifaxe nach Polaris gerichtet. (K. und G.) Schweiflänge mit blossem Auge um $11^{\mathrm{h}}\cdot5 = \alpha\gamma$ Ursæ majoris. Im Refractor mit 80-facher Vergrösserung zeigte sich eine breite fächerähnliche Ausströmung, symmetrisch zur Schweifaxe. Jedoch war an der linken Seite (im umgekehrten Bilde) eine hellere, schmalere Ausströmung und rechts eine kleinere aus dem Kerne ausgehende auf diesem Fächer projectirt. Auch jetzt ging eine Doppelparabole aus dem Kerne aus, welche die Begrenzung des Kopfes an der Schweifseite bildete. Nach der Sonne hin war der Kopf sehr verwaschen. (G.) Ungefähr 11^{h} ging ein Stern 7 Grösse dicht am Kerne durch den Kopf, ohne die geringste Abschwächung zu erleiden. (De la carte de Heis on obtient 86° 18′, + 75° 34′ pour $s = 5^{\circ}\cdot5$.)

Juli 1, $11^{\mathrm{h}}\cdot3$. Die Schweiflänge mit blossem Auge $= \alpha\gamma$ Ursæ majoris. (G.) An der vorangehenden Seite ist der Schweif mit blossem Auge schärfer und heller. Im Kometensucher ist der Himmelsgrund an der linken Seite des Schweifes dunkler, welches auch auf eine schärfere Begrenzung der vorangehenden Seite hinweist. Die Linie Kern – P ist nach α und ϕ Draconis gerichtet. (K.und G.) Von P aus weist die Axe nach ϵ Ursæ minoris hin. (D'où pour la position on a 90 Camelop. 93° 50′, + 79° 41′, pour $s = 5^{\circ}\cdot5$.) Im Refractor ist das Bild ungefähr wie am 29 Juni; der Fächer aber mehr gleichmässig. Im Schweife noch immer kein[*] dunkler Streif längs der Axe. (G.)

Juli 4, $11^{\mathrm{h}}\cdot1$ Die Schweiflänge nicht ganz gleich $\alpha\gamma$ Ursæ majoris. (G.) Die Linie Kern – P zeigt nach 0·1 des Bogens $\psi\phi$ Draconis, von ψ an gerechnet; die Tangente am Kernpunkte der Axe nach einem Punkte in der nämlichen Entfernung und auf dem nämlichen Bogen rechts von ϕ Draconis. (K. und G.) Von P. an geht die Schweifaxe ziemlich gerade nach ϵ Ursæ minoris. Der Schweif an der rechten Seite mit blossem Auge etwas heller. Im Refractor zeigt sich eine ziemlich starke Ausströmung nach der rechten Seite. (G.) (Pour la position on obtient 105° 0′, + 80° 0′, et $s = 5^{\circ}\cdot5$.)

Juli 5, $11^{\mathrm{h}}\cdot1$. Schweiflänge mit blossem Auge $= \beta\gamma$ Ursæ majoris. Die Linie Kern – P weist nach $\frac{1}{3}$ des Bogens $\psi\chi$ Draconis, von χ an. (Pour la direction Camelop. $\frac{1}{2}(102 + 106)$, 108° 43′, + 81° 4′, pour $s = 5^{\circ}\cdot5$). Von P an geht die Axe nach ψ Draconis. Die Tangente am Kernpunkte geht verlängert nach δ Ursæ minoris. (G.)

Juli 14, $11^{\mathrm{h}}\cdot1$. Mit blossem Auge ist die Schweiflänge fast der Distanz $\beta\gamma$ Ursæ minoris gleich. (s est égal à $3^{\circ}\cdot0$.) Der Kern hat die Helligkeit eines Sterns von der 4^{te} Grösse. Der Anfang des Schweifes geht scharf rechts an jenem

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[*] Les observateurs munis de bonnes lunettes ont observé un cône obscur derrière le noyau.—Br.

Sternchen der 6 Grösse vorbei das im Heis'schen Atlas auf $\frac{1}{3}$ des Bogens liegt, welcher vom Schwanzende des Drachens zum Sterne 5 Grösse am rechten Horne der Giraffe läuft. Weiter ist der Schweif auch wegen des Mondscheins nicht deutlich genug, um die Krümmung zu bestimmen. (G.) (Pour le bord postérieur: Camelop. 124, 153° 14′, + 83° 18′.)

Juli 17, $10^h{\cdot}4$, mit blossem Auge. Im Schweife befindet sich der Stern 5 Grösse (Camelop. 131). Die Schweifaxe geht südlich längs dieses Sternes in einer Distanz = $\frac{1}{2}$(Mizar — Alcor). Ungeachtet der grössten Aufmerksamkeit war keine Krümmung zu constatiren obgleich bisweilen eine Andeutung von einer Abbiegung des Schweifendes nach Süden zu bestehen schien, jedoch immer sehr zweifelhaft. (K.) (Pour la direction: 131 Camelop., mais sa déclinaison doit être diminuée de 10′, 192° 1′, + 84° 2′; à *s* calculé à l'aide de cette étoile il faut ajouter 0·3 *s*.) Schweiflänge 1·3 der Distanz vom Kometen bis zu dem genannten Stern.

Juli 18, $11^h{\cdot}1$. An diesem Abende erkannte ich den Kometen mit blossem Auge bei ausserordentlich klarem Himmel, um 9 Uhr. Der Kern erschien als ein Sternchen, nur sehr wenig schwächer als Alcor (im Operngucker); jedoch schien der Komet mit blossem Auge viel heller als ein Stern der 5 Grösse. Ich sah den Schweif später an diesem Abend nicht länger als $\beta\gamma$ Ursæ minoris (*s* est égal à 3°·0), und ohne Krümmung. Die Schweifaxe war nach einem Stern der 6 Grösse gerichtet, der etwas höher als das rechte Horn der Giraffe (Atlas Heis) sichtbar war, ein wenig links von der verlängerten Linie, welche aus dem Schwanzende des Drachen (Atlas Heis) zum genannten Sterne gezogen werden kann. (Pour la direction 135 Camelop., 206° 40′, + 83° 29′ et *s* = 3°·0.)

Juli 27, $10^h{\cdot}1$. Die Schweifaxe war nach 137 Camelopard gerichtet und ferner nach ψ Draconis. Der Schweif war gerade und mit blossem Auge so lang als $\alpha\delta$ Ursæ minoris. (La carte de Heis donne pour la direction 208° 19′, + 82° 4′; *s* = 4°·0.)

Le dessin de M. Tempel (planche I., fig. 1) se rapporte au 27 Juin, $10^h{\cdot}25$ t.m. de Greenwich. Il est fait sur la première carte de l'atlas d'Argelander (Neue Uranometrie). On voit l'étoile polaire au milieu des deux queues; l'extrémité de la queue plus courte touche l'étoile 138 Camelop. (225° 7′, + 83° 6′); pour l'extrémité de la queue plus longue l'ascension droite est celle de τ Draconis et la déclinaison de υ Draconis (289° 35′, + 71° 6′).

Par rapport à la double queue M. Tempel dit dans sa lettre: "Der Comet hatte nur am 27 Juni diesen Doppelschweif, und die folgenden Nächte war der nach links zu allein sichtbar."

L'existence des deux queues vers ce temps est parfaitement confirmée par les observations importantes de M. Lewis Boss. (The American Journal of Science, No. 128, August, 1881, p. 141), Dudley Observatory, Albany (lat. 42° 40′ N.). "The nights of June 26, June 28, July 1, 8 and 13 were unusually favourable for such studies here. The atmosphere was unusually transparent on June 26, and I then traced the tail for a distance of nearly forty degrees from the nucleus. On that night there were two branches. The longer and brighter branch was perfectly straight. The other curved, with its concavity toward greater right ascension. On the next clear night (June 28) the straight branch was of about the same length as the curved one, and was a thin and scarcely perceptible streak. On July 1, the two branches seem to have merged into one, presenting a shorter and broad fan-like appendage, perfectly straight and strongly marked on the preceding side, concave and nebulous on the following.

On all occasions the nucleus under a power of 250 has seemed to be quite distinctly defined and star-like in appearance. On June 26, its measured diameter was 7″; on July 8, this had become 2″. The latter measure reduced to the distance of June 26 becomes 3″·3, a rather surprising reduction in the diameter, if it be real."

La description de la queue du 26 Juin, faite par M. L. Boss, parait être une explication du dessin de M. Tempel du 27 Juin, à tel point le phénomène était identique pour ces deux observateurs.

Comme la clarté de la Lune influe sur les apparences de la queue, je veux indiquer ici les phases de notre satellite :

Mai 27—nouvelle Lune ; Juin 5—premier quartier ; Juin 12—pleine Lune ; Juin 18—dernier quartier ; Juin 26—nouvelle Lune ; Juillet 4—premier quartier ; Juillet 11—pleine Lune ; Juillet 18—dernier quartier ; Juillet 25—nouvelle Lune.

Les éléments de l'orbite (*Dunér :* Astr. Nachr. No. 2394, p. 283) sont :

$$\begin{aligned} T &= 1881,\ \text{Juin } 16{\cdot}442 \text{ t. m. de Greenwich.}\\ &\left.\begin{aligned} \pi - \text{☊} &= 354^\circ\ 15'{\cdot}2 \\ \text{☊} &= 270\quad\ 57{\cdot}7 \end{aligned}\right\} \text{ éq m. } 1881{\cdot}0.\\ i &= 63\quad 25{\cdot}9 \\ \lg.\ q &= 9{\cdot}865985 \\ \lg.\ m &= 0{\cdot}16115 \end{aligned}$$

La Terre a passé par le plan de l'orbite le 21 Juin, $19^h\ 38^m$ t. m. de Greenwich. Avec $\epsilon = 23^\circ\ 27'{\cdot}2$ l'on trouve :*

$$A = 192^\circ\ 6'{\cdot}9 \qquad D = +\ 23^\circ\ 51'{\cdot}2$$

Pour calculer les positions du noyau on a :

$$\begin{aligned} x &= r \cdot (9{\cdot}65082) \sin (356^\circ\ 25'{\cdot}7 + v) \\ y &= r \cdot (9{\cdot}99185) \sin (243\quad 25{\cdot}7 + v) \\ z &= r \cdot (9{\cdot}96124) \sin (328\quad 28{\cdot}1 + v) \end{aligned}$$

---

* Les formules pour les réductions au plan de l'orbite se trouvent dans les Astr. Nachr. No. 1172.

Ces positions et les positions du Soleil *a* et *d* pour les moments des observations sont—

1881, t. m. de Gr.	α	δ	*a*	*d*
Juin 24·461	84° 37′·3	+ 49° 23′·1	93° 47′·8	+ 23° 24′·4
25·490	85 41·8	53 27·3	94 51·8	23 22·7
26·490	86 52·0	57 3·8	95 54·0	23 20·5
27·434	88 6·1	60 8·6	96 53·0	23 18·1
28·406	89 30·3	63 1·5	97 53·5	23 15·3
28·461	89 36·0	63 10·2	97 57·0	23 15·1
29·376	91 5·7	65 35·8	98 53·8	23 12·0
29·484	91 16·9	65 52·0	99 0·5	23 11·7
30·437	93 1·9	68 6·3	99 59·5	23 8·1
Juillet 1·364	94 54·4	70 4·0	100 57·0	23 4·2
1·472	95 8·4	70 17·1	101 1·5	23 3·8
2·490	97 27·6	72 10·2	102 7·0	22 59·1
3·406	99 46·4	73 40·5	103 3·5	22 54·6
4·461	102 42·6	75 13·6	104 8·8	22 48·9
4·541	102 56·7	75 19·9	104 13·8	22 48·5
5·461	105 47·4	76 29·8	105 10·5	22 43·2
6·541	109 26·7	77 41·6	106 17·0	22 36·5
14·461	144 33·1	82 4·8	114 21·3	21 34·0
15·467	149 16·1	82 13·5	115 22·3	21 24·3
17·434	157 59·3	82 20·3	117 21·3	21 4·4
18·461	162 9·5	82 19·4	118 23·0	20 53·5
19·426	165 51·4	82 16·5	119 21·0	20 42·8
27·421	188 4·0	+ 81 10·3	127 16·8	+ 19 2·6

Pour les mêmes moments on a plus loin—

	v	log *r*	log ρ	*p*°	*P*	*S*
Juin 24	+ 17° 28′·9	9·87613	9·4943	− 18° 50′	61° 35′	97° 21′
25	19 38·5	9·87880	9·5115	16 32	62 55	99 52
26	21 43·0	9·88169	9·5295	14 45	64 28	102 2
27	23 38·9	9·88462	9·5475	13 36	66 5	103 55
28	25 37·0	9·88787	9·5667	11 55	67 58	105 40
28	25 43·2	9·88805	9·5677	11 49	68 6	105 45
29	27 32·3	9·89132	9·5859	10 32	70 2	107 13
29	27 45·1	9·89170	9·5880	10 22	70 16	107 22
30	29 36·5	9·89531	9·6070	9 2	72 30	108 44
Juillet 1	31 23·2	9·89900	9·6252	7 32	74 51	109 56
1	31 35·6	9·89944	9·6273	7 18	75 9	110 2
2	33 30·3	9·90366	9·6470	5 37	77 59	111 13
3	35 11·7	9·90762	9·6640	3 51	80 43	112 14
4	37 6·1	9·91235	9·6837	1 39	84 8	113 10
4	37 14·7	9·91271	9·6852	− 1 28	84 26	113 11
5	38 52·3	9·91696	9·7016	+ 0 44	87 40	114 1
6	40 44·5	9·92209	9·7204	3 34	91 46	114 47
14	53 6·7	9·96283	9·8381	32 4	129 27	119 3
15	54 31·3	9·96825	9·8509	35 53	134 26	119 25
17	57 10·5	9·97893	9·8746	42 51	143 37	120 7
18	58 30·6	9·98451	9·8863	46 6	148 2	120 26
19	59 44·1	9·98978	9·8969	48 56	151 57	120 43
27	+ 68 48·7	0·03303	9·9733	+ 63 46	175 38	122 40

En réduisant à 1881·5 les co-ordonnées des étoiles et des points pris dans l'atlas de Heis, on trouve les co-ordonnées des points des queues α′ et δ′ et l'on calcule les valeurs p et s (cette dernière est corrigée conformement aux remarques, qu'on trouve dans l'exposé des observations) et T—

	α'	δ'	p	s	T
Juin 24	83° 53′	+ 53° 26′	− 6° 10′	5° 30′	18° 49′
25	85 44	57 56	+ 0 14	4 29	20 45
25	83 15	57 56	− 16 5	4 42	42 21
26	86 7	61 55	− 5 14	4 53	31 34
27	224 35	83 0	+ 8 21	35 13	24 54
27	289 31	71 9	− 9 9	47 51	44 12
28	90 32	78 7	+ 0 49	15 5	35 50
28	90 14	65 43	+ 5 50	5 30	31 13
29	91 56	78 7	+ 0 47	12 31	41 11
29	87 13	75 35	− 5 57	5 30	52 42
30	92 38	78 7	− 0 28	11 1	49 11
Juillet 1	96 1	78 54	+ 1 24	6 50	51 51
1	94 59	79 40	− 0 10	5 30	55 12
2	98 35	78 25	+ 2 3	6 15	57 57
3	106 0	82 3	5 51	8 28	57 26
4	106 14	79 58	7 21	5 30	61 53
4	105 40	81 28	3 45	6 10	69 17
5	109 58	81 2	8 10	5 30	67 45
6	120 24	82 48	14 37	5 24	64 17
14	154 8	83 10	43 47	3 0	82 15
15	192 3	84 3	49 55	5 17	80 22
17	192 3	83 53	52 47	5 43	91 26
18	206 28	83 21	57 54	3 0	90 14
19	206 28	83 21	58 21	5 4	96 2
27	208 7	+ 81 56	+ 65 38	4 0	118 5

A l'aide des valeurs précédentes on calcule facilement les co-ordonnées Δ et ϕ pour chaque observation.

Pour réduire les angles ϕ à une même époque, on peut employer la formule de Bessel et en déduire les valeurs des réductions ayant une exactitude tout à fait satisfaisante.

On a—

$$\operatorname{tng} \phi = \frac{4 \sqrt{q} \sqrt{\xi}}{3r \sqrt{1 - \mu}}$$

et en posant—

$$\tfrac{4}{3} \cdot \frac{1}{\sqrt{q} \sqrt{1 - \mu}} = \mathrm{F},$$

on obtient—

$$\operatorname{tng} \phi = \mathrm{F} \cdot \sqrt{\xi} \cos^2 \frac{v}{2},$$

d'où—

$$d\phi = - \tfrac{1}{2} \cos^2\phi \cdot \mathrm{F} \cdot \sqrt{\xi} \cdot \sin v \cdot dv$$

$\cos^2\phi$ peut être regardé comme constant dans les limites des réductions et on a, en intégrant entre v et v_0, où v_0 est l'anomalie de l'époque,

$$\delta\phi = \tfrac{1}{2} \cos^2\phi \cdot \mathrm{F} \cdot \sqrt{\xi} (\cos v_0 - \cos v) \qquad (1)$$

et $\delta\phi$ exprimé en minutes d'arc sera :

$$\delta\phi = - \cos^2\phi \cdot \mathrm{F} \cdot \sqrt{\xi} \cdot \sin \tfrac{1}{2} (v_0 + v) \cdot \sin (v_0 - v) : \sin 1' \qquad (2)$$

Les signes des anomalies sont indiqués par la formule (1).

On va voir que la queue principale de la comète b est du second type, et nous réduirons ses ϕ à l'époque du 27 Juin avec la valeur $1 - \mu = 1$. En désignant ces angles réduits par ϕ' nous aurons la table qui suit. Pour la construction graphique des co-ordonnées admettons l'unite (distance Terre—Soleil) égale à 800 millimètres.

1881.	ϕ	$\delta\phi$	ϕ'	Δ	Δ en millim.
Juin 24·j461	+ 19° 47′	− 0° 23′	+ 19° 24′	0·0727	58·2
25·490	24 16	0 13	24 3	0·0595	47·6
25·490	1 13	0 13	1 0	0·0364	29·1
26·490	18 11	− 0 7	18 4	0·0485	38·8
27·434	32 2	0 0	32 2	0·2346	187·7
27·434	10 30	0 0	10 30	0·2617	209·4
28·406	23 54	+ 0 11	24 5	0·1236	98·9
28·461	29 28	0 7	29 35	0·0592	47·4
29·376	22 42	0 21	23 3	0·1036	82·9
29·484	10 29	0 17	10 46	0·0437	35·0
30·437	18 47	0 33	19 20	0·0891	71·3
Juillet 1·364	19 40	0 35	20 15	0·0588	47·0
1·472	16 16	0 34	16 50	0·0466	37·3
2·490	17 29	0 46	18 15	0·0536	42·9
3·406	21 26	1 5	22 31	0·0744	59·5
4·461	20 13	1 5	21 18	0·0501	40·1
4·541	12 19	1 14	13 33	0·0538	43·0
5·461	16 59	1 18	18 17	0·0504	40·3
6·541	24 16	1 21	25 37	0·0527	42·2
14·461	23 49	2 10	25 59	0·0362	29·0
15·467	28 5	2 49	30 54	0·0655	52·4
17·434	19 5	3 59	23 4	0·0752	60·2
18·461	22 23	2 42	25 5	0·0403	32·2
19·426	17 20	4 22	21 42	0·0710	56·8
27·421	+ 2 38	+ 6 53	+ 9 31	0·0774	61·9

Pour la construction graphique de la parabole on a—

v	r	r en millim.
0° 0′	0·7345	587·6
10 0	0·7401	592·1
20 0	0·7573	605·8
23 38·9	0·7667	613·2
25 0	0·7706	616·5
30 0	0·7872	629·8
35 0	0·8075	646·0
40 0	0·8318	665·4
45 0	0·8605	688·4
50 0	0·8942	715·4

Les points des queues déterminés par les co-ordonnées précédentes sont portés sur la planche I., fig. 2 (petits cercles).

Calculons maintenant les positions des particules émises par le noyau et animées des forces de différentes valeurs, en désignant ces positions sur la planche par des petites croix.

Pour la queue longue et droite, qui appartient *évidemment au premier type* (l'hydrogène pur ?) la formule de Bessel est tout à fait satisfaisante, et l'on trouve, — avec $1 - \mu = 12$, $\log \xi = 9{\cdot}41091$, $\xi = 0{\cdot}2576 = 206{\cdot}1$ millim. pour l'extrémité de l'axe de la queue, le 27 Juin, l'angle $\phi = 11^\circ\, 47'$. Ce ϕ ne diffère que de 1°·3 de ϕ observé ; la différence est insignifiante, tant plus que les particules à l'extrémité doivent appartenir non à l'axe, mais au bord antérieur. Le temps d'émission pour l'extrémité de cette queue est le 18 Juin.

Pour la queue plus courte et courbée nous calculons premièrement les positions des particules soumises à la force $1 - \mu = 1$, c'est à dire des particules qui se meuvent sur les tangentes à l'orbite avec les vitesses du noyau aux moments d'émissions.

Soient les anomalies du noyau aux moments de ces émissions—

$$v = -40^\circ, \qquad -25^\circ, \qquad -10^\circ, \qquad 0^\circ.$$

Les temps correspondants à ces anomalies seront—

M_1 = Mai 27·775, Juin 4·781, Juin 11·903, Juin 16·442

Et l'on trouve pour le temps d'observation (Juin 27·434) les co-ordonnées suivantes—

ϕ	Δ	Δ en millim.
42° 36′	0·2268	181·4
32 26	0·1335	106·8
22 23	0·0649	51·9
15 50	0·0323	25·8

Ces points se trouvent sur la courbe désignée sur la planche par $1 - \mu = 1$. On voit que la plupart des points observés *s'accordent* avec la position de cet axe de la *queue calculée.* Mais il y a des points, qui présentent une déviation très sensible et dont les particules ont été mues, par conséquent, par des forces plus grandes.

Calculons les positions des particules émises par le noyau le 4·781 Juin, quand, son anomalie était égale à -25°, et mues par les forces $1 - \mu = 2{\cdot}6$ et $1 - \mu = 2{\cdot}0$, que nous avont trouvées dans les comètes de 1744 et de 1858 V.

En employant les formules exactes du mouvement hyperbolique on trouve pour notre époque, c'est à dire pour le 27·434 Juin (voir Copernicus, No. 6)—

M_1	$1 - \mu$	$\log P$	$\log E$	F
Juin 4·781	2·6	9·9629029	0·3438620	49° 57′ 42″·4
4·781	2·0	0·1670236	0·4678854	44 42 7·4

D'où l'on obtient—

$1 - \mu$	ϕ	Δ	Δ en millim.
2·6	33° 52′	0·3232	258·6
2·0	33 39	0·2544	203·5

Ces points se trouvent sur la planche sur les axes courbes, désignés par $1 - \mu = 2{\cdot}6$ et $1 - \mu = 2{\cdot}0$.

On voit, que l'observation de M. Tempel, qui se rapporte à la queue courbée, est parfaitement représentée par la force $1 - \mu = 2$, et nous venons à la conclusion, que la *queue du II type* (le carbone, l'azote, les hydrocarbures) de la comète *b* était riche en matières comme les queues analogues des comètes de 1744 et de 1858 V, et qu'elle consistait en plusieurs conoïdes. C'est pourquoi la divergence de ses bords—abstraction faite de l'influence de la perspective—était très considérable. On peut dire en général que cette comète appartient par ses propriétés physiques à une même famille avec les comètes de 1744 et de 1858 V.

D'après mes mesures, le 1 Juillet, l'étendue de la nébulosité vers le soleil était égale à 85″, ou en mesure linéaire $\epsilon = 0{\cdot}000174$, et comme vers ce temps le I type n'existait plus, on a à l'aide de la formule $g^2 = 2\ (1 - \mu)\ \epsilon : r^2$—en posant $1 - \mu = 1{\cdot}5$ (moyenne de $1 - \mu = 1$ et $1 - \mu = 2$)—la valeur bien connue $g = 0{\cdot}03$.

J'ai dit déjà plusieurs fois, qu'il est difficile de préciser la valeur de g, mais on la trouve toujours comprise pour le second type dans des limites très étroites, près de 0·03. Pour le premier type nous l'avons trouvée égale à 0·15.

Par rapport aux différents dessins de la comète faits en Juin il faut remarquer, que la queue était alors dirigée vers la Terre et que les jets d'émission ont dû se présenter sous cette forme polypétalée dont les parties paraissent se diriger dans tous les sens, vers le soleil et à l'opposite de cet astre.

Dans la queue de la comète *b*, composée de plusieurs conoïdes, diversement déclinés du rayon vecteur, l'espace obscur derrière le noyau (ou le creux des conoïdes) a dû être estompé á une certaine distance de la tête, ce qu'on a en effet observé dans cette comète. Mais dans le voisinage du noyau—et ce noyau présentait une *condensation* prononcée—l'espace obscur se voit dans plusieurs dessins et descriptions de la comète. Dans les observations de M. Konkoly (The Observatory, No. 53, pp. 258–259), par exemple, nous lisons les remarques suivantes: "June 24 . . . It must be noted that the nucleus was situated just on the summit of the dark cone which divided the tail in two parts, and that it was strictly limited on this point . . . June 25 . . . the summit of the dark cone being situated to-day at a much greater distance from it than was the case yesterday . . . June 26 . . the dark cone, too, in the tail can but faintly be recognised to-day."

Il est intéressant maintenant de voir quelle était la queue avant le périhélie. Quelques positions de cet appendice sont données par M. Eddie (Graham's Town, Cape; $1^h\ 45^m$ à l'Est de Greenwich) et par M. Markwick (Pretoria, Transvaal, Africa; $1^h\ 55^m$ à l'Est de Greenwich).

Ces observations se trouvent dans les Monthly Notices R.A.S. (1881, No. 9, pp. 435–439). Il faut noter avant tout, que dans l'hémisphère austral on n'a commencé à voir la queue à l'oeil nu que vers le 23 Mai, mais dès le 4 Juin elle était déjà très affaiblie par la Lune, et après le 9 Juin la comète ne se voyait que près de l'horizon.

Dans l'observation de M. Eddie on lit: "Mai 31, $6^h\ 15^m$ ($4^h\ 30^m$ t. m. de Greenwich). The tail had also increased in length and visibility, and could be traced to about 12 or 13 degrees from the nucleus. It extended to 2 or 3 degrees beyond *o* Columbæ, which star it grazed with its *upper* edge . . . June 3, $6^h\ 40^m$ ($4^h\ 55^m$ t. m. de Greenwich). Tail could still be traced to *o* Columbæ, which star it grazed with its *lower* edge." La première observation est tout à fait identique avec celle de M. Markwick. "May 31, $6^h\ 30^m$ ($4^h\ 35^m$ t. m. de Gr.) The *upper* edge of the tail passed just over *o* Columbæ."

D'après ces observations on voit que *l'axe* de la queue passait par *o* Columbæ le 1 Juin, à $16^h\ 44^m$ t. m. de Gr. (Juin 1·697).

Pour ce temps les co-ordonnées du noyau sont (voir les observations de M. Ellery ; ibid., pp. 432, 433)—

Asc. dr. $5^h\ 3^m\ 57^s$, Décl. $-28^\circ\ 8'{\cdot}8$

les co-ordonnées du soleil—

Asc. dr. $4^h\ 40^m\ 52^s$, Décl. $+22^\circ\ 12'{\cdot}4$

et celle de l'étoile pour 1881·4—

Asc. dr. $5^h\ 13^m\ 12^s$, Décl. $-35^\circ\ 0'{\cdot}7$

On a ensuite—

$v = -31^\circ\ 2'{\cdot}9$ $p^\circ = 175^\circ\ 5'$
$\log r = 9{\cdot}89827$ $p = 164\ \ 35$
$\log \rho = 9{\cdot}6371$ $s = 6\ \ 49$

A cet s calculé à l'aide de *o* Columbæ il faut ajouter, conformement à l'observation, 3°, et l'on a $s = 9^\circ{\cdot}8$. Puis on calcule—

$P = 17^\circ\ 32'{\cdot}0$ d'où $\log \Delta = 8{\cdot}8856$
$S = 56\ \ 55{\cdot}0$ $\Delta = 0{\cdot}0768$
$T = 96\ \ 2{\cdot}0$ $\phi = +15^\circ{\cdot}5$

On voit d'après ces co-ordonnées, que la queue appartient au *second type.*

Mais il y a quelques indications, qui font croire, que dans des circonstances exceptionnellement favorables la queue du *premier type* se laissait aussi apercevoir avant le périhélie. Ainsi M. Eddie, en parlant de la comète, fait la remarque suivante : (ibidem) : "Last evening, May 27, I discerned a large comet about 10° or 12° above the south-western horizon . . . Mrs. Eddie saw what she considered to be a comet with *two tails* on Thursday (le 26 Mai) evening. This may probably have been the same. . . ."

M. Markwick a remarqué un changement dans la direction de la queue, qui n'est pas en accord avec l'observation ultérieure de M. Eddie. "June 1, $6^h\ 10^m$ ($4^h\ 15^m$ t.m. de Gr.) The direction of the tail was from this point (il indique ici la position du noyau) towards ζ Pictoris, the direction having apparently changed since last night" (ibid., p. 439). Et nous avons vu que pour M. Eddie, le 3 Juin, la queue passait encore par *o* Columbæ.

Pour ce temps, Juin 1·177, on a les co-ordonnées du noyau—

Asc. dr. $5^h\ 3^m\ 39^s$, Décl. $-28^\circ\ 43'{\cdot}3$

les co-ordonnées du soleil—

Asc. dr. $4^h\ 38^m\ 45^s$, Décl. $+22^\circ\ 8'{\cdot}4$

et celles de l'étoile pour 1881·4—

Asc. dr. $5^h\ 16^m\ 32^s$, Décl. $-50^\circ\ 44'{\cdot}1$

Puis on a—

$v = -32^\circ\ 2'{\cdot}2$ $p^\circ = 172^\circ\ 36'$
$\log r = 9{\cdot}90037$ $p = 174\ \ 35$
$\log \rho = 9{\cdot}6368$ $s = 22\ \ 3$

Cet s donne la distance angulaire entre le noyau et l'étoile, mais la longueur de la queue ne surpassait pas 10°, donc s est 10°.

Enfin l'on calcule—

$$\begin{array}{llll} P = 78^{\circ}\ 55'{\cdot}7 & \text{d'où} & \log \Delta = & 8{\cdot}8689 \\ S = 56\ \ \ 44{\cdot}3 & & \Delta = & 0{\cdot}0739 \\ T = 81\ \ \ 28 & & \phi = & -\ 3^{\circ}{\cdot}6 \end{array}$$

La queue serait donc en avant du prolongement du rayon vecteur. En supposant une erreur assez sensible dans la direction de la queue, causée par la distance considérable de l'étoile de comparaison (22°), on doit admettre néanmoins, que pour M. Markwick, le 1 Juin, la queue était moins déviée du rayon vecteur que le 31 Mai et le 3 Juin. Cela s'explique simplement par la visibilité de la branche du I type, qui à la distance de 10° du noyau ne se séparait pas de l'autre branche, mais rendait la queue plus large et changeait par conséquent la direction de l'axe de sa figure totale.

2. *Comète c* 1881.

Observations de M. Bruhns :—

Août 12, $12^{h}{\cdot}7$. Pour l'axe de la queue ascension droite 120° 0°, déclinaison + 54° 48′.

Août 14, $7^{h}{\cdot}2$. Pour l'angle de position 56 Lyncis, 122° 11′, + 58° 11′ ; s calculé à l'aide de l'étoile doit être diminué de 1°.

Août 15, $11^{h}{\cdot}7$. Pour l'axe de la queue 132° 6′, + 54° 42′.

Août 16, $11^{h}{\cdot}2$. Pour l'angle de position 35 Ursæ majoris, 136° 16′, 57° 19′ ; s diminué de 1° 4′.

Août 17, $7^{h}{\cdot}5$. Position de la queue 140° 0′, + 60° 24′.

Août 18, $7^{h}{\cdot}0$. 73 Ursæ majoris, 147° 28′, + 57° 30′ ; s diminué de 1°·6. A droite de la queue il parait exister une autre queue, qui à partir du noyau se dirige vers ϕ Ursæ majoris. La partie inférieure de la queue est incontestablement élargie sous forme d'éventail.

Août 19, $7^{h}{\cdot}0$. Ursæ majoris $\frac{1}{2}(85 + 94)$, 154° 26′, + 54° 23′.

Août 21, 7^{h} 0. Ursæ majoris 136, 167° 8′, + 50° 16′.

Août 22, $7^{h}{\cdot}7$. Ursæ majoris $\frac{1}{2}(161 + \chi)$, 173° 39′, + 48° 6′.

Août 23, $7^{h}{\cdot}0$. Pour la déclinaison 185 Ursæ majoris, l'ascension droite doit être diminuée de 41′, 178° 40′, + 43° 51′. Il parait comme si la seconde queue était visible.

Août 24, $7^{h}{\cdot}0$. Déclinaison de 11 Canum Venaticorum ; l'ascension droite de l'étoile doit être diminuée de 24′, 184° 43′, + 42° 10′.

Août 25, $7^{h}{\cdot}5$. La carte donne pour la position de la queue 190° 0′, + 39° 18′.

Août 26, $6^{h}{\cdot}7$. La carte donne : 191° 50′, + 34° 19′.

Août 27, $6^{h}{\cdot}7$. L'ascension droite de 71 Canum Venaticorum ; la déclinaison de l'étoile doit être diminuée de 35′, 204° 4′, + 35° 9′.

Août 28, $6^{h}\cdot5$. L'ascension droite de 69 Canum; la déclinaison de l'étoile diminuée de 35′, 203° 29′, + 28° 14′.

Août 29, $6^{h}\cdot7$. Pour la position de la queue 6 Bootis, 203° 33′, + 23° 17′; *s* calculé doit être diminuée de 1°. Brouillard; comète près de l'horizon.

Août 30, $6^{h}\cdot7$. L'ascension droite de 5 Bootis, la déclinaison de l'étoile doit être diminuée de 1°·6, 203° 26′, + 19° 5′. Brouillard; comète près de l'horizon.

M. Kapteyn donne la position suivante de la queue pour le 22 Août, $9^{h}\cdot1$: "Mit Kometensucher (10 f. Vergrösserung) der Schweif ganz gerade und gerichtet nach einem Punkte auf $\frac{3}{4}$ der Distanz Mizar—Alcor von χ Ursæ majoris, an der vorangehenden Seite dieses Sterns." D'après cette donnée on a pour la position de la queue la déclinaison de χ Ursæ majoris, l'ascension dr. de l'étoile doit être diminuée de 14′, 174° 22′, + 48° 34′.

Observations de Th. Bredichin :—

Août 15, $7^{h}\cdot5$. Pour la position de la queue 11 Ursæ majoris, 127° 27′, + 59° 35′.

Août 16, $6^{h}\cdot9$. Ursæ majoris $\frac{1}{2}(26 + 31)$, 134° 16′, + 59° 49′.

Août 17, $8^{h}\cdot2$. La carte de Heis donne : 142° 8′, + 57° 47′.

Août 19, $7^{h}\cdot8$. La carte donne : 153° 37′, + 54° 56′.

Août 20, $7^{h}\cdot2$. La queue passe près des étoiles 112 et 113 Ursæ majoris ; l'angle de position *p*, calculé à l'aide de la moyenne de ces étoiles (161° 5′, + 54° 20′), doit être augmenté de 2°.

Août 22, $7^{h}\cdot5$, Pour la position de la queue on a 161 Ursæ majoris, 172° 42′, + 47° 38′.

Août 26, $6^{h}\cdot5$. La queue passe près de l'étoile 41 Canum Venaticorum, mais son angle de position est de 3°·5 plus fort que cet angle calculé pour l'ètoile.

Phases de la Lune : Août 9—pleine Lune; Août 16—dernier quartier; Août 24—nouvelle Lune; Septembre 1—premier quartier.

D'après mes notes et les notes de M. Bruhns la queue était toujours presque droite; sa partie antérieure—plus claire et bien définie; la partie postérieure beaucoup plus faible et dégradée. La tête de la comète était grande et allongée dans la direction de l'orbite, comme le montrent les dessins de M. Tempel, qui seront publiés dans les Annales de notre observatoire (Vol. VIII, livr. 1).

Le secteur d'effluves était peu développé, et au milieu de la queue M. Tempel a vu, les derniers jours, une raie claire, dont il parle ainsi : "Sonderbar war die helle Linie mitten im Schweife, gegen 2′ breit, die nicht vom Kerne unmittelbar ausging, sondern erst nach der runden Umhüllung des Kopfes anfing."

Cette raie claire, ou plutôt ces raies claires, ne se distinguent pas malheureusement dans l'intérieur de la tête, et on n'y voit pas le mode de leur liaison avec le noyau et avec le secteur lumineux des effluves. Selon toute probabilité la raie provenait de l'effluve de la matière dirigé d'abord vers le soleil et puis récourbé en arriére.

L'existence d'une raie pareille pendant un certain temps a été remarquée dans la comète de 1861 II et mes observations (Annales; Vol. VII, 2; p. 106) et plus encore les observations de M. Schmidt nous montrent, qu'elle ne provenait pas directement du noyau, mais qu'elle était liée avec l'effluve. Je saisis l'occasion de répéter ici les remarques importantes du célèbre observateur d'Athènes : "Wenn man davon spricht, dass an der Stelle der dunklen Linie im Donati-Cometen bei dem jetzigen eine helle Linie erschienen sei, so ist das *höchst irrig,* und wird bleibend unserer Auffassung von dem Baue dieser Körper schaden. Die jetzige helle Mittellinie im Schweife war nur die eine helle Seite des innern Conoïds" (Schmidt, Astronomische Beobachtungen über Cometen ; pp. 78–79, Tafel II. fig. 5 Juli 1). . . . 'Heute (Juli 8) zeigte sich (am Sucher) sehr deutlich wieder die helle Mittellinie im untern Theile des Schweifes . . . Ich *hebe ausdrücklich hervor,* dass jetzt wie Juli 1 diese Mittelzone *gar nicht* mit dem Kerne direct, sondern mit der Lichtströmung des Kerns zusammenhing, welche an der (im Fernrohre) rechten Seite den Anfang des innern Paraboloïds bildete."

La partie antérieure de la queue pour M. Tempel était aussi plus claire que la partie postérieure : "Auffallend war ferner die scharfe vorangehende (südliche) Seite des Schweifes, während die nachfolgende sehr verwaschen war. Am 26 August, bei schönem Himmel, war diese südliche Partie des Himmels so rein und dunkel und trat gegen die scharf begrenzte Seite des Schweifes so auffallend hervor, dass es schien, als würfe der Schweif Schatten vor sich her."

Il est intéressant de noter la remarque de M. Rand Capron sur la couleur de la comète (Nature ; No. 617, p. 380) : "August 21. The nucleus and star appeared of about the same intensity, but the yellow tint of the latter was strongly contrasted with the almost intense gas blue tint of the former."

L'existence de la seconde queue plus courte, plus large et plus déclinée du rayon vecteur se confirme par les raies claires (dessin de M. Tempel, août 29) déviées un peu de la raie centrale de la queue principale, et par la note, qu'on trouve dans l'English Mechanic, No 857 : "the tail resembles a slightly opened fan."

La partie centrale de la tête de la comète *c* était beaucoup plus petite et moins dense que celle de la comète *b*. La plupart des notes de divers observateurs se résument pour ainsi dire dans la remarque de M. Edward B. Kirk (The Observatory, 1881, No. 54, p. 300) : "I think, at least, that to any observer it will be evident that there is no solid nucleus in Schäberle's comet, or at least that we do not see it." Cette opinion est confirmée par la conclusion, à laquelle est amené M. Thollon (Comptes Rendus de Paris, 1881, 22 Août, p. 383) : "La comète *c* a donné comme la précédente (*b*) un spectre de bandes correspondant exactement à celle du carbone. Ces bandes, au nombre de trois, très nettes et très brillantes, dégradées, comme toujours, du côté du violet, étaient séparées par des intervalles tout à fait obscurs et c'est dans la nuit dernière, 21 Août seulement, que j'ai pu apercevoir de faibles traces d'un spectre continu. On se souvient que la comète *b* offrait un caractère tout différent. Dans cette dernière, le spectre continu dominait de telle sorte que, jusqu'à la fin de Juin, il masquait presque entièrement le spectre de bandes. On voit que cette dernière (*c*) est *presque entièrement* formée d'éléments gazeux."

On comprend aisément qu'une pareille constitution ne peut pas rester sans influence sur la figure de la racine de la queue. Le creux du conoïde, ou l'espace obscur derrière le noyau dépend essentiellement de ce que la partie du noyau détournée du Soleil n'est pas exposée directement à l'action de ses rayons. Pour un amas de grains, de gaz et de particules assez désagrégées la queue se composera plus ou moins de filaments séparés, et le creux du conoide total en sera plus ou moins rempli en voisinage même du centre de la tête. Les effluves vers le Soleil n'auront ni l'étendue, ni les formes caractéristiques de ces jets qui se développent sur un noyau plus dense, du côté du Soleil.

Les éléments de l'orbite de la comète *c* (*Hepperger*. Astr. Nachr. No. 2390) sont :

$$\begin{aligned} T &= 1881,\ \text{août } 22{\cdot}7368 \text{ t. m. de Greenwich.} \\ \left.\begin{aligned} \pi &= 218^\circ\ 37'{\cdot}2 \\ \Omega &= 96\quad 26{\cdot}3 \end{aligned}\right\} &\ \text{éq. m. 19 août.} \\ i &= 139\quad 50{\cdot}3 \\ \lg.\, q &= 9{\cdot}80018 \\ \lg.\, m &= 0{\cdot}25986 \end{aligned}$$

La Terre a passé par le plan de l'orbite le 27 Juin. Avec $\epsilon = 23^\circ\ 27'{\cdot}2$, on trouve—

$$A = 30^\circ\ 1'{\cdot}9 \qquad D = -\ 42^\circ\ 14'{\cdot}6$$

Pour calculer les positions du noyau on a—

$$\begin{aligned} x &= r \cdot (9{\cdot}88515) \sin (113^\circ\ 46'{\cdot}9 + v) \\ y &= r \cdot (9{\cdot}96793) \sin (223\quad 14{\cdot}1 + v) \\ z &= r \cdot (9{\cdot}86939) \sin (154\quad 28{\cdot}4 + v) \end{aligned}$$

Ces positions et les positions du soleil *a* et *d* pour les moments d'observations sont—

1881, t. m. de Gr.	α	δ	*a*	*d*
Août 12·530	122° 19′·8	+ 52° 29′·0	142° 46′·0	+ 14° 42′·4
14·300	129 19·7	52 48·7	144 26·0	14 9·8
15·313	133 56·1	52 45·1	145 22·8	13 50·8
15·490	134 40·1	52 43·2	145 32·8	13 47·4
16·288	138 36·7	52 28·0	146 17·5	13 32·2
16·467	139 23·8	52 23·7	146 27·8	13 28·7
17·313	143 42·4	51 57·3	147 15·0	13 12·5
17·342	143 58·0	51 51·6	147 16·5	13 12·0
18·291	148 52·0	51 1·8	148 9·5	12 53·5
19·291	154 15·0	49 47·3	149 5·3	12 33·8
19·325	154 33·1	49 42·2	149 7·0	12 33·2
20·300	159 47·8	48 6·2	150 1·5	12 13·8
21·291	164 53·7	46 9·0	150 56·5	11 54·0
22·313	170 3·7	43 41·9	151 53·0	11 33·3
22·321	170 5·4	43 41·4	151 53·8	11 33·2
22·380	170 23·2	43 39·9	151 56·8	11 31·9
23·291	174 42·4	41 2·2	152 47·0	11 13·3
24·291	179 7·3	38 1·7	153 42·3	10 52·8
25·313	183 15·6	34 44·2	154 38·5	10 31·6
26·271	186 47·3	31 30·5	155 31·0	10 11·5
26·280	186 48·8	31 29·1	155 31·5	10 11·3
27·280	190 8·0	28 3·7	156 26·3	9 50·3
28·270	193 4·8	24 40·7	157 20·8	9 29·2
29·280	194 44·8	23 7·0	158 16·0	9 7·6
30·280	198 9·2	+ 18 3·3	159 10·5	+ 8 46·1

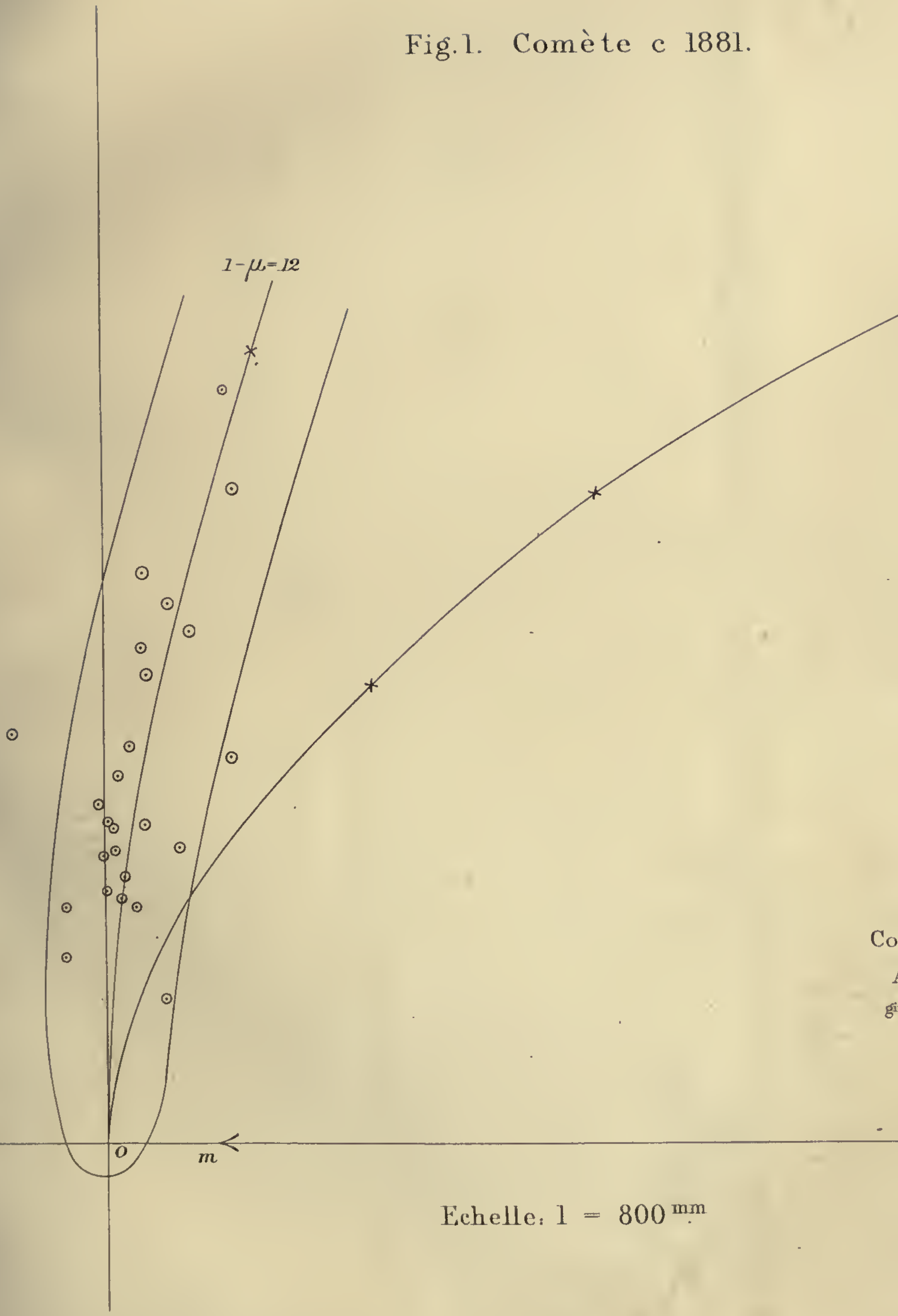
Puis on a pour les mêmes moments—

	v	lg. *r*	lg. *ρ*	*p*°	*P*	*S*
Août 12	– 27° 23′·2	9·82522	9·91487	– 30° 54′	242° 29′	56° 34′
14	22 55·4	9·81768	9·88533	22 54	246 55	52 33
15	20 12·4	9·81376	9·86806	17 28	249 43	49 59
15	19 50·4	9·81326	9·86563	16 37	250 10	49 36
16	17 38·5	9·81052	9·85221	11 49	252 24	47 25
16	17 15·9	9·81008	9·84983	10 53	252 49	46 57
17	15 0·2	9·80764	9·83657	5 31	255 2	44 33
17	14 49·7	9·80748	9·83567	– 5 9	255 11	44 25
18	12 21·8	9·80524	9·82199	+ 1 7	257 22	41 38
19	9 38·1	9·80326	9·80820	8 18	259 15	38 32
19	9 26·4	9·80312	9·80733	8 45	259 20	38 22
20	6 45·4	9·80170	9·79537	16 4	260 23	35 15
21	4 7·1	9·80074	9·78530	23 32	260 21	32 10
22	1 13·8	9·80024	9·77647	31 37	258 46	28 58
22	1 12·5	9·80023	9·77667	31 38	258 46	28 57
22	– 1 5·7	9·80022	9·77520	31 56	258 42	28 48
23	+ 1 26·2	9·80024	9·77091	39 6	255 30	26 14
24	4 12·8	9·80076	9·76759	46 40	250 1	23 52
25	7 2·3	9·80182	9·76706	54 2	242 9	22 8
26	9 43·5	9·80330	9·76922	60 38	233 15	21 22
26	9 44·9	9·80334	9·76923	60 38	233 13	21 22
27	12 25·6	9·80530	9·77413	66 45	223 27	21 29
28	15 5·9	9·80774	9·78144	72 14	214 24	22 28
29	15 47·5	9·80846	9·78728	74 14	209 59	22 58
Août 30	+ 20 25·2	9·81406	9·80260	+ 81 28	200 11	26 14

En réduisant à 1881·7 les co-ordonnées des étoiles et des points pris dans l'atlas de Heis, on trouve les co-ordonnées des points de la queue *α′* et *δ′* et on calcule les valeurs *p*, *s* (ces valeurs sont corrigées conformement aux remarques qu'ont trouve dans l'exposé des observations) et *T*—

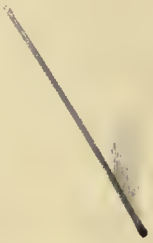
	α′	*δ′*	*p*	*s*	*T*
Août 12	120° 32′	+ 54° 44′	– 24° 38′	2° 30′	85° 39′
14	122 43	58 7	32 37	5 29	102 12
15	128 0	59 30	23 40	7 30	94 1
15	132 37	54 37	31 47	2 15	103 40
16	134 47	59 43	14 49	7 34	86 58
16	136 47	57 13	16 18	3 40	89 3
17	140 30	60 17	10 43	8 34	85 50
17	142 37	57 40	7 7	5 51	82 31
18	147 55	57 22	– 4 38	5 15	82 57
19	154 53	54 15	+ 4 44	4 29	78 0
19	154 3	54 48	– 3 14	5 6	84 9
20	161 29	54 12	+ 6 12	11 38	80 25
21	167 31	50 8	22 41	4 21	71 25
22	173 4	47 29	26 48	4 21	71 10
22	174 1	47 57	31 20	5 4	69 29
22	174 43	48 25	30 48	5 38	69 46
23	179 0	43 42	48 32	4 9	66 17
24	185 3	42 1	46 54	6 2	67 51
25	190 19	39 9	49 48	7 9	68 20
26	195 4	36 26	54 37	8 29	68 39
26	192 9	34 10	57 39	5 13	68 41
27	204 22	35 1	56 34	13 58	69 2
28	203 47	28 6	68 4	10 10	71 1
29	203 51	23 9	87 58	7 24	77 21
Août 30	203 44	+ 18 57	+ 79 29	5 22	75 53

Fig. 1. Comète c 1881.



Echelle: 1 = 800^{mm}.

Fig. 2.



Comet c (Schäberle)
1881
August 24th $9^{h}30^{m}$ L.M.T.
as seen in a Field-Glass
giving a power of 4 Diameters.

A l'aide des valeurs précédentes on calcule facilement les co-ordonnées Δ et ϕ pour chaque observation. La queue de la comète *c* étant du premier type, nous réduirons ses ϕ à l'époque du périhélie, quand $v_0 = 0°$—avec la valeur $1 - \mu = 12$, et en désignant ces angles réduits par ϕ' nous aurons la table qui suit. L'unité est égale à 800 millimètres—

1881.	ϕ	$\delta\phi$	ϕ'	Δ	Δ en millim.
Août 12·530	− 11° 20′	+ 0° 17′	− 11° 3′	0·0359	28·7
14·300	+ 18 27	16	+ 18 43	0·0770	61·6
15·313	9 37	15	9 52	0·0983	78·6
15·490	23 3	7	23 10	0·0300	24·0
16·288	4 24	12	4 36	0·0940	75·2
16·467	7 53	8	8 1	0·0453	36·2
17·313	7 12	9	7 21	0·1026	82·1
17·342	2 42	7	2 49	0·0698	55·8
18·291	7 29	5	7 34	0·0608	48·6
19·291	4 19	3	4 22	0·0507	40·6
19·325	14 45	2	14 47	0·0582	46·6
20·300	11 28	2	11 30	0·1260	100·8
21·291	0 54	0	0 54	0·0477	38·2
22·313	4 53	0	4 53	0·0468	37·4
22·321	0 18	0	0 18	0·0548	43·8
22·380	+ 1 8	0	+ 1 8	0·0605	48·4
23·291	− 8 57	0	− 8 57	0·0453	36·2
24·291	− 0 14	1	− 0 13	0·0641	51·3
25·313	+ 3 57	2	+ 3 59	0·0752	60·2
26·271	5 35	3	5 38	0·0889	71·1
26·280	2 47	3	2 50	0·0556	44·5
27·280	9 36	7	9 43	0·1446	115·7
28·270	+ 4 3	9	+ 4 12	0·1080	86·4
29·280	− 12 26	8	− 12 18	0·0793	63·4
30·280	+ 2 5	+ 0 13	+ 2 18	0·0601	48·1

Les points déterminés par ces co-ordonnées sont portés sur la planche (petits cercles). Avec la valeur $1 - \mu = 12$, on trouve pour l'axe de la queue, à la distance du noyau $\Delta = 0{\cdot}15$, $\phi = 10° 38'$. Ce point est désigné sur la planche par une petite croix et par l'inscription $1 - \mu = 12$. Les bords antérieurs et postérieurs sont calculés à l'aide de $g = 0{\cdot}15$ et $G = \mp 90°$, pour $\Delta = 0{\cdot}15$, et on a trouvé $\phi = 4° 50'$ et $\phi = 16° 5'$.

Il est évident que la queue appartient au *premier type.*

Calculons maintenant pour le temps de l'époque les positions des particules émises par le noyau et animées de la force $1 - \mu = 1$, en désignant ces positions sur la planche par des petites croix.

Soient les anomalies du noyau aux moments de ces émissions—

$$v = \quad -45° \quad -55° \quad -65°$$

Les temps correspondants à ces anomalies seront M_1 = Août 4·683, Juillet 30·336 Juillet 23·918, et l'on trouve pour le temps de l'époque, août 22·737, quand l'anomalie du noyau est 0°, les co-ordonnées suivantes—

ϕ	Δ	Δ en millim.
30° 20′	0·1010	80·8
37 22	0·1540	123·2
44 29	0·2209	176·8

Ces points se trouvent sur l'axe de la queue du second type, désigné par l'inscription $1 - \mu = 1$.

La direction de cet axe par rapport à l'axe de la queue principale est précisément la même, dans laquelle M. Bruhns a cru voir la seconde queue, plus courte et plus faible.

Les particules qui se trouvent à l'extrémité de la queue (principale) le 22·7 août, se sont détachées du noyau le 17 août.

L'écart de la position de la queue observée le 29 août et vue sur la planche à gauche du bord antérieur calculé de la queue, s'explique simplement par le mauvais temps et la proximité de la comète de l'horizon (voir l'exposé des observations de M. Bruhns).

Dans les dessins de M. Tempel (août 26, 29 et 30) on voit que les raies claires du milieu de la queue s'affaiblissent près de l'atmosphère de la comète et ne se discèrnent pas dans son intérieur. Peut être que la lumière de leurs particules, dont la portion principale provient du soleil, est un peu absorbée dans son trajet vers ces particules par la partie relativement plus dense de la tête.

La queue de la comète *c* par ses positions par rapport au soleil et surtout par les positions de sa figure par rapport aux observateurs, ressemble beaucoup à la queue de la comète de 1858 V, et ces appendices présentent une grande différence et en même temps une analogie remarquable. La comète Donati a développé une queue immense du second type, très courbée et riche en matières, animées de différentes forces répulsives entre $1 - \mu = 1$ et $1 - \mu = 2{\cdot}4$. De son bord antérieur se détachait une autre queue, presque rectiligne et très faible, soumise à une grande force répulsive. Dans la comète *c*, au contraire, la queue rectiligne faisait sa partie principale ; la queue courbée du second type était à peine perceptible et n'était pas sortie, pour ainsi dire, de son état rudimentaire.

On doit admettre, que les substances constituantes sont les mêmes dans ces deux comètes, mais que leur quantité rélative est différente.

La comète *c* peut être inscrite dans cette famille de comètes, à laquelle appartiennent la comète de Halley, la comète de 1861 II etc. . . . .

Th. Bredichin.

Moscou, novembre 1881.

N.B.—L'existence de la seconde queue dans la comète *c* est maintenant mise hors de doute par le dessin de cette comète fait à Forest Lodge, Maresfield, par M. le Capitaine W. Noble (Monthly Notices of the R.A.S. Vol, XLII. No. 1, November, 1881. Physical observations of Schäberle's Comet). Le dessin est accompagné de la note suivante : "My sketch of this comet represents it as viewed in a field-glass, magnifying 4 diameters, on August 24, from 9^h to 10^h L.M.T. I drew it for the strange, faint *second tail,* or bifurcation of the first one."

Sur ce dessin la queue principale est étroite, rectiligne et claire ; la seconde queue

est plus courte, plus large et beaucoup plus faible. Il est très important pour nous, que la position de cet appendice secondaire, par rapport à la queue principale, et sa courbure s'accordent avec la direction de l'axe calculé de la queue du second type, qui est designé sur la planche par l'inscription $1 - \mu = 1$.

Br.

Déc. 20, 1881.

We are indebted to Captain Noble for the sketch of Comet *c*, reproduced on Plate II.—Ed.

BEOBACHTUNGEN DES COMETEN *g* 1881 (Swift).

1881.	M.Z. Dresden	$\alpha' - \alpha$	$\delta' - \delta$	α app.	l.f.p.	δ app.	l.f.p.	Vergl.	*
Dec. 16,	$9^h\ 13^m\ 29^s$	$+3^m\ 42^s{\cdot}67$	$+\ 2'\ 29''{\cdot}9$	$23^h\ 38^m\ 30^s{\cdot}10$	9·549	$+36°\ 6'\ 44''{\cdot}8$	0·536	12:7	a
„ 18,	9 36 26	—1 29·71	+ 2 35·2	23 37 25·39	9·577	+34 7 3·4	0·600	14:7	b
„ 18,	11 20 6	—1 31·31	— 1 57·9	23 37 23·79	9·650	+34 2 30·3	0·725	18:9	b

Mittlere Oerter der Vergleichssterne für 1881·0.

a	$23^h\ 34^m\ 33^s{\cdot}46$	$+\ 0^s{\cdot}04$	$+\ 36°\ 3'\ 36''{\cdot}0$	$+\ 0''{\cdot}5$	Weisse 23^h 734–5
b	23 38 51·12	+ 0·04	+ 34 3 49·9	+ 0·5	— - 816

Eine rohe Vergleichung mit der Ephemeride des Herrn Dr. Palisa ergab (B.—R.) :—

Dec. 16	$+\ 3^s$	$+\ 0'{\cdot}5$
18	+ 2	+ 0·4
18	+ 3	+ 0·4

Am 16ten December stand der Comet zwischen mehreren Sternen 11 und 12 Grösse. Am 18ten stand der Comet ganz nahe an dem Stern 7 mag., D.M. + 34°·4763. An beiden Abenden war das Wetter stürmisch und nebelig; der Comet war sehr schlecht zu sehen. Die Beobachtungen wurden mit dem 12 zölligen Aequatoreal und Fadenmicrometer angestellt.

B. von Engelhardt.

ELEMENTS OF COMET *g* 1881 (SWIFT).

Dun Echt Circulars, Nos. 43 and 44, contain the following:—

Writing from Währing on Dec. 14, 1881, Mons. J. Palisa sends the following elements of Comet *g* 1881 (Swift)—

$$\begin{array}{rl} T & = 1881, \text{ November } 20{\cdot}0359 \text{ Berlin M. T.} \\ \left.\begin{array}{r} \pi \\ \Omega \\ i \end{array}\right. & \left.\begin{array}{l} = 299^\circ\ 25'\ 25'' \\ = 181\ \ 21\ \ 41 \\ = 144\ \ 48\ \ \ 6 \end{array}\right\} \text{ Mean Equinox } 1881{\cdot}0 \\ \log q & = 0{\cdot}284788 \end{array}$$

Computed from observations on Nov. 22, 27, and Dec. 12. Deviations of the middle place:—

$$\Delta\lambda \cos\beta = -\,9''; \qquad \Delta\beta = +\,6''.$$

Equations for the Equator.

$$\begin{aligned} x &= (9{\cdot}999959)\, r \sin(\ 26^\circ\ 56'\ 58'' + v) \\ y &= (9{\cdot}716395)\, r \sin(115\ \ 39\ \ 42\ \ + v) \\ z &= (9{\cdot}931456)\, r \sin(117\ \ 25\ \ 39\ \ + v) \end{aligned}$$

Aus den Beobachtungen Cambridge (Mass.) Nov. 17, Paris Nov. 27 und Wien Dec. 12 habe ich folgende Bahn berechnet:—

$$\begin{array}{rl} T & = 1881, \text{ November } 19{\cdot}7402 \text{ m. Z. Berlin} \\ \left.\begin{array}{r} \pi - \Omega \\ \Omega \\ i \end{array}\right. & \left.\begin{array}{l} = 117^\circ\ 52'\ 13'' \\ = 181\ \ 19\ \ 51 \\ = 144\ \ 49\ \ 17 \end{array}\right\} \text{ m. Aeq. } 1882{\cdot}0 \\ \log q & = 0{\cdot}28468 \end{array}$$

Die Darstellung der mittleren Beobachtung wird:—

$$\Delta\lambda \cos\beta = +\,4''; \qquad \Delta\beta = +\,6''.$$

DR. OPPENHEIM.

Berlin, Dec. 18, 1881.

According to the "Science Observer," No. 35, a wrong declination (70° 42′ 47″·1 instead of 70° 47′ 2″·3) for Nov. 20 was used in computing the orbit given in Dun Echt Circular No. 40 (*Copernicus* Vol. I, p. 238). The orbit became erroneous in consequence, but was nevertheless of great value as it enabled European observers to find the comet.

RECENT LITERATURE.

On the Relative Accuracy of Different Methods of Determining the Solar Parallax; by Wm. Harkness.* The author points out that all the known methods of finding the distance of the sun belong to one or other of three classes:

I. Trigonometrical methods, II. Gravitational methods, and III. Photo-tachymetrical methods.

To the first class belong *observations of Mars* either by the meridional method at two stations in the northern and southern hemispheres, the most noteworthy results of which for the solar parallax are—

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* Amer. Jour., Nov., 1881 (20 pp.)

1672	J. D. Cassini,	9″·5	1865	E. J. Stone,	8″·943
1751	Lacaille,	10·38	1865	A. Hall,	8·842
1835	Henderson,	9·028	1867	Newcomb,	8·855
1856	Gilliss and Gould,	8·495	1879	Downing,	8·960,
1863	Winnecke,	8·964			

or from the diurnal parallax, instances of which are—

1672	J. D. Cassini,	10″·2	1857	W. C. Bond,	8″·605
1672	Flamstead,	10	1877	Maxwell Hall,	8·789
1719	Bradley and Pound,	10·5	1879	Gill,	8·78

Both these methods have been applied to asteroids—

In 1875 to Flora by Galle, ...	$\pi =$ 8″·879
1877 Lord Lindsay and Gill (from Juno in 1874)	8·765

There are great difficulties in the way of obtaining satisfactory results from Mercury and Venus by these methods.

Transits of Venus, until quite recently, were regarded as furnishing by far the best measures of the sun's distance. The contact observations of 1761 yielded results of which Pingré's 10″.10 and Planman's 8″·49 are the extremes; the transit of 1769 furnished 11 values varying from Planman 8″·43 to Stone 8″·91. Encke's well-known result from both transits is 8″·571. Lastly we have from the Transit of 1874, Airy 8″·760 and Tupman 8″·846. The differences mainly arise from the varying interpretations put upon the descriptions of the phenomena of contact. From 61 selected observations of the Transit of 1874 discussed by Col. Tupman the author finds the probable error of an observed contact to be $\pm\, 4^{s}{\cdot}59$ corresponding to ± 0″·15 in the distance of centres. Actual errors of 20^{s} or 30^{s} are not uncommon.

Observations of the Transit of 1874 with heliometers have not yet been published.

As the reductions of the U.S. transit of Venus photographs taken at Kerguelen Island, Hobart Town and Peking are not quite completed, Prof. Harkness examines the results obtained from 119 American photographs of the Transit of Mercury in 1878 by the long focus apparatus that was used in 1874 for the transit of Venus. As respects the measures of the photographs the constant error of reading is practically zero for the mean of the three stations, Cambridge, Washington, and Ann Arbor. The probable error of a position of Mercury from two sets of readings made upon a single photograph is within a circle whose radius is ± 0″·553.

II. Gravitational Methods. The author first derives the expression—

$$\text{Solar parallax} = p = 609{\cdot}434 \sqrt[3]{\frac{\mathrm{E}}{\mathrm{S}+\mathrm{E}}},$$

where E and S are the masses of the Earth and Sun, and the constant is derived from the length of the sidereal year, the size and figure of the earth and the force of gravity on its surface. This equation with Le Verrier's values of the earth's mass derived from the secular variations of Venus and Mars gives—

$p = 8''\cdot862$, from latitudes of Venus at Transits of 1761 and 1769;
$p = 8\cdot868$, from 106 years' meridian observations of Venus, and
$p = 8\cdot875$, from the occultation of ψ^2 Aquarii by Mars, Oct. 1, 1672.

The uncertainty of the parallax is estimated at possibly $\pm\ 0''\cdot07$, As these secular variations continually increase they will eventually give the sun's parallax with the utmost precision.

Designating the Parallactic Inequality by Q, and the moon's mass by M, there results the equation—

$$p = [8\cdot837088]\ Q\ \frac{1 + M}{1 - M}$$

where the logarithmic constant mainly depends on the moon's parallax and the ratio of the mean motions of the sun and moon.

Taking five of the best values of Q and varying the moon's mass from $\frac{1}{80}$ to $\frac{1}{83}$ Prof. Harkness finds p to vary between the limits of $8''\cdot776$ and $8''\cdot910$, and to be but little affected by the assumed mass of the moon.

For p in terms of the Lunar Inequality, L, the author finds the equation—

$$p = 0\cdot0164564\ L\ \left(\frac{E + M}{M}\right)$$

in which the constant is that of lunar parallax lessened by $28''\cdot18$ (a term of Delaunay's, depending upon the difference of mean longitudes of the sun and moon). Various values of L are $6''\cdot50 + 0''\cdot02$ from observations at Greenwich, Paris and Königsberg (Le Verrier); and two values by Professor Newcomb, $6''\ 56 \pm 0''\cdot04$ from Greenwich observations and $6''\cdot51 \pm 0''\cdot07$ from Washington data. These, combined with the same values of M as before, give p between $8''\cdot664$ and $9''\cdot068$. Setting the uncertainty of L at $\pm\ 0''\cdot02$ and that of the reciprocal of the moon's mass at $\pm\ 0\cdot5$ the probable error of p comes out $\pm\ 0''\cdot06$.

III. Photo-tachymetrical methods.

Given the velocity of light upon the earth's surface the sun's parallax may be found in two ways:

1st. By combining the light equation with the measured velocity in terms of the earth's equatoreal radius.

2nd. Assuming the ratio of the earth's orbital velocity to that of light to be represented by the constant of aberration and combining that constant with the velocity of light.

To determine p within $\pm\ 0''\cdot01$, the probable error of the velocity of light per second must not exceed $\pm$ 240 Kilometers, or that of the light equation $\pm\ 0^s\cdot40$ or of the constant of aberration $\pm\ 0''\cdot016$. The probable error of p seems to be at least $\pm\ 0''\cdot05$.

The results are—

Velocity and Light Equation,	8″·72 to 8″·89
Velocity and Aberration,	8·73 to 8·90

This method is also embarrassed by a number of serious theoretical difficulties, prominent among which is that we have no sure means of passing from the velocity of light at the earth's surface to its velocity in inter-planetary space.

Collecting the various limiting values of the solar parallax the author goes on to say that :—

"By adopting suitable weights almost any value from 8″·8 to 8″·9 could be obtained; and no matter what the result actually was, it would always be open to a suspicion of having been cooked in the weighting. We only know that the parallax seems to lie between 8″·75 and 8″·90, and is probably about 8″·85. Attack the problem as we will, the results cluster around this central value. All the methods give a probable error of about ± 0″·06, and no one of them seems to possess decided superiority over the others. We have nearly exhausted the powers of our instruments, and further advance can only be made at the cost of excessive labour.

"In the beginning of the eighteenth century the uncertainty of the solar parallax was fully two seconds; now it is only about 0″·15. To narrow it still further we require a better knowledge of the masses of the earth and moon, of the moon's parallactic inequality, of the lunar equation of the earth, of the constants of nutation and aberration, of the velocity of light, and of the light equation. All these investigations can be carried on at any time, but there are others equally important which can only be prosecuted when the planets come into the requisite positions. Among the latter are observations of Mars when in opposition at its least distance from the earth, and transits of Venus.

"In 1874 all astronomers hoped and believed that the transit of Venus which occurred in December of that year would give the solar parallax within 0″·01. These hopes were doomed to disappointment, and now, when we are approaching the second transit of the pair, there is less enthusiasm than there was eight years ago. Nevertheless, the astronomers of the twentieth century will not hold us guiltless if we neglect, in any respect, the transit of 1882. Observations of contacts will, doubtless, be made in abundance, but our efforts should not cease with them. We have seen that the probable error of a contact observation is ± 0″·15, that there may always be a doubt as to the phase observed, and that a passing cloud may cause the loss of the transit. On the other hand, the photographic method cannot be defeated by passing clouds, is not liable to any uncertainty of interpretation, seems to be free from systematic errors, and is so accurate that the result from a single negative has a probable error of only ± 0″·55. If the sun is visible for so much as fifteen minutes during the whole transit thirty-two negatives can be taken, and they will give as accurate a result as the observation of both internal contacts. In view of these facts can it be doubted that the photographic method offers as much accuracy as the contact method, and many more chances of success?

"The transit of 1882 will not settle the value of the solar parallax, but it will contribute to that result, directly as a trigonometrical method, and indirectly through the gravitational methods with which the final solution of the problem must rest. As our knowledge of the earth's mass may be made to depend upon quantities which continually increase with the time, it will ultimately attain great exactness, and then the solar parallax will be known with the same exactness. Long before that happy day arrives the present generation of astronomers will have passed over to the silent majority, but not without the satisfaction of knowing that their labours will contribute to that fullness of knowledge which shall be the heritage of their successors."

Observations of the Transit of Mercury, 1881, *November* 7, *at Mount Hamilton, California.* (Communicated by the Lick Trustees.) By Edw. S. Holden and

S. W. Burnham (American Journal of Science, January, 1882). Professor Holden and Mr. Burnham were invited by the Lick Trustees to observe the Transit of Mercury on the 7th of November, with the smaller instruments which have lately been set up at the Lick Observatory. Professor Holden observed with a Clark comet-seeker of 4 inches aperture and a power of 50 diameters, the highest available. Captain R. S. Floyd, president of the Lick Trustees, observed with a small telescope, aperture $2\frac{1}{2}$ inches and power 56. Mr. Burnham observed with the 12-inch Clark equatorial (its aperture being reduced to six inches) and a power of 150 diameters. The assumed position of Mount Hamilton is—

$$\phi = +\,37^\circ\ 21'\ 3''$$
$$\lambda = +\,2^h\ 58^m\ 14^s{\cdot}6 \text{ from Washington.}$$

The computed time of first contact is (using the elements of the American Ephemeris), $2^h\ 9^m\ 42^s{\cdot}3$.

The local mean times as observed were—

First Contact.

Holden	$2^h\ 11^m\ 2^s{\cdot}3$	
Burnham . . .	$2\ \ 10\ \ 41{\cdot}1$	; H. − B. = + $21^s{\cdot}2$

Second Contact.

Holden	$2^h\ 12^m\ 25^s{\cdot}8$	
Floyd	$2\ \ 12\ \ 15{\cdot}0$	; H. − F. = + $10^s{\cdot}8$
Burnham . . .	$2\ \ 12\ \ 5{\cdot}7$	; H. − B. = + $20{\cdot}1$

Duration of Ingress.

Holden .	$1^m\ 23^s{\cdot}5$	Burnham .	$1^m\ 24^s{\cdot}6$

"*Zur Theorie des Encke'schen Cometen. Von O. Backlund.*" (Mém. de l'Acad. Imp. de St. Pétersbourg, 7^{me} Série, T. 28, No. 6, 83 pp.) This paper forms a continuation of Von Asten's memoir on the absolute Jupiter-perturbations of Encke's comet. It gives the general expressions of the perturbations of the first order of the so-called Hansen's elements, the mean anomaly, the logarithm of the radius vector and the sine of the latitude for the part of the orbit between the true anomalies 180° and 190°, so far as this is possible without determining the corresponding expressions for the remaining part of the orbit. As Von Asten's calculations refer to the part of the orbit between 170° and 180°, the principal difficulty in deducing the absolute Jupiter-perturbations may now be considered as overcome, as the comet in the rest of the orbit never approaches Jupiter. While Von Asten followed Gyldén's older method, Dr. Backlund has adopted the methods given in the same author's "Receuil de Tables contenant les développements numériques à employer dans le calcul des perturbations des comètes."

ON SOME INTERPOLATION FORMULÆ FOR DOUBLE STARS.

By Professor T. N. THIELE.

The great majority of Double Stars, whose movement is so perceptible that observations at various times cannot be immediately compared, move however too slowly to render it practicable to carry out actual determinations of their orbits for some centuries to come. For this purpose seven mutually independent determinations are requisite, and the observations are relatively of so little accuracy, that even when half the orbit has been described under observation, one or several of the elements may remain almost indeterminable.

But while thus in most instances any knowledge of the actual conditions of the movements must at present be held inattainable, it is yet very possible to satisfy our more immediate requirements by such computations of the observations as may throw light on the errors made in observing.

For this purpose the formulæ for the determination of an elliptic orbit can certainly be used, if, whenever an equation occurs which becomes indeterminate, on any reasonable assumption respecting the magnitude of the errors, use is made of the right thus obtained to assume the quantity sought. Still this would practically restrict the application of this method to double stars to those cases where the whole of the observations could be said to represent six mutually independent values, so that an assumption respecting a single element of the orbit (in almost every case the period) would remove all the indeterminateness of the equations to be solved. But where several arbitrary assumptions would have to be made, the computations must be made by interpolation formulæ in place of the exact methods.

From what has been said it follows, that for double stars a number of interpolation formulæ should be provided, and that the most complicated of these should contain five constants to be determined from the observations.

It may be insisted on that these interpolation formulæ shall be easy of computation, at least as easy as the elliptical formulæ in their normal application. But beyond this there is something more that must be demanded, which on the whole has not been taken into account in the interpolation formulæ that hitherto have been used in this problem; namely, that the properties which characterize the actual movement in a short interval of time should also be reproduced in the formula. Each such peculiarity that characterizes the elliptic orbit (*e.g.*, maxima or minima) should find a representation in at least one of the interpolation formulæ, and on the other hand the interpolation formula ought generally not to have any prominent features (*e.g.*, an inflection in the orbit) which do not occur in elliptic orbits. The interpolation formula must be conceivable as an imperfect development of the true movement in the form of a series. As the true movement has but seven arbitrary constants, if the series has a larger number they must be reduced by taking into account the relations between the constants in the interpolation formula. But a special condition may require that a relation shall also be established between a smaller number of constants in the first member of the series, than are taken account of in the interpolation formula, and just such a condition here exists.

There are always two kinds of observations, viz., measured distances r and directions (position angles), R at given times t; in each case, therefore, there ought to be employed simultaneously two interpolation formulæ—

$$r = f(t) \quad \text{and} \quad \mathrm{R} = \mathrm{F}(t).$$

But in elliptic motion there is also the law of proportionality between the areas and the times—

$$\tfrac{1}{2} r^2 \frac{d\mathrm{R}}{dt} = k; \tag{1}$$

between the two simultaneous interpolation formulæ there must therefore exist a relation, the differential equation—

$$2f'(t)\,\mathrm{F}'(t) + f(t)\,\mathrm{F}''(t) = 0, \tag{2}$$

the very same, which in its greatest generality expresses, that the observed, projected motion is in accordance with some attractive force or other between the two stars. In the representation of the apparent motion of double stars only such pairs of interpolation formulæ $r = f(t)$ and $\mathrm{R} = \mathrm{F}(t)$ are applicable, as fulfil the condition given in (2). In some measure either $r = f(t)$ or $\mathrm{R} = \mathrm{F}(t)$ may be chosen arbitrarily, but this choice determines the other function with, however, the addition of one arbitrary constant in $f(t)$ or of two in $\mathrm{F}(t)$. Arbitrary choice may also be made of the formula for the apparent attractive force as function of the distance r and then both formulæ of interpolation can be determined with the introduction of four constants in all.

The interpolation formula can thus be found on the assumption, that the attraction is the same in the apparent as in the true orbit, inversely proportional to the square of the distance, and in this way formulæ are obtained for the elliptic motion about the chief star as focus, which will be very advantageous, especially when the plane of the orbit is approximately perpendicular to the line of sight; I shall, however, not delay myself with these approximations, the less so as they are sufficiently well known, although hitherto not applied in this way.

On the other hand there is reason for discussing some other formulæ of far simpler computation, and which often can be applied even in cases when the observed motion has been large enough to induce astronomers to attempt an actual computation of an orbit. They are all subject to the assumption that the attraction in the apparent orbit is inversely proportional to the cube of the distance. But their title to be used is not to be sought in this property. Imagine the square of the apparent distance developed in a series according to ascending powers of the time—

$$r^2 = l + mt + nt^2 + \ldots\ldots$$

then, remarkably enough, this series will often be found more convergent than the series for r, R or $x = r \cos \mathrm{R}$ or $y = r \sin \mathrm{R}$. I take account only of the first three members in my interpolation formula, and set therefore for $f(t)$ the expression—

$$r = \sqrt{l + mt + nt^2}\,; \qquad (3)$$

whence for F (t) by (2) or (1)—

$$\begin{aligned} \mathrm{R} &= 2k \int \frac{dt}{l + mt + nt^2} \\ &= \mathrm{Q} + \frac{2k}{n\,(v - u)} \log \operatorname{nat} \frac{t - v}{t - u}, \qquad (4) \end{aligned}$$

where u and v are the roots of the equation $0 = l + mt + nt$.
In case the roots are complex numbers—

$$\begin{aligned} v &= w + si \\ u &= w - si, \end{aligned}$$

(3) takes the form—

$$r = \sqrt{n}\,\sqrt{s^2 + (t - w)^2} \qquad (3')$$

and (4) becomes—

$$\mathrm{R} = \mathrm{P} + \frac{2k}{ns} \operatorname{arc} \left(\operatorname{tang} \frac{t - w}{s}\right). \qquad (4')$$

In the special case when $v = u$—

$$r = \sqrt{n}\,(t - u) \qquad (3'')$$

and then—

$$\mathrm{R} = \mathrm{Q} - \frac{2k}{n} \cdot \frac{1}{t - u} \cdot \qquad (4'')$$

Finally, if one root is infinite, $n = 0$, $nu = -m$, $v = -\frac{l}{m}$, and we have—

$$r = \sqrt{l + mt} \qquad (3''')$$

and—

$$\mathrm{R} = \mathrm{W} + \frac{2k}{m} \log \operatorname{nat} \left(t + \frac{l}{m}\right). \qquad (4''')$$

It follows from (3) that these interpolation formulæ cannot give to the distance more than one maximum (3) and (4), or one minimum (3′) and (4′). On the other hand, the apparent orbit can have as many as two of each kind, it follows therefore that these formulæ are restricted to cases where the observations comprise at the most one maximum or minimum. Further, the orbit given by the formula is strictly symmetrical with respect to a maximum or minimum, and as, in general, this does not hold for the exact laws, the use of the formula is also restricted on this account.

None of the orbits given by the formulæ have inflexions, and the curvature of the orbit is always turned towards the chief star except in the solitary case of (3′) and (4′), when $\frac{ns}{2k} > 1$. The case when $\frac{ns}{2k} = 1$, giving uniform rectilinear motion, is therefore the extreme applicable limit. The distance can never become infinite for a finite t, but on the other hand it may easily happen that $r = 0$, but only simultaneously with $R = \infty$, so that the companion can only reach the chief star after having circled round it an infinity of times.

For practical computation and for solving the problems we shall treat of, the interpolation formulæ are best written in the following form—

For (3) and (4) we have—

$$\left.\begin{aligned} r\,10^{\alpha R} &= a + bt \\ r\,10^{-\alpha R} &= c + dt \end{aligned}\right\}\text{I.}$$

Or when n is positive—

$$\left.\begin{aligned} r\,10^{\alpha(R-Q)} &= \sqrt{n}\,(t-u) \\ r\,10^{\alpha(Q-R)} &= \sqrt{n}\,(t-v) \end{aligned}\right\}\text{I}a\,;$$

on the contrary for n negative—

$$\left.\begin{aligned} r\,10^{\alpha(R-Q')} &= \sqrt{-n}\,(t-u) \\ r\,10^{\alpha(Q'-R)} &= \sqrt{-n}\,(v-t) \end{aligned}\right\}\text{I}b,$$

where—

$$k = \frac{n(u-v)}{4\alpha}\,\text{Log}\,e = \frac{bc-ad}{4\alpha}\,\text{Log}\,e.$$

For (3′) and (4′) we have—

$$\left.\begin{aligned} r\cos\beta R &= \mathfrak{a} + \mathfrak{b}t \\ r\sin\beta R &= \mathfrak{c} + \mathfrak{d}t \end{aligned}\right\}\text{II}$$

or—

$$\left.\begin{aligned} r\cos\beta\,(R-P) &= \sqrt{n}\cdot s \\ r\sin\beta\,(R-P) &= \sqrt{n}\cdot(t-w) \end{aligned}\right\}\text{II}a,$$

where—

$$k = \frac{ns}{2\beta} = \frac{\mathfrak{a}\mathfrak{d} - \mathfrak{b}\mathfrak{c}}{2\beta};$$

it is here assumed that R is given in arc. In each pair of these formulæ there enter 5 constants.

From (3‴) and (4‴)

$$\left.\begin{aligned} r^2 &= l + mt \\ R &= A + B\log(l+mt) \\ &\text{or—} \\ 10^{\frac{R-A}{2B}} &= r \end{aligned}\right\}\text{III.}$$

According to (3″) and (4″)—

$$\left.\begin{aligned} r &= a' + b't \\ rR &= c' + d't \end{aligned}\right\}\text{IV.}$$

In II. we have formulæ for rectilinear uniform motion—

$$\left.\begin{aligned} r\cos R &= \mathfrak{a} + \mathfrak{b}t \\ r\sin R &= \mathfrak{c} + \mathfrak{d}t \end{aligned}\right\}\text{V},$$

interpolation formulæ with four constants.

Finally included under IV. we have the formulæ $r = a'$, $R = A + Bt$ and $r = a' + b't$, $R = A$ as quite special cases, and these are without doubt the sole formulæ with only three constants that can be required.

It is very easy to compute ephemerides by these formulæ, nor is it difficult to determine the constants from observations. But in dealing with double stars, it is not possible as in the case of planets or comets, to lay down a definite form for the computations which can in some measure be followed in all instances. For double stars it is necessary to give the computer free choice between many different methods, and the result will greatly depend on his being led by experience and delicate tact to the choice of the right method. The cause of this is first and foremost, that the method of least squares is powerless in face of the large systematic errors which deform micrometric measurements. Sir John Herschel, as is well known, proposed to replace the successive treatment according to the method of least squares by a preliminary graphic interpolation and computation of the one co-ordinate from the other by use of the law of proportionality between areas and times. The device is good, although not quite free from leaving too much room for arbitrary treatment. But it should not be applied solely to the measures of position, but also to the distances, as I applied it long ago (see Undersögelse af Omlöbsbevægelsen i Dobbeltstjernesystemet γ Virginis), and it is best of all to dispense with these introductory operations. They were necessary so long as the exclusive aim was to compute all seven elements of the orbit without regard to the sufficiency of the observations. By the use of interpolation formulæ graphic interpolation can be dispensed with. By reducing the number of constants, usable first approximations can be derived from the raw observations, and it is easy through the formation and amelioration of normal places gradually to approach the goal.

But the impossibility of treating all double stars by one and the same method, has another, fully as essential cause in the large and, as it appears, irregular distinction between the motions of the various pairs. No element can be approximately known beforehand, all excentricities, all inclinations, &c., are found about equally represented, and even, where the form of the apparent orbit is the same, the different magnitude of the orbit (the difference between corresponding apparent angles) demands an absolutely different mode of computation. The difference occasions that in one instance measures of distance are preferred to those of position for the determination of elements that can be deduced from both kinds, and in another that just the opposite course is taken. But *one* part of the observations can always be considered free from error, namely, the times at which the observations are made. It is therefore a great (and unfortunately, frequent) error, to use or recommend methods of computing orbits, which essentially rest on the combination of distances and positions, in which the times of observation are treated as unknown. The observational foundation for every computation of an orbit or interpolation formula should at all times consist in combinations of the two kinds, Time-Distance and Time-Position, and the difference in the methods should show itself in the different number of these kinds of combinations that are considered as given. There must necessarily be included at least one observation of time-distance for the determination of the unit of distance. For the computation of an orbit it is further necessary that at least one observation has the form time-position; our interpolation formulæ demand even, that at least two observations shall have this form, as not only the fundamental direction, but also the unit of position has to be fixed by observations of this co-ordinate.

For most methods it is advantageous, and for some almost necessary, that the epochs of some of the measures of distance and position angle should exactly coincide. This is in so far unfortunate, that the success of a computation depends, not only on one kind of observation not obtaining an undue preponderance over the other, but also on the single observations or normal places being taken from such epochs that they express as well as possible the whole series of observations; thus, *e.g.*, a number of position angles are to be preferred at epochs when the distance was small and the angular velocity great, while observations of distance should chiefly be chosen from times when the distance itself was great or its variation rapid. It may be difficult enough fully to satisfy these demands when the choice is free, but the difficulty increases much when combinations of the form Time-distance *and* position are to be used.

Rules of computation shall now be communicated for my various interpolation formulæ, arranged chiefly according to the varying number of employed distances r, and positions R, and secondarily in accordance with the various sets of formulæ I, II, III, IV or V that are used.

Three Distances and Two Positions.

When three distances and two positions are used, the epochs can be chosen without any restriction. The computation itself shows whether the interpolation formulæ are of the form I or II. From the three distances r_1, r_2 and r_3, which correspond to the times t_1, t_2 and t_3, l, m and n are computed by the three equations—

$$\begin{aligned} r_1^2 &= l + mt_1 + nt_1^2 \\ r_2^2 &= l + mt_2 + nt_2^2 \\ r_3^2 &= l + mt_3 + nt_3^2. \end{aligned}$$

From the sign of $m^2 - 4ln$ it is seen whether the equation—

$$0 = l + mt + nt^2$$

has real or imaginary roots.

1. In the first case these roots are computed from—

$$\begin{aligned} v &= (-m + \sqrt{m^2 - 4ln}) : 2n \\ u &= (-m - \sqrt{m^2 - 4ln}) : 2n. \end{aligned}$$

When $n > 0$, the times u and v must both fall either before or after the epochs of all the observations, not only of t_1, t_2 and t_3 but also of t_4 and t_5, which belong to the positions R_4 and R_5. Then from the equations—

$$2\alpha(R_4 - Q) = \log(t_4 - u) - \log(t_4 - v)$$
$$2\alpha(R_5 - Q) = \log(t_5 - u) - \log(t_5 - v)$$

—where log means Briggs' logarithms,—are found the constants α and Q, whence the final formulæ—

$$\left.\begin{aligned} r\,10^{\alpha(R-Q)} &= \sqrt{n}\,(t-u) \\ r\,10^{\alpha(Q-R)} &= \sqrt{n}\,(t-v) \end{aligned}\right\}\text{Ia}$$

in which the $\sqrt{n}$ is to be taken with the same sign as $t - u$ and $t - v$.

If, on the contrary, $n < 0$, the time u will fall before, v after the whole of the epochs of observation, and then α and Q' are to be computed by the equations—

$$2\alpha(R_4 - Q') = \log(t_4 - u) - \log(v - t_4)$$
$$2\alpha(R_5 - Q') = \log(t_5 - u) - \log(v - t_5)$$

in the formulæ—

$$\left.\begin{aligned} r\,10^{\alpha(R-Q')} &= \sqrt{-n}\,(t-u) \\ r\,10^{\alpha(Q'-R)} &= \sqrt{-n}\,(v-t) \end{aligned}\right\}\text{Ib.}$$

In this case for the maximum of distance $t_m = \frac{1}{2}(u + v)$, $R_m = Q'$ and $r_m = \frac{1}{2}\sqrt{-n}\,(v - u)$.

2. If, on the contrary, $0 = l + mt + nt^2$ has imaginary roots, n will at all times be positive, and we can take—

$$w = -\frac{m}{2n}$$
$$s = \sqrt{4ln - m^2} : 2n.$$

Then β and P are computed from the times t_4 and t_5 for the observations of position by the two equations—

$$\beta(R_4 - P) = \text{arc}\left(\tan = \frac{t_4 - w}{s}\right)$$
$$\beta(R_5 - P) = \text{arc}\left(\tan = \frac{t_5 - w}{s}\right).$$

We have thus all the constants for the formulæ—

$$\left.\begin{aligned} r\cos\beta(R-P) &= \sqrt{n}\cdot s \\ r\sin\beta(R-P) &= \sqrt{n}\,(t-w) \end{aligned}\right\}\text{IIa.}$$

There is a minimum distance for $t_m = w$, $R_m = P$ and $r_m = \sqrt{n}\cdot s$.

These computations show plainly when there is reason for trying the application of one of the simpler interpolation formulæ III, IV or V with four constants, and which of them should be used.

Two Distances and Two Positions.

3. When in the preceding calculations with three distances and two positions the constant n has turned out very small, it is well to try, if by leaving out one

distance, formulæ III can be found to represent the observations. First l and m are to be computed from—

$$r_1^2 = l + mt_1$$
$$r_2^2 = l + mt_2,$$

and then A and B from—

$$R_3 = A + B \log (l + mt_3)$$
$$R_4 = A + B \log (l + mt_4).$$

4. If, on the other hand, $m^2 - 4nl$ is small, positive or negative, formulæ IV should be tried by computing a' and b' from—

$$r_1 = a' + b't_1$$
$$r_2 = a' + b't_2$$

and c' and d' from—

$$(a' + b't_3)\, R_3 = c' + d't_3$$
$$(a' + b't_4)\, R_4 = c' + d't_4.$$

And in general the calculation should at once be made in this way, whenever the observed changes have been very small both in distance and position.

5. Lastly if β has differed but little from 1 in the computation of II from three distances and two positions, or in case inaccuracies in the observations should have caused β to become greater than 1, so that the convex side of the orbit would turn towards the principal star, it becomes necessary to compute a rectilinear path according to V.

If simultaneous observations of distance and position have been available, this chance will not cause any difficulty. It is only necessary to transform the polar into rectangular co-ordinates and compute the constants in the resulting linear equations.

Two Distances and Three Positions.

Cases where two distances and three positions must necessarily be used for the determination of the constants in the interpolation formulæ will often occur, especially when older observations have to be used, in which only the position angles are in any degree reliable, or when the distance during part of the time of observation has been very small.

6. If both measures of distance are simultaneous with two of the measures of position, it will not be possible to avoid indirect methods, but approximate formulæ can be given. It is easy to see from the formulæ I and II that the whole difficulty will rest in determining α or β. An equation can then be formed with the given values t_1, r_1, R_1; t_2, r_2, R_2; t_3, R_3 in which provisionally only α or β occurs as unknown. To form this, suppose the unknown distance r_3 provisionally retained in the equations I and II, until the unknown constants a. b, c, d or $\mathfrak{a}$, $\mathfrak{b}$, $\mathfrak{c}$, $\mathfrak{d}$ are eliminated. Eliminating r_3 next we have—

$$\frac{t_3 - t_1}{t_3 - t_2} \cdot \frac{r_2}{r_1} = \frac{10^{\alpha(R_3 - R_1)} - 10^{\alpha(R_1 - R_3)}}{10^{\alpha(R_3 - R_2)} - 10^{\alpha(R_2 - R_3)}}$$

or—

$$\frac{t_3 - t_1}{t_3 - t_2} \cdot \frac{r_3}{r_1} = \frac{\sin\{\beta\,(R_3 - R_1)\}}{\sin\{\beta\,(R_3 - R_2)\}}.$$

From these α or β are now to be determined by trial. But since in the interpolation formulæ it is always assumed that the motion has not been particularly large, α or β can be approximately computed, and that moreover from one and the same series. Suppose R expressed in degrees, then we can put—

$$\left(\frac{\alpha}{\text{Log}\,e}\right)^2 = -\left(\frac{\beta\pi}{180}\right)^2 = \gamma$$

and then—

$$\frac{t_3 - t_1}{r_1\,(R_3 - R_1)} : \frac{t_3 - t_2}{r_2\,(R_3 - R_2)} = \frac{N_1}{N_2} = \frac{1 + \frac{1}{6}\gamma(R_3 - R_1)^2 + \frac{1}{120}\gamma^2(R_3 - R_1)^4 + \ldots}{1 + \frac{1}{6}\gamma(R_3 - R_2)^2 + \frac{1}{120}\gamma^2(R_3 - R_2)^4 + \ldots}$$

whence follows as an approximate formula for γ,—

$$\frac{1}{\gamma} + \frac{1}{30}\{(R_3 - R_1)^2 + (R_3 - R_2)^2\} = \frac{1}{12}\,\frac{N_1 + N_2}{N_1 - N_2}\{(R_3 - R_1)^2 - (R_3 - R_2)^2\}$$

$$= \frac{1}{6\ \text{log nat}\left(\frac{N_1}{N_2}\right)}\{(R_3 - R_1)^2 - (R_3 - R_2)^2\}.$$

If the ratio $N_1 : N_2$ does not differ from 1 more than to make the two expressions on the right of this equation agree, considerable reliance may be placed on the correctness of this determination of γ. But if they do not agree, either of the values or an intermediate one can be chosen, with which to compute γ and thence a provisional value of α or β.

6*a.* If γ is positive, we are led to the formula I, and have as a first approximation $\alpha = \sqrt{\gamma}\ 10^{9\cdot637784}$. The given form for the exact equation for α is not very convenient for computation. We therefore introduce the function—

$$\phi(x) = \frac{10^x - 10^{-x}}{2x},$$

for the logarithms of which the first of the following tables is computed, and then have strictly—

$$\frac{t_3 - t_1}{r_1\,(R_3 - R_1)} : \frac{t_3 - t_2}{r_2\,(R_3 - R_2)} = \frac{N_1}{N_2} = \frac{\phi\{\alpha(R_3 - R_1)\}}{\phi\{\alpha(R_3 - R_2)\}},$$

which is well adapted for the successive correction of the values of α. When α is once found, the other four constants are easily computed by the equations—

$$r_1\,10^{\alpha R_1} = a + bt_1, \qquad r_1\,10^{-\alpha R_1} = c + dt_1,$$
$$r_2\,10^{\alpha R_2} = a + bt_2, \qquad r_2\,10^{-\alpha R_2} = c + dt_2.$$

6*b.* If γ comes out negative, form II must be used. Expressing R in degrees, we have as a first approximation for β, the formula $\beta = \sqrt{-\gamma}\ 10^{1\cdot758123}$. In this case the formula for β can be applied in its unaltered form—

$$\frac{t_3 - t_1}{t_3 - t_2} \cdot \frac{r_3}{r_1} = \frac{\sin\{\beta(R_3 - R_1)\}}{\sin\{\beta(R_3 - R_2)\}},$$

but here too a device can be used which is analogous to that mentioned in the preceding case, and with the function $\psi(x) = \frac{\sin x}{x}$, for the logarithms of which a table follows, compute by the formula—

$$\frac{t_3 - t_1}{r_1\,(\mathrm{R}_3 - \mathrm{R}_1)} : \frac{t_3 - t_2}{r_2\,(\mathrm{R}_3 - \mathrm{R}_2)} = \frac{\mathrm{N}_1}{\mathrm{N}_2} = \frac{\psi\,\{\beta(\mathrm{R}_3 - \mathrm{R}_1)\}}{\psi\,\{\beta(\mathrm{R}_3 - \mathrm{R}_2)\}},$$

until the definite value of β is found. The four remaining constants are then computed from—

$$r_1 \cos \beta \mathrm{R}_1 = \mathfrak{a} + \mathfrak{b} t_1, \qquad r_1 \sin \beta \mathrm{R}_1 = \mathfrak{c} + \mathfrak{d} t_1,$$
$$r_2 \cos \beta \mathrm{R}_2 = \mathfrak{a} + \mathfrak{b} t_2, \qquad r_2 \sin \beta \mathrm{R}_2 = \mathfrak{c} + \mathfrak{d} t_2.$$

In both cases of this method with two distances and three position angles, of which two are simultaneous with the distances, there is one exceptional case, which must be guarded against. It is obvious that the solution will be indeterminate it two of the three times of observation lie very near together, but beyond this, as is shown by the equations for α and β, an indeterminate solution is obtained in a case which one is easily tempted to employ, namely, when $\mathrm{R}_3 = \frac{1}{2}(\mathrm{R}_1 + \mathrm{R}_2)$. In this case α or β cannot be determined, but there exists between the given values the relation $r_1\,(t_2 - t_3) + r_2\,(t_1 - t_3) = 0$.

Here too the computations show easily, when there is a prospect of satisfying the observations by the simpler formulæ III, IV or V. But it will depend on the special circumstances, whether in such a case the computation should be made as above with two distances and two positions or by the following methods.

One Distance and Three Positions.

7. To compute the constants in formula III from a single distance and three position angles, B must first be sought as determined by three equations of the form—

$$l + mt = 10^{\frac{\mathrm{R} - \mathrm{A}}{\mathrm{B}}},$$

then—

$$\frac{t_3 - t_2}{t_2 - t_1} = \frac{10^{\frac{\mathrm{R}_3}{\mathrm{B}}} - 10^{\frac{\mathrm{R}_2}{\mathrm{B}}}}{10^{\frac{\mathrm{R}_2}{\mathrm{B}}} - 10^{\frac{\mathrm{R}_1}{\mathrm{B}}}}$$

or—

$$\frac{t_3 - t_2}{\mathrm{R}_3 - \mathrm{R}_2} : \frac{t_2 - t_1}{\mathrm{R}_2 - \mathrm{R}_1} = 10^{\frac{\mathrm{R}_3 - \mathrm{R}_1}{2\mathrm{B}}} \frac{\phi\left(\frac{\mathrm{R}_3 - \mathrm{R}_2}{2\mathrm{B}}\right)}{\phi\left(\frac{\mathrm{R}_2 - \mathrm{R}_1}{2\mathrm{B}}\right)}.$$

As the ϕ's do not here vary much, and as the chief effect of a change of B falls on the purely exponential factor, it is easy to get the right value of B by computing the ϕ's with an assumed value for B and by improving this in the succeeding value for $10^{\frac{\mathrm{R}_3 - \mathrm{R}_1}{2\mathrm{B}}}$. When B is known $l\,10^{\frac{\mathrm{A}}{\mathrm{B}}}$ and $m\,10^{\frac{\mathrm{A}}{\mathrm{B}}}$ are computed from two observations of position by the equations—

$$\left(l\,10^{\frac{\mathrm{A}}{\mathrm{B}}}\right) + \left(m\,10^{\frac{\mathrm{A}}{\mathrm{B}}}\right) t_1 = 10^{\frac{\mathrm{R}_1}{\mathrm{B}}}$$
$$\left(l\,10^{\frac{\mathrm{A}}{\mathrm{B}}}\right) + \left(m\,10^{\frac{\mathrm{A}}{\mathrm{B}}}\right) t_2 = 10^{\frac{\mathrm{R}_2}{\mathrm{B}}}.$$

Finally the observation of distance—

$$r_4^2 = l + mt_4 = 10^{\frac{R_4 - A}{B}}$$

gives the, as yet unknown, constant A and the common unit for l and m.

8. With the same distribution of data to compute by formula IV, it is to be noted that the unknown distances r_1, r_2 and r_3 are respectively proportional to—

$$\frac{R_3 - R_2}{t_3 - t_2}, \quad \frac{R_3 - R_1}{t_3 - t_1} \text{ and } \frac{R_2 - R_1}{t_2 - t_1}.$$

9. Equally easy and quite analogous is the computation of the constants for uniform rectilinear motion by the formula V. Here the distances r_1, r_2 and r_3 are respectively proportional to—

$$\frac{\sin(R_3 - R_2)}{t_3 - t_2}, \quad \frac{\sin(R_3 - R_1)}{t_3 - t_1} \text{ and } \frac{\sin(R_2 - R_1)}{t_2 - t_1}.$$

One Distance and Four Positions.

10. The computation with a single distance and four observations of position ought only to be undertaken when the distance has been small and therefore seldom or never accurately measured. If the motion in position-angle has not been very considerable, it may easily happen that a preparatory graphical rounding-off will be necessary to lessen the influence of the errors on the numerous data of computation of the same kind following each other in close succession.

We found above, in the case of two distances and three positions, an equation of the form—

$$\frac{t_3 - t_2}{t_2 - t_1} \cdot \frac{r_1}{r_3} = \frac{10^{\alpha(R_3 - R_2)} - 10^{\alpha(R_2 - R_3)}}{10^{\alpha(R_2 - R_1)} - 10^{\alpha(R_1 - R_2)}} = \frac{\sin\{\beta(R_3 - R_2)\}}{\sin\{\beta(R_2 - R_1)\}},$$

to this we now add another of the same form—

$$\frac{t_4 - t_3}{t_1 - t_4} \cdot \frac{r_1}{r_3} = \frac{10^{\alpha(R_4 - R_3)} - 10^{\alpha(R_3 - R_4)}}{10^{\alpha(R_1 - R_4)} - 10^{\alpha(R_4 - R_1)}} = \frac{\sin\{\beta(R_4 - R_3)\}}{\sin\{\beta(R_1 - R_4)\}},$$

from these we eliminate the relation between the unknown distauces r_1 and r_3 and thus find an equation in which α or β is the only unknown—

$$\frac{(t_3 - t_2)(t_1 - t_4)}{(t_4 - t_3)(t_2 - t_1)} = \frac{10^{\alpha(R_3 - R_2)} - 10^{\alpha(R_2 - R_3)}}{10^{\alpha(R_4 - R_3)} - 10^{\alpha(R_3 - R_4)}} \cdot \frac{10^{\alpha(R_1 - R_4)} - 10^{\alpha(R_4 - R_1)}}{10^{\alpha(R_2 - R_1)} - 10^{\alpha(R_1 - R_2)}}$$

$$= \frac{\sin\{\beta(R_3 - R_2)\} \sin\{\beta(R_1 - R_4)\}}{\sin\{\beta(R_4 - R_3)\} \sin\{\beta(R_2 - R_1)\}}.$$

Writing here for the anharmonic relations of the times and the positions—

$$\mathsf{T} = \frac{(t_3 - t_2)(t_1 - t_4)}{(t_4 - t_3)(t_2 - t_1)} \text{ and } \mathsf{R} = \frac{(R_3 - R_2)(R_1 - R_4)}{(R_4 - R_3)(R_2 - R_1)},$$

the equation for α or β becomes—

$$\frac{\mathsf{T}}{\mathsf{R}} = \frac{\phi\{\alpha(R_3 - R_2)\} \phi\{\alpha(R_1 - R_4)\}}{\phi\{\alpha(R_4 - R_3)\} \phi\{\alpha(R_2 - R_1)\}} = \frac{\psi\{\beta(R_3 - R_2)\} \psi\{\beta(R_1 - R_4)\}}{\psi\{\beta(R_4 - R_3)\} \psi\{\beta(R_2 - R_1)\}}$$

and we have the developement in series common to both cases—

$$\frac{\mathsf{T}}{\mathsf{R}} = \frac{1 + \frac{\gamma}{12}(\mathsf{A}^2 + \mathsf{B}^2) + \frac{\gamma^2}{360}(\mathsf{A}^4 + \mathsf{A}^2\mathsf{B}^2 + \mathsf{B}^4) + \ldots}{1 + \frac{\gamma}{12}(\mathsf{A}^2 + \mathsf{C}^2) + \frac{\gamma^2}{360}(\mathsf{A}^4 + \mathsf{A}^2\mathsf{C}^2 + \mathsf{C}^4) + \ldots}$$

$$\begin{aligned} \mathsf{A} &= \mathrm{R}_4 - \mathrm{R}_3 + \mathrm{R}_2 - \mathrm{R}_1 \\ \mathsf{B} &= \mathrm{R}_4 + \mathrm{R}_3 - \mathrm{R}_2 - \mathrm{R}_1 \\ \mathsf{C} &= \mathrm{R}_4 - \mathrm{R}_3 - \mathrm{R}_2 + \mathrm{R}_1 \end{aligned}$$

From this follows as an approximate formula—

$$\begin{aligned} \frac{1}{\gamma} + \frac{6\mathsf{A}^2 + \mathsf{B}^2 + \mathsf{C}^2}{120} &= \frac{1}{24}\,\frac{\mathsf{T} + \mathsf{R}}{\mathsf{T} - \mathsf{R}}(\mathsf{B}^2 - \mathsf{C}^2) \\ &= \frac{1}{12}\,\frac{\mathsf{B}^2 - \mathsf{C}^2}{\log \text{ nat } \frac{\mathsf{T}}{\mathsf{R}}}. \end{aligned}$$

From these values of γ, as in the case of two distances and three positions, we may compute preliminary values for α or β, according as γ is positive or negative; these provisional results are then to be tested and amended by the exact equations.

10*a*. When α has finally been determined, we may compute values for at least two of the distances r_1, r_2 or r_3 with the unknown factor (123) in the equations—

$$\begin{aligned} (123)\, r_1 &= \frac{(\mathrm{R}_3 - \mathrm{R}_2)\,\phi\,\{\alpha(\mathrm{R}_3 - \mathrm{R}_2)\}}{t_3 - t_2} \\ (123)\, r_2 &= \frac{(\mathrm{R}_1 - \mathrm{R}_3)\,\phi\,\{\alpha(\mathrm{R}_1 - \mathrm{R}_3)\}}{t_1 - t_3} \\ (123)\, r_3 &= \frac{(\mathrm{R}_2 - \mathrm{R}_1)\,\phi\,\{\alpha(\mathrm{R}_2 - \mathrm{R}_1)\}}{t_2 - t_1} \end{aligned}$$

10*b*. If β had been determined, we have—

$$\begin{aligned} [123]\, r_1 &= \frac{\sin\,\{\beta(\mathrm{R}_3 - \mathrm{R}_2)\}}{t_3 - t_2} \\ [123]\, r_3 &= \frac{\sin\,\{\beta(\mathrm{R}_2 - \mathrm{R}_1)\}}{t_2 - t_1}. \end{aligned}$$

k and Two Distances and Two Positions.

11. Under certain assumptions k, the constant ratio between the area and the time, can be considered as known, and may be used in the determination of the constants in the interpolation formulæ, *e.g.*, after a preliminary graphical equalisation. The methods based thereon are scarcely convenient except in the sole case, that in addition to k, the corresponding values both of distance and position are given for two epochs.

11*a*. According to I we have—

$$\phi\,\{\alpha(\mathrm{R}_2 - \mathrm{R}_1)\} = \frac{2k(t_2 - t_1)}{r_1\, r_2\, (\mathrm{R}_2 - \mathrm{R}_1)}\,\phi\,(0),$$

so that also under the above assumptions α can be computed direct by the tables of function ϕ.

After α is found, the remaining constants are computed as in the case of two distances and three positions.

11*b*. From II we have—

$$\psi\,\{\beta(R_2 - R_1)\} = \frac{2k(t_2 - t_1)}{r_2\, r_1\, (R_2 - R_1)}\,\psi\,(0).$$

β can thus be found quite analogously to α as determined above. From β the remaining constants can also be found as in the case of two distances and three positions.

The method 11 can be used in exact computations of an orbit to interpolate in the ephemeris. Particularly when the excentricity is large, there is a good deal of work connected with solving a large number of Kepler's equations to find the excentric anomalies. A small number of exactly computed positions in the ephemeris will suffice if these are connected pairwise by the interpolation formula computed by method 11.

Table of the Function $\text{Log}\,\frac{10^x - 10^{-x}}{2\,x} = \text{Log}\,\phi(x)$

x		0	1	2	3	4	5	6	7	8	9	Δ
·00	·36	222	222	222	222	222	223	223	223	224	225	0
·01		225	226	227	228	229	230	231	233	234	235	2
·02		237	238	240	242	244	246	248	250	252	254	2
·03		256	258	261	263	266	269	271	274	277	280	3
·04		283	286	289	293	296	299	303	306	310	314	3
·05		317	321	325	329	333	338	342	346	351	355	5
·06		360	364	369	374	379	384	389	394	399	404	5
·07		409	415	420	426	432	437	443	449	455	461	6
·08		467	473	479	486	492	498	505	512	518	525	7
·09		532	539	546	553	560	567	575	582	590	597	8
·10		605	612	620	628	636	644	652	660	668	677	8
·11		685	693	702	711	719	728	737	746	755	764	9
·12		773	782	791	801	810	820	829	839	849	858	10
·13		868	878	888	898	908	919	929	939	950	961	10
·14		971	982	993	*004	*014	*025	*037	*048	*059	*070	12
·15	·37	082	093	105	116	128	140	152	163	175	187	13
·16		200	212	224	236	249	261	274	287	299	312	13
·17		325	338	351	364	377	391	404	417	431	444	14
·18		458	472	485	499	513	527	541	555	570	584	14
·19		598	613	627	642	656	671	686	701	716	731	15
·20		746	761	776	792	807	823	838	854	869	885	16
·21		901	917	933	949	965	981	998	*014	*030	*047	16
·22	·38	063	080	097	114	130	147	164	181	199	216	17
·23		233	250	268	285	303	321	338	356	374	392	18
·24		410	428	446	464	483	501	520	538	557	575	19
·25		594	613	632	651	670	689	708	727	747	766	20
·26		786	805	825	844	864	884	904	924	944	964	20
·27		984	*004	*025	*045	*065	*086	*107	*127	*148	*169	21
·28	·39	190	211	232	253	274	295	316	338	359	381	21
·29		402	424	446	468	489	511	533	555	578	600	22
·30		622	644	667	689	712	735	757	780	803	826	23

Table of the Function Log $\frac{10^x - 10^{-}}{2x}$ = Log $\phi(x)$

x	0	1	2	3	4	5	6	7	8	9	Δ
·30	·39 622	644	667	689	712	735	757	780	803	826	23
·31	849	872	895	918	942	965	988	*012	*035	*059	24
·32	·40 083	106	130	154	178	202	226	250	274	299	24
·33	323	348	372	397	421	446	471	496	521	546	25
·34	571	596	621	646	672	697	722	748	774	799	26
·35	825	851	877	903	929	955	981	*007	*033	*060	26
·36	·41 086	112	139	166	192	219	246	273	300	327	27
·37	354	381	408	435	463	490	518	545	573	600	28
·38	628	656	684	712	740	768	796	824	853	881	28
·39	909	938	966	995	*024	*052	*081	*110	*139	*168	29
·40	·42 197	226	255	285	314	343	373	402	432	461	30
·41	491	521	551	581	611	641	671	701	731	761	31
·42	792	822	853	883	914	945	975	*006	*037	*068	31
·43	·43 099	130	161	192	224	255	286	318	349	381	31
·44	412	444	476	508	540	572	604	636	668	700	32
·45	732	765	797	829	862	894	927	960	993	*025	33
·46	·44 058	091	124	157	190	224	257	290	324	357	34
·47	391	424	458	491	525	559	593	627	661	695	34
·48	729	763	797	832	866	901	935	970	*004	*039	35
·49	·45 074	108	143	178	213	248	283	318	354	389	35
·50	424	460	495	531	566	602	638	673	709	745	36
·51	781	817	853	889	925	961	998	*034	*070	*107	36
·52	·46 143	180	217	253	290	327	364	401	438	475	37
·53	512	549	586	624	661	698	736	773	811	849	37
·54	886	924	962	*000	*038	*076	*114	*152	*190	*228	38
·55	·47 266	305	343	381	420	458	497	536	574	613	39
·56	652	691	730	769	808	847	886	926	965	*004	40
·57	·48 044	083	123	162	202	241	281	321	361	401	40
·58	441	481	521	561	601	641	682	722	762	803	40
·59	843	884	924	965	*006	*047	*087	*128	*169	*210	41
·60	·49 251	293	334	375	416	458	499	540	582	623	42
·61	665	707	748	790	832	874	916	958	*000	*042	42
·62	·50 084	126	168	211	253	295	338	380	423	466	42
·63	508	551	594	637	679	722	765	808	851	895	43
·64	938	981	*024	*068	*111	*155	*198	*242	*285	*329	44
·65	·51 373	416	460	504	548	592	636	680	724	768	45
·66	813	857	901	946	990	*034	*079	*124	*168	*213	45
·67	·52 258	302	347	392	437	482	527	572	617	663	45
·68	708	753	798	844	889	935	980	*026	*071	*117	46
·69	·53 163	209	254	300	346	392	438	484	530	577	46
·70	623	669	715	762	808	855	901	948	994	*041	47
·71	·54 088	135	181	228	275	322	369	416	563	510	48
·72	558	605	652	699	747	794	842	889	937	984	48
·73	·55 032	080	128	175	223	271	319	367	415	463	48
·74	511	560	608	656	704	753	801	850	898	947	48
·75	995	*044	*093	*141	*190	*239	*288	*337	*386	*435	49
·76	·56 484	533	582	631	681	730	779	829	878	927	50

Table of the Function $\text{Log}\,\frac{10^x - 10^{-x}}{2\,x} = \text{Log}\,\phi(x)$

x		0	1	2	3	4	5	6	7	8	9	Δ
·76	·56	484	533	582	631	681	730	779	829	878	927	50
·77		977	*027	*076	*126	*176	*225	*275	*325	*375	*425	50
·78	·57	475	525	575	625	675	725	775	826	876	926	51
·79		977	*027	*078	*128	*179	*229	*280	*331	*382	*432	51
·80	·58	483	534	585	636	687	738	789	840	891	943	51
·81		994	*045	*097	*148	*200	*251	*303	*354	*406	*457	52
·82	·59	509	561	613	665	716	768	820	872	924	976	53
·83	·60	029	081	133	185	237	290	342	395	447	500	52
·84		552	605	657	710	763	815	868	921	974	*027	53
·85	·61	080	133	186	239	292	345	398	452	505	558	54
·86		612	665	718	772	825	879	933	986	*040	*094	54
·87	·62	148	201	255	309	363	417	471	525	579	633	54
·88		687	742	796	850	904	959	*013	*068	*122	*177	54
·89	·63	231	286	340	395	450	505	559	614	669	724	55
·90		779	834	889	944	999	*054	*109	*165	*220	*275	56
·91	·64	331	386	441	497	552	608	663	719	775	830	56
·92		886	942	997	*053	*109	*165	*221	*277	*333	*389	56
·93	·65	445	501	557	614	670	726	782	839	895	951	57
·94	·66	008	064	121	177	234	291	347	404	461	518	56
·95		574	631	688	745	802	859	916	973	*030	*087	58
·96	·67	145	202	259	316	374	431	488	546	603	661	57
·97		718	776	833	891	949	*007	*064	*122	*180	*238	58
·98	·68	296	354	411	469	527	586	644	702	760	818	58
·99		876	935	993	*051	*110	*168	*226	*285	*343	*402	59
1·00	·69	461										

Table of the Function $\text{Log}\,\frac{\sin x}{x} = \text{Log}\,\psi(x).$

x		·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	
0°	8·24	188	188	188	188	187	187	187	187	186	186	0
1		186	185	185	184	183	183	182	181	181	180	1
2		179	178	177	176	175	174	173	172	170	169	1
3		168	167	165	164	162	161	159	158	156	154	2
4		152	151	149	147	145	143	141	139	137	135	2
5		133	130	128	126	123	121	119	116	114	111	3
6		108	106	103	100	097	095	092	089	086	083	3
7		080	077	073	070	067	064	060	057	054	050	3
8		047	043	039	036	032	028	025	021	017	013	4
9		009	005	001	*997	*993	*989	*984	*980	*976	*971	4
10	8·23	967	963	958	953	948	944	940	935	930	925	4
11		921	916	911	906	901	896	891	885	880	875	5
12		879	864	859	853	848	843	837	832	826	820	6
13		814	809	803	797	791	785	779	773	767	761	6

Table of the Function $\text{Log}\,\frac{\sin x}{x} = \text{Log}\,\psi(x)$.

x		·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Δ
13°	8·23	814	809	803	797	791	785	779	773	767	761	6
14		755	748	742	736	730	723	717	710	704	697	7
15		690	684	677	670	664	657	650	643	636	629	7
16		622	615	608	600	593	586	578	571	564	556	7
17		549	541	533	526	518	510	503	495	487	479	8
18		471	463	455	447	439	430	422	414	406	397	8
19		389	380	372	363	355	346	337	329	320	311	9
20		302	293	284	275	266	257	248	239	230	220	9
21		211	202	192	183	173	164	154	144	135	125	10
22		115	105	096	086	076	066	056	046	035	025	10
23		015	005	*994	*984	*974	*963	*953	*942	*932	*921	11
24	8·22	910	899	889	878	867	856	845	834	823	812	11
25		801	790	779	767	756	744	733	722	710	698	11
26		687	675	664	652	640	628	616	604	592	580	12
27		568	556	544	532	520	507	495	483	470	458	13
28		445	433	420	407	395	382	369	356	343	330	13
29		317	304	291	278	265	252	238	225	212	198	13
30		185	171	158	144	131	117	103	089	076	062	14
31		048	034	020	006	*992	*977	*963	*949	*935	*920	14
32	8·21	906	892	877	863	848	833	819	804	789	774	15
33		759	745	730	715	700	684	669	654	639	624	16
34		608	593	577	562	546	531	515	500	484	468	16
35		452	436	421	405	389	373	356	340	324	308	16
36		292	275	259	242	226	209	193	176	160	143	17
37		126	109	092	076	059	042	025	007	*990	*973	17
38	8·20	956	939	921	904	886	869	851	834	816	798	17
39		781	763	745	727	709	691	673	655	637	619	18
40		601	582	564	546	527	509	490	472	453	435	19
41		416	397	378	360	341	322	303	284	264	245	19
42		226	207	188	168	149	129	110	090	071	051	20
43		031	012	*992	*972	*952	*932	*912	*892	*872	*852	20
44	8·19	832	812	791	771	751	730	710	689	669	648	21
45		627	607	586	565	544	523	502	481	460	439	21
46		418	396	375	354	332	311	289	268	246	225	22
47		203	181	159	138	116	094	072	050	028	005	22
48	8·18	983	961	939	916	894	871	849	826	804	781	23
49		758	736	713	690	667	644	621	598	575	552	24
50		528	505	482	458	435	411	388	364	341	317	24
51		293	269	246	222	198	174	150	126	101	077	24
52		053	029	004	*980	*955	*931	*906	*882	*857	*832	25
53	8·17	807	782	758	733	708	682	657	632	607	582	26
54		556	531	506	480	455	429	403	378	352	326	26
55		300	274	248	222	196	170	144	118	091	065	26
56		039	012	*986	*959	*932	*906	*879	*852	*825	*799	27
57	8·16	772	745	718	691	663	636	609	582	554	527	28
58		499	472	444	416	389	361	333	305	277	249	28
59		221	193	165	137	109	080	052	024	*995	*967	

Table of the Function $\text{Log}\,\dfrac{\sin x}{x} = \text{Log}\,\psi(x)$

x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Δ
59°	8·16 221	193	165	137	109	080	052	024	*995	*967	28
60	8·15 938	909	881	852	823	794	765	736	707	678	29
61	649	620	590	561	532	502	473	443	414	384	29
62	354	325	295	265	235	205	175	145	115	084	30
63	054	024	*993	*963	*932	*902	*871	*840	*810	*779	30
64	8·14 748	717	686	655	624	593	562	530	499	468	31
65	436	405	373	342	310	278	246	215	183	151	32
66	119	087	054	022	*990	*958	*925	*893	*860	*828	32
67	8·13 795	762	730	697	664	631	598	565	532	499	33
68	466	432	399	366	332	299	265	232	198	164	33
69	130	096	062	028	*994	*960	*926	*892	*858	*823	34
70	8·12 789	754	720	685	650	616	581	546	511	476	34
71	441	406	371	336	300	265	230	194	159	123	35
72	087	052	016	*980	*944	*908	*872	*836	*800	*764	36
73	8·11 727	691	655	618	582	545	508	472	435	398	37
74	361	324	287	250	213	175	138	101	063	026	37
75	8·10 988	951	913	875	837	799	762	723	685	647	38
76	609	571	532	494	456	417	378	340	301	262	38
77	223	184	145	106	067	028	*989	*949	*910	*871	39
78	8·09 831	791	752	712	672	632	592	552	512	472	40
79	432	392	351	311	270	230	189	149	108	067	40
80	026	*985	*944	*903	*862	*821	*779	*738	*697	*655	41
81	8·08 613	572	530	488	446	405	363	321	278	236	42
82	194	152	109	067	024	*981	*939	*896	*853	*810	42
83	8·07 767	724	681	638	595	551	508	464	421	377	43
84	334	290	246	202	158	114	070	026	*981	*937	43
85	8·06 893	848	803	759	714	669	624	579	534	489	44
86	444	399	354	308	263	217	172	126	080	034	45
87	8·05 989	943	896	850	804	758	711	665	619	572	45
88	525	479	432	385	338	291	244	196	149	102	47
89	054	007	*959	*912	*864	*816	*768	*720	*672	*624	48
90	8·04 576	527	479	431	382	333	285	236	187	138	48
91	089	040	*991	*942	*892	*843	*794	*744	*694	*645	49
92	8·03 595	545	495	445	395	345	294	244	193	143	50
93	092	041	*991	*940	*889	*838	*787	*735	*684	*633	51
94	8·02 581	530	478	426	375	323	271	219	166	114	52
95	062	010	*957	*905	*852	*799	*746	*694	*641	*587	52
96	8·01 534	481	428	374	321	267	214	160	106	052	53
97	8·00 998	944	890	835	781	726	672	617	562	508	54
98	453	398	343	287	232	177	121	066	010	*954	55
99	7·99 898	843	787	730	674	618	562	505	449	392	56
100	335										57

OBSERVATIONS OF COMETS 1881 *b*, *f*, and *g*.

Made at Dun Echt Observatory with the Filar Micrometer of the 15·06-inch Refractor.
(Communicated by the Earl of CRAWFORD and BALCARRES.)

Comet 1881 *b*.

Date—1881	Dun Echt Mean Time	Comet − *		α Comet	δ Comet	Observer
		$\Delta\alpha$	$\Delta\delta$			
June 27,	$11^{h}\ 5^{m}\ 30^{s}$	$-12^{m}\ 14^{s}{\cdot}70$	$+\ 9'\ 55''{\cdot}7$	$5^{h}\ 52^{m}\ 46^{s}{\cdot}57$	$+60^{\circ}\ 11'\ 37''{\cdot}9$	R. C.
July 14,	12 19 34	+18 55·01	+11 54·6	9 38 54·08	+82 2 50·2	,,
,, 18,	12 43 4	− 2 24·23	− 2 12·7	10 48 29·83	+82 16 47·6	,,
,, 26,	10 52 27	+ 3 43·99	−11 8·0	12 23 23·17	+81 18 2·1	,,
Oct. 16,	8 31 49	− 3 37·38	− 3 58·6	18 12 4·79	+70 43 54·9	,,
Nov. 15,	12 34 25	− 0 58·55	− 6 19·8	20 10 35·01	+66 0 25·6	J. G. L.
,, 16,	8 17 38	+ 1 46·06	− 1 37·4	20 13 33·61	+65 51 57·1	R. C.
,, 23,	11 10 18	+ 1 0·26	− 6 13·1	20 38 42·81	+64 37 37·9	J. G. L.
	11 48 14	− 48·66	− 3·1	20 38 47·32	+64 37 18·0	,,
Dec. 19,	10 17 51	+ 31·33	+ 2 56·6	22 0 0·69	+60 19 55·3	R. C.
,, 21,	12 38 9	− 6·64	− 5 51·8	22 5 56·98	+60 1 11·3	J. G. L.

ADOPTED MEAN PLACES OF COMPARISON-STARS FOR 1881·0.

1881	α	Redn.	δ	Redn.	Authorities
June 27,	$6^{h}\ 4^{m}\ 59^{s}{\cdot}52$	$+\ 1^{s}{\cdot}75$	$+60^{\circ}\ 1'\ 46''{\cdot}0$	$-\ 3''{\cdot}8$	A. Ö. 6602
July 18,	10 50 56·49	− 2·43	+82 19 2·9	− 2·6	Carrington 1628
,, 26,	12 19 42·17	− 2·99	+81 29 10·1	0	,, 1845
Oct. 16,	18 15 44·40	− 2·23	+70 47 30·3	+23·2	A. Ö. 18145–6
Nov. 15,	20 11 33·95	− 0·39	+66 6 11·2	+34·2	,, 20303–4
,, 16,	20 11 47·94	− 0·39	+65 53 0·4	+34·1	,, 20313
,, 23,	20 37 42·47	+ 0·08	+64 43 14·8	+36·2	Piazzi XX. 295
	20 39 35·84	+ 0·14	+64 36 44·8	+36·3	Bonn. Beob. VI. + 64°·1460
Dec. 19,	21 59 27·83	+ 1·53	+60 16 18·7	+40·0	A. Ö. 23322
,, 21,	22 6 1·99	+ 1·63	+60 6 22·9	+40·2	Rümker 9940

On July 14 the app. place of the comparison-star was taken from "Mittl. u. scheinb. Oerter . . . . von 539 Sternen":—

July 14, | 1 Hev. Draconis | app. $\alpha = 9^{h}\ 19^{m}\ 59^{s}{\cdot}07$ | app. $\delta = +\ 81^{\circ}\ 50'\ 55''{\cdot}6$

Remarks:

July 14. Observations interrupted by clouds.

Oct. 16. The comet has a trace of a tail on the following side and is very much condensed in the middle.

Nov. 15. Diameter of comet = 1′·4. It is very difficult to set exactly on the central part, which is only slightly brighter than the surrounding area. With the wires fully lit up it is just possible to make out the place of the comet on the wires.

Nov. 16. The comparison-star A. Ö. 20313 is very full red orange.

Nov. 23. Comet faint; about 1′ in diameter.

Nov. 24. 9^{h} G. M. T. The comet inside a quadrilateral of small stars; $\delta = +\ 64^{\circ}\ 27'$. Diameter in $\alpha \begin{Bmatrix} = 12^{s} = 1'{\cdot}3 \\ = 14 \ = 1{\cdot}4 \end{Bmatrix}$ to the extreme limit of the coma. R. C.

Dec. 19. Comet very faint; about 1′ in diameter. Observations difficult owing to the nearness of small stars. Measured from the nf component of a double-star, A. Ö. 23322 = D.M. + 60°·2331. The components are finely coloured full orange and bluish-green.

Dec. 21. Comet very faint.

Comet 1881 *f* (Denning).

Date—1881	Dun Echt Mean Time	Comet – *		α Comet	δ Comet	Observer
		Δα	Δδ			
Oct. 9,	16^{h} 45^{m} 10^{s}	$-\ 0^{m}$ $8^{s}{\cdot}53$	$+\ 8'\ 25''{\cdot}5$	9^{h} 34^{m} $30^{s}{\cdot}25$	$+14^{\circ}\ 48'\ 1''{\cdot}4$	R. C.
,, 10,	16 1 26	+ 2 0·58	+ 9 19·1	9 36 39·39	+14 48 54·8	J. G. L.
,, 12,	14 58 14	+ 6 17·99	+ 10 24·0	9 40 56·85	+14 49 59·5	R. C.
,, 14,	16 26 43		+ 10 5·9		+14 50 19·8	J. G. L.
,, 14,	16 50 25	+ 58·26		9 45 23·86		,,

ADOPTED MEAN PLACES OF COMPARISON-STARS FOR 1881·0.

1881	α	Redn.	δ	Redn.	Authorities
Oct. 9,	9^{h} 34^{m} $36^{s}{\cdot}00$	$+\ 2^{s}{\cdot}78$	$+\ 14^{\circ}\ 39'\ 51''{\cdot}2$	$-15''{\cdot}3$	B. W. IX. 705–6
,, 10,	9 34 36·00	+ 2·81	+ 14 39 51·2	– 15·5	,, ,,
,, 12,	9 34 36·00	+ 2·86	+ 14 39 51·2	– 15·7	,, ,,
,, 14,	9 44 22·75	+ 2·85	+ 14 40 30·3	– 16·4	,, 929

Remarks:

Oct. 9. Diameter about 1′·5. Twilight; wires difficult to see.

Oct. 10. Comet pretty bright; much brighter in the middle. Light clouds frequently passing.

Oct. 12. Comet 1′ in diameter; considerably brighter in the middle.

Oct. 14. The differences of R. A. and Decl. had to be taken separately, on account of the comet being rendered nearly invisible by the double set of wires.

The comet has a small nucleus, apparently just as bright as on Oct. 10.

Comet 1881 *g* (Swift).

Date—1881	Dun Echt Mean Time.	Comet — *		α Comet	δ Comet	Observer
		Δα	Δδ			
Nov. 22,	15^{h} 17^{m} 10^{s}	$-\ 2^{m}$ $4^{s}{\cdot}15$	$-\ 12'\ 34''{\cdot}2$	0^{h} 52^{m} $22^{s}{\cdot}31$	$+\ 67^{\circ}\ 50'\ 23''{\cdot}9$	J. G. L.
,, 23,	8 47 41	– 1 28·72	+ 4 21·5	0 45 49·91	+ 66 48 17·6	R. C.
,, 24,	7 55 19	+ 3 0·02	– 6 23·0	0 38 5·21	+ 65 23 57·5	J. G. L.
,, 25,	11 29 0	– 5 53·04	+ 3 5·8	0 30 2·66	+ 63 42 4·8	,,
,, 28,	8 17 54	– 9·16	+ 12·7	0 14 17·06	+ 59 24 55·8	,,
Dec. 9,	8 41 5	– 6·17	+ 4 47·7	23 45 7·65	+ 44 1 40·5	,,
,, 19,	7 33 54	+ 24·14	+ 5 54·1	23 37 1·53	+ 33 12 45·0	R. C.
,, 25,	11 9 21	– 12·22	+ 3 9·4	23 35 39·67	+ 28 3 12·8	J. G. L.

ADOPTED MEAN PLACES OF COMPARISON-STARS FOR 1881·0.

1881	α	Redn.	δ	Redn.	Authorities
Nov. 22,	0^{h} 54^{m} $18^{s}{\cdot}96$	$+\ 7^{s}{\cdot}50$	$+\ 68^{\circ}\ 2'\ 20''{\cdot}0$	$+\ 38''{\cdot}1$	Bonn. Beob. VI + 67°·89
,, 23,	0 47 11·57	+ 7·06	+ 66 43 17·2	+ 38·9	A. Ö. 844.
,, 24,	0 34 58·65	+ 6·54	+ 65 29 40·5	+ 40·0	U. S. Naval Obs. Cat. No. 327
,, 25,	0 35 49·37	+ 6·33	+ 63 38 19·0	+ 40·0	A. Ö. 630
,, 28,	0 14 20·79	+ 5·43	+ 59 24 1·6	+ 41·5	D. M. + 59°·37, connected with A. Ö. 212
	0 12 56·56		+ 59 25 55·3		A. Ö. 212.
Dec. 9,	23 45 9·59	+ 4·23	+ 43 56 12·3	+ 40·5	B. W. XXIII, 929
,, 19,	23 36 33·50	+ 3·89	+ 33 6 13·5	+ 37·4	,, 781
,, 25,	23 35 48·09	+ 3·80	+ 27 59 28·0	+ 35·4	D.M. + 27°·4598 connected with B. W. XXIII, 776
	23 36 28·66		+ 28 3 40·2		B. W. XXIII, 776

Remarks:

Nov. 22. Observed with 6-inch Simms Refractor, as it was deemed inadvisable to open the dome of the 15·06-inch Refractor on account of the strong gale then blowing.

Nov. 24. The comet is fairly bright; diameter about 1′.

Nov. 25. Comet round; diameter = 1′·5.

Nov. 28. There is a bare possibility that there is an error of a wire-interval in this observation.

Dec. 9. Comet faint; but not very difficult to observe.

Dec. 25. Comet faint.

The observers' initials, R. C. and J. G. L., indicate respectively R. Copeland and J. G. Lohse.

Corrections to the Observations of Comet *c*, 1881.

Vol. I., page 236, line 3 from bottom, column $\Delta\delta$, *for* $- 0' \ 18''{\cdot}9$ *read* $+ 0' \ 18''{\cdot}9$.
line 2 from bottom, columns $\Delta\alpha$, α Comet and δ Comet,
for $- 0^{m} \ 2^{s}{\cdot}22$ $5^{h} \ 56^{m} \ 19^{s}{\cdot}46$ $+ 41^{\circ} \ 55' \ 29''{\cdot}1$
read $- 0^{m} \ 2^{s}{\cdot}18$ $5^{h} \ 56^{m} \ 19^{s}{\cdot}50$ $+ 40^{\circ} \ 55' \ 29''{\cdot}1$

RECENT LITERATURE.

"*Account of Observations of the Transit of Venus,* 1874 *December* 8, *made under the Authority of the British Government, and of the Reduction of the Observations. Edited by Sir George Biddell Airy,* K.C.B. *Printed for H. M. Stationery Office,* 1881." (532 pp. 4to with 17 plates.) This publication is divided into five parts containing respectively the observations made by the expeditions to the Sandwich Islands, Egypt, Rodriguez, Kerguelen Island and New Zealand. In the preface a rapid historical sketch of the preparations and of the reduction of the observations is given. The latter was entrusted to Col. Tupman who in the autumn of 1880, on leaving the country, handed over to Sir George Airy the calculations and introductions for each station and the printed sheets for the observation-districts of the Sandwich Islands and Egypt. The observations and reductions for these two districts had been printed in great detail, but as they could serve as specimens of the operations, Sir George Airy determined to print the remainder in the condensed form suggested in his paper in the *Monthly Notices*, XXXV., p. 288. The reduced observations are found near the end of each part, the equations being given in the shape which admits of combination in the easiest way for the computer's operations when he has decided on the recorded phase of contact of limbs which he thinks best to adopt. In an appendix is given a general account of the photographs with sufficient details on the instruments. Most of the plates serve to illustrate the different phenomena of contact.

ON THE THEORY OF THE SIDEREAL SYSTEM PROPOSED BY MR. MAXWELL HALL.

BY W. E. PLUMMER.

To the 43rd Volume of the Memoirs of the Royal Astronomical Society Mr. Maxwell Hall contributed an elaborate paper on the Sidereal System, in which it is attempted to connect Proper Motion with Annual Parallax, and with the observed radial velocity of the fixed stars, and to determine the form of the solar orbit and the law of force by which it is governed, as well as the position and distance of the centre of the Sidereal Universe. Mr. Hall lays down the proposition, that whatever be the attracting power, if sufficiently great to retain the Sun in its orbit, it must have the same effect on the neighbouring stars, so that the Sun and the nearer stars must move in orbits about the same point: the centre of the sidereal system. Mr. Hall was perfectly aware that the inaccuracy, in some respects, of the data at his command necessarily introduced some insecurity into the final result at which he arrived, but was tempted to publish his theory in its incomplete form, trusting that the interest of the inquiry would induce astronomers to supply data in that particular direction, which could be made available for the purposes of the investigation.

The theory, as given, is susceptible of two tests, the materials for one of which will be supplied very slowly, the other and perhaps the less reliable with considerable rapidity. The one depends upon the determination of the annual parallax of stars, and from the labour involved in the necessary observations, a considerable time must elapse before we are in possession of a sufficient number of trustworthy determinations to be able to apply a decisive and final test in this direction. The other test is supplied by the observation of the radial velocities of the stars, or their motion in the line of sight, and as this element is readily, if not very precisely determined by spectroscopic means, a mass of information, more or less reliable, is already in our hands, and with which the theory may be conveniently compared. The greater portion of this latter class of information, including the continued series made at the Royal Observatory, Greenwich, has been made since Mr. Hall published his investigation, and it will, therefore, not be undesirable to compare these recent results with those derived from his theory, and see whether it will bear the strain which our recently acquired information will put upon it, and further to examine whether the theory be in such a state of completeness that the constants involved may be improved by recognised mathematical methods.

It will not be necessary here to enter into any lengthy description of the methods employed by Mr. Hall, seeing that they are given with such complete detail in the

43rd Vol. of the Memoirs; it will be sufficient to mention the results which he finally adopted, and which involve the constants that have been used in the following computations. Mr. Hall has rejected the hypothesis of a material central Sun, and has supposed the Sun and stars to move in circular orbits round their common centre of gravity, of which the position when viewed from the Sun and referred to the equinox of 1850 is—

R.A. 9° 15′
N.P.D. 63 28,

and consequently the direction cosines, expressed in the notation of Mr. Hall by L, M, and N, are respectively—

[– 9·94601]
[– 9·15806]
[– 9·64992]

The velocity (v) of the Sun in its orbit is expressed by 9·9430, the unit of measurement being the radius of the Earth's orbit, and the value of the parallax of the centre of the system (Π) has been ascertained to be $0''·00665$.

For the apex of the Solar System, referred also to the Equinox of 1850, the position—

R.A. 259° 51′
N.P.D. 56 21

is chosen, which is a mean of the two results given by Sir George Airy in Vol. XXVIII. of the Memoirs, combined with the positions found by Argelander and Mr. Galloway. The direction cosines of this point are—

$$\lambda = [-\ 9·16642]$$
$$\mu = [-\ 9·91350]$$
$$\nu = [+\ 9·74360]$$

By the aid of these constants the annual parallax and the change in distance from the Sun, expressed in terms of the radius of the Earth's orbit, have been computed by a tentative process, and the results are given in the following tables. I originally intended to confine myself to the comparison of the Spectroscopic results with the theory, because Mr. Hall, having in the Monthly Notices for December, 1878, returned to this question, had apparently considered those instances of observed parallax which were either inadvertently omitted in the original memoir or were unknown at the date of its publication. Since this latter date, however, some instances of noticeable parallax have been detected, and, moreover, as in some instances quoted by Mr. Hall, the proper motions have not been derived from the most trustworthy sources, I have incorporated into one table the whole of the comparisons not given in the first paper. In one instance (Lal. 21185) the parallax computed by Mr. Hall seems to be erroneous. At least I am not able to reproduce his figures. On page 132, Vol. XXXIX. of the Monthly Notices, log *dl* is given as 0·18865. From the proper motions cited, combined with the co-ordinates of the star, log *dl* is 0·40545. The resulting parallax is of course greatly different from that given on the same page. For the purposes of comparison I have followed strictly the notation given in the paper in the 'Memoirs,' and have arranged the tables not quite so fully in matter of detail, but strictly on the same lines as those there given.

TABLE I.

No.	Star's Name.	Mag.	R.A. 1850.	N.P.D. 1850.	Proper Motion in R.A.	Proper Motion in N.P.D.	Authority for Proper Motion.
1	η Cassiopeiæ, .	4	10° 1′	32° 59′	+ 1″·98	+ 0·49	G.
2	μ Cassiopeiæ, .	5–6	14 35	35 49	+ 5·79	+ 1·56	G.
3	Castor, . . .	2–1	111 15	57 47	− 0·20	+ 0·079	D.
4	Groomb. 1618, .	6–5	150 33	39 47	− 2·085	+ 0·501	A.
5	Lal. 21185, . .	7–8	163 47	53 2	− 0·675	+ 4·72	A.
6	Lal. 21258, . .	8·5	164 28	45 42	− 6·00	− 0·94	A.
7	Oeltzen 11677, .	9·5	168 0	23 20	− 7·605	− 0·21	...
8	β Centauri, . .	1	208 19	149 39	− 0·152	+ 0·053	N.
9	γ Draconis, . .	2–3	268 16	38 29	+ 0·038	+ 0·037	D.
10	Bradley 3077, .	6	346 31	33 40	+ 3·77	− 0·27	A.
11	85 Pegasi, . .	6	358 35	63 43	+ 0·98	+ 0·985	A.

The magnitudes given in this table have been taken, wherever it has been possible, from the Atlas Coelestis Novus (1872) of Dr. Heis, and the letters in the column headed 'Authority for Proper Motion' have reference to the following catalogues :—

A. Untersuchungen über die Eigenbewegungen von 250 Sternen (Bonn Observations, Vol. VII.)

D. Sterntafeln, enthaltend die Positionen von 46 Fundamental Sternen, von Dr. O. Danckwortt (Vierteljahrsschrift der Astronomischen Gesellschaft, 1881).

G. Greenwich 9 year catalogue of 2263 stars for 1872.

N. Catalogue of 1098 standard clock and zodiacal stars, prepared under the direction of Simon Newcomb.

The proper motion of Oeltzen 11677 is given by Dr. Geelmuyden in Astronomische Nachrichten, No. 2287.

From the data given in Table I. combined with the constants given on page 3, has been computed the parallax (π) given in the following Table—

TABLE II.

No.	Star's Name.	Computed Parallax π.	Observed Parallax.	$d\pi = O - C.$	Authority for observed Parallax.
1	η Cassiopeiæ, . .	0″·065	0″·154	+ 0″·089	O. Struve
2	μ Cassiopeiæ, . .	0·199	0·342	+ 0·143	O. Struve
3	Castor, . . .	fails	0·198	...	Johnson
4	Groomb. 1618, . .	fails	0·334	...	Ball
5	Lal. 21185, . .	1·119	0·501	− 0·618	Winnecke
6	Lal. 21258, . .	fails	0·271	...	Krüger
7	Oeltzen 11677, .	1·219	0·27	− 0·95	Geelmuyden
8	β Centauri, . .	0·002	0·213	+ 0·211	Moesta
9	γ Draconis, . .	0·014	0·092	+ 0·078	...
10	Bradley 3077, .	0·127	0·069	− 0·058	Brünnow
11	85 Pegasi, . .	0·072	0·054	− 0·018	Brünnow

Of the amount of agreement or discordance between the results presented in this latter table, individual judgment must decide. It is not possible to get any result in three cases out of the eleven stars considered, and in three others the want of accordance is considerable and may be thought greater than any possible error in the determination of the parallax warrants. At the same time it must be remembered that most of the stars here given are remarkable for large and irregular proper motion, and, moreover, it is not contended that all stars are necessarily of the same dynamical system as the Sun. Neither are the stars all equally well situated for the application of the method. Castor, one of the decided failures, was excluded by Mr. Hall from his first catalogue (nor is it referred to in the 'Addenda') on account of its unfavourable position, and, as far as I know, the value of the observed parallax given by the late Mr. Johnson has not been confirmed. The Parallax ot β Centauri is also extremely doubtful,* the difference between the determinations of Maclear and Moesta (0″·257) is greater than that between Moesta's value and that computed from the theory. The parallax of Oeltzen 11677, though probably very much smaller than that given by the theory, is regarded by Dr. Geelmuyden as extremely doubtful. On the other hand, the two accordant values which Dr. Winnecke has found for the parallax of Lal. 21185, mark this star as one whose parallax has been best determined, and yet here we have a considerable disagreement in the two results. In the case of Groombridge 1618 we have a complete failure. The comparative proximity of these two stars to the Sun makes the failure in these two cases important.

In the Table given on page 191 of the Memoirs, Vol. XLIII., fifteen stars are compared. (I exclude Lalande 21258, for which an erroneous value is given, owing to the proper motion in North Polar Distance being affected with a positive sign) and of these, three having been used for the derivations of the constants must be excluded, so that there remain but twelve distinct test cases. Of these, four give either a negative or inadmissible result, so that 33 per cent. have failed. This proportion large as it is, does not prevent Mr. Hall prosecuting the inquiry hopefully, and though the proportion of failures in the instances now quoted is something larger it may still be thought that the evidence is inconclusive. Mr. Hall, indeed, with the greater portion of this later evidence before him, contends that "these additions are for the most part satisfactory and confirm the Theory as formerly explained."

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* In a recent number of 'Copernicus' there is, however, reference to an exhaustive paper by Dr. Elkin on the parallax of this star (Vol. I., p. 73).

We have now to consider the evidence supplied by Spectroscopic Observations. This method of observation had been applied to only a few stars at the date of the publication of the Theory, or of the 'Addenda,' and of those few, one-half were considered inadmissible by Mr. Hall as tests of his deductions. Admitting the accuracy of the observations, two objections may be raised to the acceptance of the evidence which the Spectroscope affords. One objection is, that in a great many instances, the observed Proper Motion is so small as to be untrustworthy, both in amount and direction, and that consequently very small and possible alterations would lead to very different results. Instances of this will be found in the computed values of Π, given by Mr. Hall in the Monthly Notices, and those given in this paper that have been derived from slightly different data. The other is that this theory is only intended to apply to those stars which being near to the Sun may fairly be supposed to be brought within the influence of the same immense attractive power (whatever be its character) that retains the Sun in its orbit. The computed parallax is in most of the cases here submitted so small that this latter objection has considerable force. Of the uncertainty introduced by the delicacy and difficulty of the observation it is not necessary to remark; the probable error of the measure frequently bearing no inconsiderable proportion to the total amount of displacement measured.

In the Greenwich "Spectroscopic and Photographic Results, 1880," recently published, is collected the whole of the measures which have been made at the Royal Observatory during the six years 1875–1880. This catalogue, containing the measures of the displacement of certain lines in the spectrum of 90 stars, is the only one I have consulted for the purposes of this comparison. Of these 90 stars, some are excluded as having been already submitted to examination by Mr. Hall, but the greater portion have so small a proper motion that they are useless for this purpose. There remain but 40 stars for further examination, and the necessary data for computing the radial velocity with the deduced results are given in the following tables—

Table III.

No.	Star's Name.	Mag.	R.A. 1850.		N.P.D. 1850.		Proper Motion in R.A.	Proper Motion in N.P.D.	Authority for Proper Motion.
1	α Andromedæ, .	2	0°	9′	61°	44′	+ 0″·1551	+ 0″·1669	N
2	β Cassiopeiæ, .	2	0	18	31	41	+ 0·945	+ 0·19	G
3	α Cassiopeiæ, .	(Var.)	8	0	34	17	+ 0·1017	+ 0·0364	½(N + D)
4	β Andromedæ, .	2–3	15	20	55	11	+ 0·2277	+ 0·1221	N
5	β Arietis, . .	3	26	36	69	56	+ 0·0886	+ 0·1179	N
6	α Arietis, . .	2	29	41	67	15	+ 0·2124	+ 0·142	½(N + D)
7	α Ceti, . .	2–3	43	37	86	30	− 0·0169	+ 0·0929	½(N + D)
8	α Persei, . .	2	48	24	40	41	+ 0·074	+ 0·410	D
9	β Tauri, . .	2	79	12	61	31	+ 0·0886	+ 0·1863	½(N + D)
10	γ Geminorum, .	2–3	97	16	73	29	+ 0·0397	+ 0·0475	N
11	β Canis Minoris, .	3	109	45	81	25	− 0·060	+ 0·05	G
12	Castor, . . .	2–1	111	15	57	47	− 0·200	+ 0·0788	½(N + D)
13	Pollux, . . .	1–2	114	2	61	37	− 0·7001	+ 0·0610	½(N + D)
14	α Hydræ, . .	2	140	3	98	1	− 0·0120	− 0·0325	½(N + D)
15	ε Leonis, . .	3	144	20	65	32	− 0·0648	+ 0·0217	N
16	β Ursæ Maj., . .	2–3	163	11	32	49	+ 0·150	− 0·03	G
17	α Ursæ Maj., . .	2	163	37	27	27	− 0·273	+ 0·0799	½(N + D)
18	θ Leonis, . .	3–4	166	36	73	45	− 0·075	+ 0·06	G
19	χ Ursæ Maj., . .	4–3	174	32	41	23	− 0·195	− 0·02	G
20	δ Ursæ Maj., . .	4–3	182	0	32	8	+ 0·225	+ 0·04	G
21	γ Virginis, . .	3–2	188	31	90	38	− 0·5593	+ 0·0065	N
22	ε Ursæ Maj., . .	2	191	51	33	13	+ 0·195	+ 0·05	G
23	δ Virginis, . .	3	192	1	85	47	− 0·45	+ 0·09	G
24	α Canum Ven., .	3–2	192	15	50	52	− 0·2942	− 0·0483	N
25	ε Virginis, . .	3–2	193	41	78	14	− 0·270	− 0·03	G
26	ζ Ursæ Maj., . .	2	199	28	34	17	+ 0·255	+ 0·04	G
27	η Ursæ Maj., . .	2	205	24	39	56	− 0·1636	+ 0·0243	½(N + D)
28	η Bootis, . .	3	206	53	70	51	− 0·0726	+ 0·3608	N
29	β Ursæ Min., . .	2	222	48	15	14	− 0·104	+ 0·045	½(N + D)
30	ε Serpentis, . .	3–4	235	50	85	4	+ 0·150	− 0·07	G
31	γ Herculis, . .	3–4	243	49	70	29	− 0·06	− 0·06	G
32	ζ Herculis, . .	3–2	248	54	58	7	− 0·510	− 0·45	G
33	α Ophiuchi, . .	2	261	59	77	20	+ 0·1173	+ 0·2207	½(N + D)
34	ξ Draconis, . .	3–4	267	44	33	6	+ 0·21	− 0·06	G
35	γ Draconis, . .	2–3	268	16	38	29	+ 0·038	+ 0·037	D
36	δ Aquilæ, . .	3–4	289	29	87	11	+ 0·2415	− 0·0758	N
37	ε Cygni, . .	3–2	310	2	56	35	+ 0·435	− 0·32	G
38	α Cephei, . .	3–2	318	45	28	3	+ 0·3331	− 0·0413	½(N + D)
39	Fomalhaut, . .	1–2	342	20	120	25	+ 0·3640	+ 0·1660	½(N + D)
40	β Pegasi, . .	(Var.)	344	8	62	44	+ 0·210	− 0·15	G

TABLE IV.

No.	Computed π.	ρ_0.	Computed $\frac{d\rho}{dt}$.	Observed $\frac{d\rho}{dt}$.	O – C.	Probable error of the observation.	No. of observed measures.
1	0″·015	66·57	– 1·81	– 12·1	– 10·29	± 1·20	18
2	0·032	31·40	– 7·01	+ 12·8 ?	+ 19·81		3
3	0·002	553·	– 0·14	+ 7·9 ?	+ 8·04		4
4	0·015	68·20	+ 0·25	+ 4·1	+ 3·85		6
5	0·011	87·72	+ 4·85	– 4·5	– 9·35		8
6	0·017	59·63	+ 6·00	– 5·2	– 11·20		7
7	fails	. .	. .	– 11·7 ?	. .		4
8	0·014	72·43	+ 3·61	+ 7·9	+ 4·29		4
9	0·027	37·73	+ 13·01	– 5·2	– 18·21		7
10	0·008	123·3	– 3·20	+ 3·4	– 6·60		4
11	fails			– 5·2 ?	. .		3
12	fails			+ 9·3	. .	± 1·17	15
13	fails			– 9·3	. .	± 0·93	34
14	fails			+ 13·1	. .		5
15	fails			– 4·5	. .		6
16	0·108	9·22	– 2·41	+ 8·3	+ 10·71	± 1·37	16
17	0·079	12·67	– 11·53	– 7·2	+ 4·33		4
18	0·002	615·	– 2·59	+ 2·8 ?	+ 5·39		3
19	0·006	171·2	+ 2·21	+ 12·1 ?	+ 9·89		2
20	0·065	15·47	– 7·10	+ 3·8	+ 10·90		5
21	0·031	32·71	– 0·40	+ 3·1 ?	+ 3·50		8
22	0·043	23·09	– 7·47	+ 5·5	+ 12·97	± 1·00	10
23	0·021	47·12	– 3·61	+ 19·7 ?	+ 23·31		2
24	0·013	77·69	– 0·51	– 10·3 ?	– 9·79		4
25	0·014	73·94	– 1·68	– 9·6	– 7·92		4
26	0·054	18·62	– 4·62	+ 7·6	+ 12·22		6
27	0·004	244·	– 13·04	– 4·1	+ 8·94		5
28	0·028	35·09	– 12·39	– 11·7 ?	+ 0·69		2
29	0·002	417·	– 18·38	+ 10·0	+ 28·38		6
30	No result.		. .	+ 11·4	. .		4
31	0·003	289·	– 8·66	– 10·3 ?	– 1·64		4
32	0·058	17·16	– 7·94	+ 7·9	+ 15·84		3
33	0·017	57·64	– 7·72	+ 2·8	+ 10·52	± 1·62	19
34	0·037	27·32	– 18·22	+ 3·4 ?	+ 21·62		2
35	0·014	70·69	– 17·56	– 5·9	+ 11·66	± 0·96	10
36	0·109	9·21	– 15·16	+ 6·2 ?	+ 21·36		2
37	0·049	20·59	– 11·21	+ 5·5	+ 16·71		8
38	0·016	64·49	– 13·92	– 21·4 ?	– 7·48		2
39	0·021	48·42	– 2·97	+ 1·7 ?	+ 4·67		5
40	0·062	16·20	– 1·05	+ 6·9	+ 7·95		9

The separate columns of these tables require but little explanation. The remarks that were made on Table I. are equally applicable to Table III. In Table IV., ρ_0 gives the distance of the star in terms of the distance of one whose parallax is one second of arc: a very convenient unit of measurement. The sign minus, in the column $\frac{d\rho}{dt}$, implies that the star is approaching the Sun, and the velocity is expressed in terms of mean radii of the Earth's orbit per annum. To obtain the velocity in miles per second, the quantity usually given in the catalogues, it is necessary to multiply $\frac{d\rho}{dt}$ by 2·9. The probable error of the observations (expressed in the same unit) is given wherever it was determined by the Greenwich Authorities, and the symbol (?) attached sometimes to the observed velocity is that used in the Greenwich volume.

I think the study of the column (O − C) is disappointing, and that it will generally be admitted that no connexion can be traced between the observed and computed quantities. It might have been expected that in 33 cases in which it was possible to compute the velocity, some sixteen would have agreed in direction, since the signs + and − were presumably likely to occur with equal frequency, and if the theory was to receive any support from spectroscopic observations a far greater proportion than 50 per cent. would have been required. As a matter of fact only twelve agree in sign, and these are by no means the twelve that have been most frequently observed and whose measures are consequently the most reliable. The only two which agree, nearly in amount as well as in direction, are γ Herculis and η Bootis, and both these are marked with the symbol of uncertainty.

Before leaving this subject it will be not uninteresting to place in juxtaposition with the parallaxes computed by this method those that Dr. Klinkerfues* computed from the motion in the line of sight obtained by Dr. Huggins in his earlier researches.—

	Klinkerfues.	Hall.
	π	π
β Ursæ Majoris	0″·010	0″·108
δ Ursæ Majoris	0·024	0·065
ϵ Ursæ Majoris	0·030	0·043
ζ Ursæ Majoris	0·043	0·054

But one other point remains to be considered, namely, whether it be practicable to derive any improved values of the constants, which when introduced into the equations shall make the agreement between theory and observation more conspicuous and satisfactory. A preliminary question, however, is whether the theory is in such a state of completeness, and whether the quantities are known with sufficient approximation to permit a uniform and general method being applied, for if the position or distance of the centre, or the velocity in the orbit, should be greatly in error it is by no means certain that least squares, or any analogous method, would necessarily tend to correct their supposed values. The constants used have been determined, it will be remembered, from a discussion of the proper motion and parallax of three stars only, viz., α Centauri, 61 Cygni, and Sirius. It was Mr. Hall's intention to make use of ten stars, and if these ten had been employed, the result would no doubt have been entitled to greater confidence. It is by no means clear why this method was abandoned. Mr. Hall says that the equations founded upon two of the stars (which I understand to be Lalande 21258 and 34 Groombridge) failed to give a consistent result when compared with the other eight. Of these two, one, Lal. 21258, had been introduced through an error, and should not have been included. The other could have been rejected on the same grounds that had already been sufficient to suppress four or five others. Mr. Hall, however, preferred to reject "the generality of the method" and to depend upon the three stars already mentioned, which give three equations, from which corrections to L, V, and Π can and have been obtained. The equation derived is as follows—

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* Nachrichten von der Königl. Gesellschaft der Wissenschaften und der G. A. Universität zu Göttingen, 1873, Juni.

α Centauri . . . $+ 3{\cdot}799\,\Delta L + 0{\cdot}093\,\Delta V + 8{\cdot}261\,\Delta \Pi + 0''{\cdot}107 = 0$
61 Cygni . . . $- 0{\cdot}359\,\Delta L + 0{\cdot}042\,\Delta V - 0{\cdot}656\,\Delta \Pi - 0{\cdot}008 = 0$
Sirius . . . $- 4{\cdot}538\,\Delta L + 0{\cdot}037\,\Delta V + 16{\cdot}447\,\Delta \Pi + 0{\cdot}035 = 0$

The independent term in these equations is derived by subtracting the parallax computed from the proper motion from the observed parallax. A considerable discretion is allowed to the investigator in deciding on the observed parallax. Mr. Hall has accepted, in the three instances, respectively, $0''{\cdot}936$, $0''{\cdot}422$, and $0''{\cdot}210$ perfectly legitimate assumptions, but not necessarily exact. From a discussion of the most reliable determinations of stellar parallax, quoted in Nature (Vol. XV., p. 86), the following values were obtained—

α Centauri . . .	$0''{\cdot}928$
61 Cygni . . .	0·553
Sirius . . .	0·193

quantities which do not differ materially from those quoted by Mr. Hall, but which, if substituted in the equation given above, give the following values of the corrections—

$$\Delta L = + 0{\cdot}0176$$
$$\Delta V = - 2{\cdot}6297$$
$$\Delta \Pi = + 0{\cdot}0097$$

whereas the corrections derived in the first instance were—

$$\Delta L = - 0{\cdot}0144$$
$$\Delta V = - 0{\cdot}0266$$
$$\Delta \Pi = - 0{\cdot}0060$$

The great variation in the derived corrections is sufficient to show the insecurity of the method employed, and does indeed suggest another reason for distrusting the results at which Mr. Hall has arrived. It is evident that very slight alterations, in the adopted parallax, might have given rise to either an impossible cosine or to a negative value of Π. Any attempt to compute minute corrections to this preliminary result seem to be premature and would probably be disappointing.

University Observatory, Oxford,
1881, Feb. 28.

POSITIONS OF NEBULÆ. Series II.

BY C. H. F. PETERS.

In a former communication to this Journal (Vol. I., p. 51), I have given the positions of some nebulæ as read from part of my manuscript charts. Before continuing that series I have thought it well to publish a short list of nebulæ determined occasionally, either by the filar micrometer or by the ring micrometer, or simply after the manner of d'Arrest by putting nebula and comparison star into the centre of the field and reading off the circles. These various classes of determinations are indicated in the fourth column by *fil.*, *r*, or only by a number in brackets, and they may besides be distinguished by the degree of accuracy with which the Right Ascensions and Declinations are given, no significance being, however, claimed for the last figure.

The positions are given for 1860·0, and where not otherwise stated in the notes, their agreement with those of the "General Catalogue" is close. The figures following the authority for the star are the measured differences in α and δ (corrected for refraction, but not for precession, &c.), taken in the sense *nebula minus star.*

Gen.Cat.	R.A.	Decl.	Compar.	Date.	Notes.
440	1^{h} 48^{m} 0^{s}·7	− 9° 42′ 0″	4 r.	1881, Jan. 20,	About this neb. and the two following ones see Astr. Nachr. 2365.
441	1 49 27·9	− 9 44 58	4 r.	1881, Jan. 20,	—
Nova	1 53 53·8	− 9 41 4	2 r.	1865, Feb. 27,	Found independently by Tempel in 1879, see Astr. N. 2347.
„	54·1	40 56	4 r.	1881, Jan. 20,	—
574	2 31 38·5	− 7 16 58	4 r.	1880, Dec. 29,	Met with in sweeping for a comet reported to have been found by Mr. Cooper in England (A.N. 2361).
847	4 17 38	+ 64 32·1	(2)	1860, May 14,	—
1202	5 29 37·8	− 6 48 33	2 r.	1861, Mar. 28,	LL 10396 = W. 5^{h}·609, + 5^{m} 4^{s}·87 + 32″·8.
1923	9 39 31	− 30 31·3	(1)	1859, Apr. 3,	L, 8′, R, central planetary disk.
1949	9 43 54·83	+ 69 43 21·6	1 fil.	1863, June 9,	M. 81. * Arg. + 69°·540 + 2^{m} 19^{s}·02 + 1′ 23″·5.
1950	9 44 4	+ 70 22 18	2 fil.	1863, June 9,	M. 82. A streak 7′ long, 35″ broad, mbp, in the M a knot with a B point. * DM + 70°, 587 + 26^{s}·50, + 2′ 12″·2.
Nova	9 48 12	− 25 2·3	(1)	1859, Apr. 3,	L, pB, irr. oblong.
„	47 54	3·0	(2)	1880, Mar. 2,	—
1981	9 52 3	− 26 14·3	(1)	1859, Apr. 3,	—
2301	10 58 38·9	+ 0 43 9	6 r.	1860, Feb. 21,	* W 10^{h} 1019 = Lam. 3186 = Y 4624 + 1^{m} 47^{s}·82, − 13″·8.
3092	12 28 5	− 3 1·0	(1)	1859, Mar. 2,	—
3198	12 37 57	+ 0 17·7	(1)	1859, Mar. 2,	L.
3214	12 40 0	− 1 58·3	(1)	1859, Mar. 2,	S, a little oblong.
3221	12 41 7	− 2 33·8	(1)	1859, Mar. 2,	vF.
3337	12 52 0	− 14 16·2	(2)	1859, Apr. 27,	B.
3420	13 1 31	− 14 46·2	(1)	1859, Apr. 27,	S, F, a star surrounded by nebulosity.
3465	13 7 55	− 15 38·2	(1)	1859, Apr. 27,	Globular.

Gen. Cat.	R.A.			Decl.			Compar.	Date.	Notes.
5865	17^h	25^m	37^s·7	+ 7°	9′	50″	2 r.	1859, Aug. 20,	S, F. * W. 17^h·494 = Lam. 2552 = Schj. 6282. − 1^m 8^s·62 ; − 6′ 56″·6.
,,		25	37·6		9	59	2 r.	1859, Aug. 21,	* Same. − 1^m 8^s·75 ; − 6′ 47″·8.
4308	17	26	24·30	+ 16	29	33·4	5 fil.	1878, July 8,	* W. 17^h·817, 818. − 59^s·58 ; + 4′ 17″·4. The position of Gen. Cat. 26^m 3^s·6 ; 30′ 57″·6 rests upon a single observation of Sir W. Herschel.
4325	17	37	51	+ 3	15	12	2 r.	1859, Aug. 21,	L, F. *DM + 3°·3481 (undetermined). + 1^m 16^s·4 ; + 1′ 18″.
,,		38	0·2		14	17	1 r.	1859, Sept. 18,	* Lam. 2707. − 4^m 4^s·75 ; + 5 16″·3.
Nova	17	45	54·6	+ 4	12	9	4 r.	1859, Aug. 21,	vS, BM. * Lam. 2763. − 1^m 3^s·16 ; + 6″·4.
4369	17	56	39·7	− 0	17	43	2 r.	1859, Sept. 23,	* Cop. and Börg. 4913 = Lam. 5967 = Santini 975. − 2^m 16^s·12 ; + 9′ 32″·5. [W. 17^h·1240 is more than 2′ too small in Decl.]
4473	19	4	5·5	+ 0	48	8	1 r.	1859, Sept. 25,	Round, large, cometary. * Lam. 6623 and Lam. 6650=Schj. 7194.
,,		4	5·0		48	16	4 r.	1859, Sept. 26,	Beautiful night ; neb. like a cl. of very fine stars. * W. 19^h·40 = Lam. 6639. + 44^s·94 ; + 1′ 54″·2.
,,		4	2·00		48	5·6	2 fil.	1880, Oct. 28,	The neb. faint, in haze near the horizon. * W. 19^h·40. + 41^s·79 ; + 1′ 40″·4.
4487	19	11	36·08	+ 6	17	8·3	10 fil.	1880, Nov. 26,	L 5′, not vF, though pale : Sir John calls it a planetary neb. * Schj. 7252. + 1^m 51^s·59 ; − 3′ 58″·1.
4510	19	36	4·24	− 14	28	52·9	5; 2 fil.	1864, July 15,	Planetary neb. * Lam. 2984 = Schj. 7545. + 55^s·41 ; + 4′ 34″·3.
4851	22	44	22·3	− 6	17	50	1 r.	1860, Nov. 7,	F. * W. 22^h·995 (only 1 comp., neb. north and star south in ring).
4879	22	53	7·9	+ 15	13	46	8 r.	1881, May 23,	* DM. + 15°·2742 (det. from 3 Bessel stars). − 9^s·97 ; + 23″·9.
Nova	22	54	9·2	− 7	6	21	1 r.	1860, Nov. 7,	S, B, close to an 11^m star np. * Y. 10028.
4928	23	10	48·3	− 5	24	53	12 r.	1865, Aug. 13,	* Anon. dct. by comp. with W. 23^h·280 (Radc. obs. and Pulkowa). − 59^s·20 ; − 18″·6.
4956	23	17	47·7	+ 14	30	24	16 r.	1861, Oct. 28,	vF. Mistaken for Encke's comet. − * DM. + 14°·4982 (det. from W. 23^h·312 = R. 11084). + 1^m 30^s·37 ; + 1″·3.
6206	23	24	6·05	+ 20	38	20·4	7 fil.	1865, Oct. 13,	vS. (I fell in with this neb. in hunting for the lost heads of Biela's comet.) * W. 23^h·422. + 3^m 30^s·16 ; + 55″·0. The position of Gen. Cat. is 4^s in Æ and 2′ in Decl. wrong.
4993	23	29	4·50	+ 1	22	50·7	5; 3 fil.	1865, Sept. 19,	* 16 Piscium. − 10^s·13 ; + 3′ 17″·8.

On page 53 of Vol. I. of this Journal I beg to correct:

No. 5629. Decl., instead of 24′·7 read 22′·7.
No. 5630. Æ, instead of 23^s read 13^s.

The note to *Nova* 12^h 16^s 42^s; − 2° 27′·7 should be cancelled, as on 1881, May 5, I have seen and drawn upon my chart also the nebula G. C. 2911.

NOTE SUPPLÉMENTAIRE SUR LES COMÈTES *b* ET *c* (III. et IV.) 1881.

PAR TH. BREDICHIN.

Sur les dessins de M. Tempel (comète *c*), le 26, 29 et 30 août, on voit près de la tête de la comète, à son côté postérieur par rapport au mouvement orbital, un faible appendice, une nébulosité conique.* La direction longitudinale de cet appendice fait un angle très grand (80°) avec la direction de la queue principale et l'axe de sa figure n'est point tangente au rayon vecteur près du noyau. Dans sa lettre à moi, M. Tempel s'exprime ainsi par rapport à cette nébulosité : " Der wunderbare Haken war sehr schwach, aber doch sah ich ihn auch mit Amici I ; die südliche Begrenzung dieses Hakens war schärfer und deutlicher als die nördliche."

A l'aide du dessin et de l'échelle donnée par M. Tempel on trouve l'étendue de cet appendice en crochet égale (le 30 août) à 19′, et le rayon angulaire de la tête dans la direction vers le Soleil égal à 6′ 30′′. La distance entre le noyau et la Terre étant ce jour 0·63475, on trouve pour la longueur de l'appendice 0·0035 et pour le rayon de la tête $\epsilon = 0{\cdot}0012$.

On peut montrer par le calcul, que la nébulosité en question était la partie postérieure du conoïde du troisième type, dilaté et estompé. Cette dilatation provenait de ce que les particules de la queue du premier type, qui était assez dense et claire, en sortant de la sphère d'action du noyau et en rencontrant sur leur chemin les particules du troisième type, beaucoup moins nombreuses et douées des vitesses initiales beaucoup plus faibles,—ont dû leur communiquer quelque partie de la vitesse initiale du I type. Or l'accroissement de la vitesse initiale g mène directement à la dilatation du conoïde de la queue.

L'examen de toutes les comètes que nous avons étudiées, nous a conduit à admettre, que les valeurs de g pour les trois types sont égales approximativement à 0·15 (I), 0·03 (II) et 0·015 (III).

Si l'on calcule à l'aide de la formule $g^2 = 2\ (1 - \mu)\ \epsilon^2 : r^2$ et avec la valeur donnée plus haut de ϵ la vitesse initiale g pour la comète *c*, on obtient pour le I type $g = 0{\cdot}26$, pour le II type $g = 0{\cdot}075$ et pour le III type $g = 0{\cdot}04$. La queue du premier type dans cette comète était très étroite et par conséquent il n'y a aucune raison de supposer dans ses particules l'existence d'une pareille vitesse initiale anomale, qui surpasserait la vitesse ordinaire de la quantité 0·11. Il est beaucoup plus naturel d'admettre,—ce que nous avons fait déjà avec succès pour la comète de 1825, IV (Memorie della società degli spettroscopisti Italiani, 1881, Dicembre),—que l'action des particules du I type sur les particules des types II et III augmente quelque peu les vitesses initiales de ces dernières. Dans cette action je vois la raison de ce que la queue du II type de la comète *c*, dessinée par M. Noble, à son origine même est beaucoup plus large que la racine de la queue principale du I type.

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* Dont la base est le diamètre de la tête.

Ainsi il ne sera point hasardé de supposer, que la vitesse initiale du troisième type 0·015, grâce à l'action des particules douées de la vitesse 0·15, se soit changée en $g = 0{\cdot}04$, que nous avons déduite de l'observation.

C'est avec cette vitesse 0·04 et avec la force $1 - \mu = 0{\cdot}3$ qu'il faut calculer les positions des particules du troisième type. Mais pour avoir affaire avec des dimensions plus considérables et plus commodes à la construction graphique, nous ferons le calcul avec la vitesse $g = 0{\cdot}1$. D'après ces résultats il sera facile de faire des conclusions pour le cas de $g = 0{\cdot}04$. Pour avoir une notion de la position de l'axe de la queue du troisième type, nous ferons aussi le calcul pour les particules qui auraient sorti de la sphère d'action du noyau avec la vitesse initiale $g = 0$.

Le calcul sera fait à l'aide des formules exactes du mouvement hypérbolique (Copernicus ; Vol. I. pp. 107, 108).

Posons que les particules se sont séparées du noyau dans le moment M_1 = août 12·530 (pour ce temps nous avons la première position de la queue principale) et soit l'époque de l'observation M = août 30·280 (le temps de la dernière observation de la queue).

Pour ces moments on a :—

		v	$\log r$	$\log \rho$
1881, Août	12·j530	− 27° 23′ 12″	9·8252200	9·91487
„	30·280	+ 20 25 12	9·8140600	9·80260

Et à l'aide des formules indiquées, on trouve—

$\log m = 0{\cdot}4690700$; $H^2 = 2{\cdot}9909600$; $\beta = 103°\ 41'\ 36''$; $\log K = 8{\cdot}1581304$

Pour les directions des vitesses initiales soient : $G = 0$ (émission en ligne droite vers le Soleil) ; $G = +\ 13°\ 41'\ 36''$ (direction normale à l'orbite du noyau à l'époque M_1) et $G = -\ 13°\ 41'\ 36''$ (direction symétrique avec la précédente par rapport au rayon vecteur). Pour la distance du sommet *o* du paraboloïde (pour $g = 0{\cdot}1$) on a $\epsilon = 0{\cdot}0071$.

Les détails du calcul sont—

	b	*a*	*c*	*d*
g	0·1	0·1	0·1	0·0
G	0° 0′ 0″	+ 13° 41′ 36″	− 13° 41′ 36″	0° 0′ 0″
H_1^2	3·082840	3·000960	3·160070	...
γ	3° 10′ 18″	3° 18′ 33″	2° 51′ 47″	...
β_1	106 51 54	107 0 9	106 33 23	...
m	...	...	...	2·857143
m_1	2·944900	2·866700	3·018690	...
$\log E$	0·275018	0·257415	0·291339	0·260028
V_1	− 25° 43′ 29″	− 26° 18′ 20″	− 24° 56′ 1″	− 21° 10′ 3″
ψ	57 56 10	56 26 21	59 15 3	56 40 10
$\log P$	0·256106	0·243783	0·268259	0·256096
$\log b$	9·849838	9·887350	9·817178	9·892073
J	− 9·5050	− 9·8313	− 9·1239	− 8·0539
t	8·2450	7·9187	8·6261	9·6961
N	0·086566	0·073037	0·101382	0·087984
F	12° 35′ 52″	9° 50′ 3″	13° 35′ 42″	13° 43′ 29″
V	22 33 12	18 12 56	23 40 37	25 9 45
$\log R$	9·818408	9·809493	9·822464	9·833332
(r, R)	− 0° 28′ 17″	+ 3° 17′ 8″	− 0° 48′ 14″	+ 1° 28′ 36″

Les co-ordonnées polaires des particules pour les temps M—

	b	a	c	d
ϕ	$-$ 39° 39′·7	102° 1′·0	$-$ 36° 2′·8	30° 53′·8
Δ	0·00849	0·03779	0·01584	0·03419

Les points calculés *a, b, c* et *d* sont portés sur la planche ci-jointe. La courbe qui passe par les points *a o c* représente le contour de la queue du III type pour la vitesse initiale $g = 0·1$ et pour les limites données plus haut de l'angle G qui contiennent la masse principale d'émission. La ligne *d* indique l'axe de cette queue ; les courbes adjacentes à la ligne *a o e* correspondent aux angles G plus grands et plus petits des G limites, introduits dans le calcul.

Les courbes pointillées *gg, ff, ee* correspondent aux contours des queues pour des différentes vitesses entre 0·0 et 0·1.

Les queues du premier et du second type sont aussi portées sur la planche et y désignées par I et II.

Pour les vitesses initiales entre 0·0 et 0·1 les valeurs calculées de ϵ seront ($1 - \mu$ étant 0·3)—

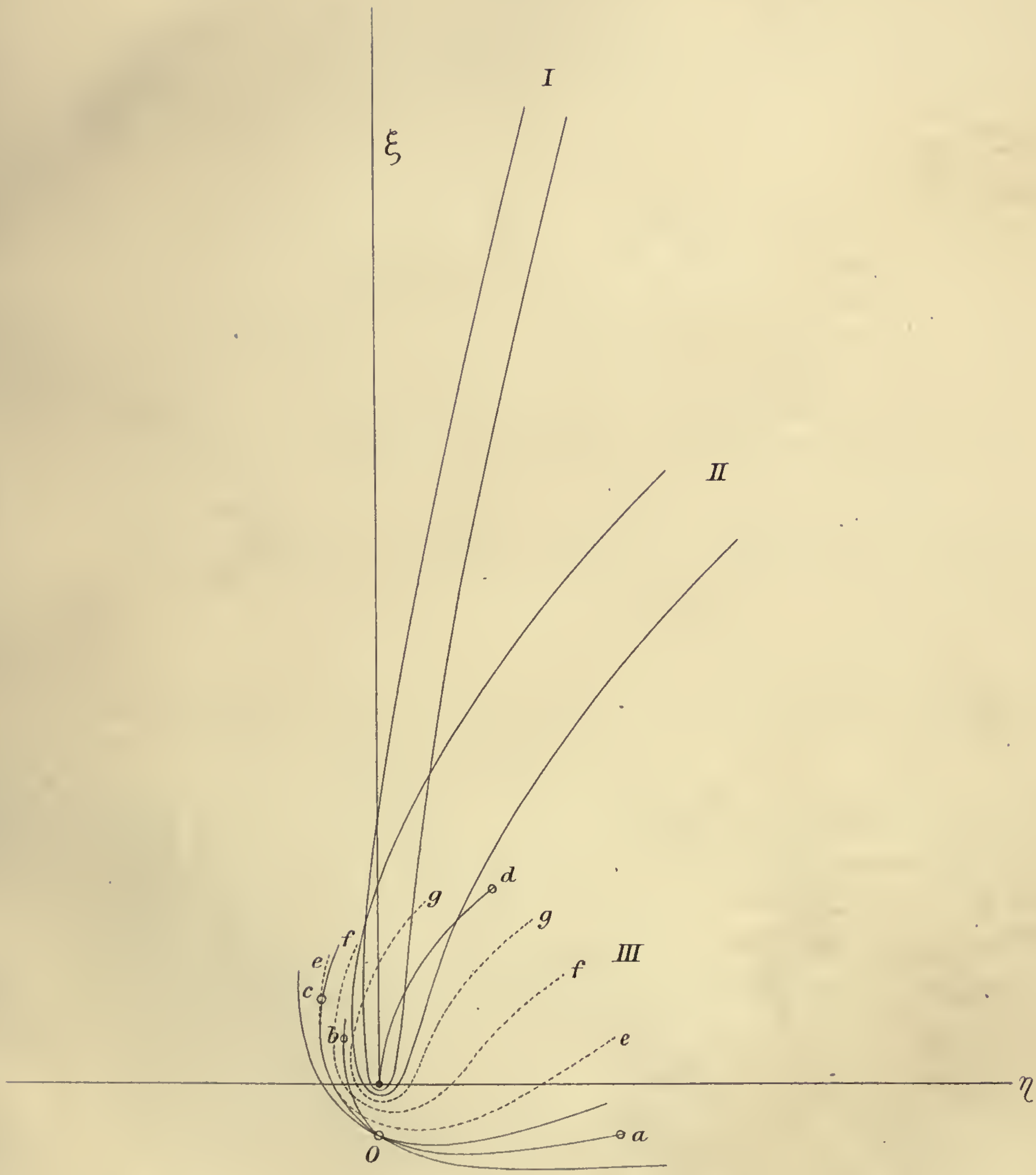
g	ϵ
0·02	0·0003
0·04	0·0012
0·06	0·0025
0·08	0·0045
0·10	0·0071

et il est évident, que pour $g = 0·04$ le contour de la queue se trouvera dans l'intérieur de la courbe *c o a*. L'étendue acquise par la queue ($g = 0·1$) dans l'intervalle entre le 12 et le 30 d'août est égale approximativement à la distance *o a*, c'est à dire à 0·038. Près de la partie antérieure de la tête la queue du III type, même pour la vitesse $g = 0·1$ sort à peine des contours des queues du I et du II type. Mais la faiblesse de lumière ne rend accessibles à la vision que les particules beaucoup moins éloignées de la tête, et en effet la longueur de l'appendice nébuleux d'après M. Tempel n'est que de 0·004. Et quant à la partie antérieure de la queue du III type, surtout pour la vitesse 0·04 (consulter la planche), ses particules seront tout à fait confondues avec les particules des queues du premier et du second type.

Par conséquent, ce sont les particules de la moitié postérieure du conoïde du troisième type, les particules placées entre *o d* et *o a*, qui ont pu devenir accessibles à la vision près de la partie postérieure de la tête, ou près des racines des queues du I et du II type, sous l'aspect d'une nébulosité si faible, que sa forme échappe à l'observation et au calcul.

On peut dire seulement que la partie plus éloignée de l'axe du conoïde doit être relativement plus claire, d'après la règle générale de la distribution de la clarté dans les conoïdes des queues. Et cela s'accorde avec la remarque de M. Tempel que "die südliche Begrenzung dieses Hakens war schärfer und deutlicher als die nördliche."

Comète c 1881. (Note).



Echelle: 1 = 1000 mm.

L'existence des nébulosités élargies du II et du III type autour de la racine de la queue principale parait être aussi la cause d'un certain gonflement de cette racine, qui a dû être remarqué principalement par ceux, qui n'ont pas vu les queues secondaires. Et en général, l'action des particules avec les vitesses g plus grandes sur les particules aux vitesses plus faibles joue un rôle important dans la structure des têtes bulbiformes. Quand la bulbe qui entoure la racine de la queue n'est pas l'atmosphère ordinaire, mais la racine plus large de la queue d'un type inférieur, alors le contour de cette bulbe aura la figure paraboloïdale et il sera ouvert à l'opposite du Soleil.

La comète *b*,—d'après le dessin de M. Tempel, du 27 juin,—nous montre un gonflement sur le contour postérieur de la queue, tout près de la tête. Par l'effet de la perspective cet appendice parait être soudé au bord de la queue du II type ; mais sa longueur peu considérable par rapport à la longueur de la queue principale nous porte à admettre, que c'est aussi une queue du III type. Pour confirmer cette opinion, on n'a qu'à examiner attentivement la disposition et la longueur relatives des axes des queues des trois types, qui sont représentés par les constructions théorétiques des comètes, qu'on voit sur les planches jointes à mon article imprimé dans le "Copernicus" (Vol. I., pp. 99–115), et principalement dans les figures 3 et 4.

La queue secondaire de la comète *b* 1877 doit être envisagée du même point de vue que l'appendice en crochet de la comète *c* 1881.

Conclusion. Mes recherches sur toutes les comètes, dont les observations j'ai pu trouver dans la littérature astronomique et dont le nombre est 36, me mettent en état de calculer d'avance pour chaque grande comète qui paraîtrait dans l'avenir les positions et la figure de ses queues de tous les trois types.

Il est évident, que la quantité rélative des substances caudales de différents types ne peut pas être déterminée d'avance et par conséquent c'est seulement l'observation qui pourra nous montrer et la clarté rélative des types et l'absence possible de tel ou tel d'entre eux.

Mais en tout cas, la position à une époque donnée et la forme générale de celle des queues, qui deviendra accessible à la vision, sera en accord avec sa position et sa figure calculées d'avance.

TH. BREDICHIN.

Moscou, 1882, Février.

OBSERVATIONS DES COMÈTES *b* ET *c* 1881, FAITES À LOUVAIN.

PAR LE DR. F. TERBY.

(Extrait des Mémoires in 8vo de l'Académie Royale de Belgique.)

Comète b.—La queue de cette comète avait une limite presque rectiligne au bord oriental; en l'examinant avec attention on découvrait pourtant, de ce côté, une légère convexité; cette région était aussi la plus brillante, la meilleure définée. Du côté occidentale la limite de la queue était peu accusée, visiblement concave et la lumiére était trés affaiblie. Il en résultait que l'appendice comètaire, à première vue, semblait réduit à un jet rectiligne et brillant correspondant au bord oriental.

La longueur de la queue a eu pour nous son maximum à la fin de Juin. Voici la série des résultats obtenus concernant cet élément—

Date		Heure		Longeur	
Juin	23	11^h	35^m	5°	
	26	11	45	9	30′
	29	12	0	9	30
	30	11	40	10	
Juillet	1	11	15	9	30
	2	11	10	9	30
	4	12	0	7	
	7	10	50	4	
	10	11	0	3	30
	11	11	15	2	
	12	11	30	1	30
	13	13	15	2	45
	15	10	15	1	15
	16	10	50	3	30
	17	10	0	2	20
	18	10	0	2	20

L'influence de la lumière lunaire s'est fait sentir surtout du 7 au 15 Juillet. Les deux dernières observations ont été faites avant le lever de la lune.

Le 23 Juin, 10^h 15^m – 11^h 35^m; noyau de première grandeur, de dimensions plus sensibles que α Aurigæ, mais moins étincelant.

26 Juin, 10^h 30^m – 11^h 45^m. Un grossissement de 180 fois montre une gerbe verticale dans la chevelure; il y a une région obscure derrière le noyau par rapport au Soleil.

29 Juin, 11^h – 12^h; le noyau est très voisin de Radcliffe 1661. Le mouvement de la comète par rapport à cette petite étoile est aussi sensible que celui de l'aiguille des minutes d'une montre. A 11^h 40^m le noyau est muni de deux aigrettes, dont la plus longue, l'orientale, se dirige exactement vers Radcliffe 1661. On voit encore la région sombre derrière le noyau, dans l'axe de la queue.

30 Juin, 10^h 30^m – 11^h 40^m; queue à l'occident de 22 Camelop. (Hev.); elle s'étend un peu plus haut que 24 Camelop., en passant à l'orient de cet astre.

1 Juillet, 10^h – 11^h 15^m; queue jusqu'à 23 Camelop.

2 Juillet, 10^h 15^m – 11^h 10^m; la queue dépasse un peu 24 Camelop. et touche cette étoile par son bord occidental.

4 Juillet, 11^h – 12^h; la queue va jusqu'à B.A.C. 2326, le bord oriental passe par Σ 973, à l'occident de 24 Camelop.

7 Juillet, $10^h\ 35^m$ — $10^h\ 50^m$; la queue se montre jusqu'à B.A.C. 2677, le bord occidental passe par 28 Camelop.

10 Juillet, 10^h – 11^h ; la queue se dirige vers la droite de 29 Camelop., et s'étend jusqu'à la moitié de la distance de cette étoile.

11 Juillet, $10^h\ 15^m$ – $11^h\ 15^m$; la queue va vers 29 Camelop.

12 Juillet, $10^h\ 30^m$ – $11^h\ 30^m$; la queue se dirige à l'occident de 29 Camelop. Le bord occidental est à droite de 1 H. Draconis (P. Camelop., Bode).

13 Juillet, $10^h\ 20^m$ – $13^h\ 15^m$; j'observe la conjonction du noyau avec 1 Draconis ; le noyau est blanc-bleuâtre et l'étoile jaunâtre. A $11^h\ 30^m$ la queue passe à l'orient de B.A.C. 3528 et ne dépasse pas cette étoile.

15 Juillet, 10^h – $10^h\ 15^m$; queue jusqu'à B.A.C. 3528, passant à l'occident.

16 Juillet, $10^h\ 35^m$ – $10^h\ 50^m$; queue dirigée à l'orient de B.A.C. 4281.

17 Juillet, $9^h\ 55^m$ – 10^h ; la queue se dirige vers B.A.C. 4281, jusqu'à la moitié de la distance entre le noyau et cette étoile, et en s'inclinant encore un peu à l'orient.

18 Juillet, $9^h\ 50^m$ – 10^h ; la queue va à l'occident de B.A.C. 4281 ; elle est fortement concave au bord occidental et rectiligne à l'orient.

Comète c.—17 Août, $8^h\ 45^m$ – $10^h\ 30^m$; longueur minime de la queue : 3° ; par moments un prolongement très faible semble s'étendre jusqu'à une distance double, à l'orient de 44 Lyncis. Le bord occidental est le plus tranché, le plus brillant ; le bord oriental a une courbure plus prononcée. Un grossissement de 38 fois montre un noyau très visible, mais celui-ci disparait et se confond complétement avec la chevelure sous le grossissement de 80 et de 120 fois.

Le 19 Août, $10^h\ 30^m$ – $11^h\ 15^m$, longueur minime de la queue : 4° 30′. Le bord occidental est concave et moins tranché que le 17 ; la queue va en s'élargissant dans la direction de 36 Ursæ majoris.

Le 21 Août, $8^h\ 45^m$ — $9^h\ 15^m$; comète fort rapprochée de ψ Ursæ majoris. La lumière blanche-bleuâtre du noyau se distingue de la teinte jaunâtre de ψ autant que la lumière électrique se distingue de la flamme du gaz d'éclairage. La queue se dirige exactement vers une étoile dont les co-ordonnées sont : $\alpha = 11^h\ 9^m{\cdot}9$, $\delta = +\ 50°\ 8'$ (Uranométrie de M. Houzeau). La région la plus brillante se dirige à l'occident de cette étoile, c'est le bord le plus net, il est concave ; le bord oriental de la queue, dirigé à droite de la même étoile, est plus vague, également concave et parait être une seconde queue distincte un peu plus courte. Longueur de la queue = 4° 20′.

22 Août, $9^h\ 50^m$ — $10^h\ 15^m$ et $10^h\ 30^m$. L'apparence d'une queue secondaire s'accentue de plus en plus ; le bord occidental passe par Σ 1561, c'est de ce côté que la queue atteint la plus grande extension (4° 15′) ; le bord oriental passe par Σ 1542 et s'étend seulement jusqua'à 3° 30′ du noyau. Le bord gauche est le plus brillant, presque rectiligne, et le reste de l'appendice cométaire semble formé par des queues

secondaires dont la plus visible est au bord oriental, passant par Σ 1542 ; ces queues sont plus courtes et concaves à droite.

24 Août, $8^h 55^m$ et $10^h 15^m - 10^h 30^m$. La queue rectiligne s'étend, à $10^h 15^m$, exactement dans la direction de *d* Canum Venaticorum ; c'est la partie la plus brillante de la comète. A droite de cette queue rectiligne on voit, à l'aide des jumelles, une nébulosité faible paraissant formée de plusieurs queues dont la plus orientale et la plus longue s'étend, avec une forte courbure, dans la direction de 67 Ursæ majoris, en passant un peu au dessous de Σ 1622 (2 Canum Venaticorum). La queue mesurait environ 5° 25′ dans la direction de *d* Canum et 6° dans celle de 67 Ursæ.

Louvain, le 22 Février, 1882.

THE CHROMOSPHERE IN 1881.

By the Rev. S. J. PERRY.

The measures of the chromosphere made during the past years show that solar activity is still on the increase, but the results for the several months of the year 1881 do not exhibit such a marked difference as those of 1880. The observations, of which a summary is presented in the following table, were generally taken with a dispersion of four prisms of 60°, and Mr. W. McKeon was almost invariably the observer.

	Number of days of Observation.	Mean height of Chromosphere. excluding Prominences.	Mean extent of Prominence arc.	Mean height of Prominences.	Highest Prominence.
January,	3	8″·0	23° 10′ 0″	20″·4	43″·6
February,	1	7·3	32 30 0	19·2	45·7
March,	4	8·0	32 28 8	23·3	94·8
April,	9	8·2	24 32 20	25·3	101·2
May,	12	8·2	37 8 17	21·9	63·2
June,	2	7·7	31 35 0	24·1	58·9
July,	4	8·1	31 47 30	27·7	93·9
August,	1	8·1	35 30 0	21·4	57·6
September,	5	8·3	40 32 6	33·0	82·4
October,	7	8·4	49 53 43	27·1	86·7
November,	0	...	...	...	...
December,	3	8·1	27 15 40	27·3	63·2
Mean for 1881		8·0	33 18 26	24·6	71·9
„ „ 1880		7·9	23 21 23	23·5	67·4

The prominences last year were, in general, considerably higher than those of 1880, and the difference would be much more noticeable in the mean results if the exceptionally high readings on November 4th, 1880, and on April 18th, 1881, were alone excluded.

Stonyhurst Observatory,
Feb. 11th, 1882.

HISTORICAL NOTE CONCERNING THE DISCOVERY OF NEPTUNE.

In the Astronomische Nachrichten No. 580 a letter was inserted from Encke to Schumacher, describing how Le Verrier's hypothetical planet had been found on the 23rd September, 1846, by Dr. Galle, then Assistant at the Berlin Observatory. Neither in this letter nor afterwards did Encke think it necessary to mention that a young student and volunteer at the Observatory, d'Arrest, had taken an important part in finding the planet, nor did Dr. Galle publicly acknowledge the assistance he had received from d'Arrest.

So far as I am aware, this assistance was for the first time alluded to in print in my obituary notice of d'Arrest, published in the Vierteljahrsschrift der Astronomischen Gesellschaft, Vol. XI (February, 1876.) The following year a longer (though incorrect) account was given in the German edition of Gyldén's Outlines of Astronomy.* It is there stated, that when Le Verrier's letter reached the Berlin Observatory, d'Arrest prepared immediately a map of the part of the heavens to be examined, but that he had hardly pointed the telescope to the place when Dr. Galle came, looked in the telescope and—saw the planet.

After the publication of this account Dr. Galle thought it necessary to write a note, in Astr. Nachr. No. 2134, explaining how he came to find Neptune. In this note he states, that he had in 1845 sent Le Verrier a copy of a paper of his (Triduum Roemerianum), which Le Verrier did not acknowledge till Sept. 18, 1846, when he wrote thanking for the paper and at the same time suggesting, that Galle should search for the unknown planet, which would probably be easily distinguished by its disk.† When this letter arrived at the Berlin Observatory on the 23rd of September and Galle announced his intention of looking for the planet, d'Arrest expressed a wish to take part in the search. When the night came, the question arose, what maps were available, and none but Harding's very insufficient Atlas was thought of, until *d'Arrest first suggested* that it might be worth looking among the Berliner Akademische Sternkarten to see whether Hora XXI was among those already finished. On looking among a pile of maps in Encke's hall (Vorzimmer), Dr. Bremiker's map of Hora XXI (already engraved and printed at the beginning of 1846 but not yet distributed) was soon found, and when Dr. Galle compared this with the heavens, he very soon noticed a star of the eighth magnitude, which was not on the map, and the following night the motion of the star proved that it was in fact the planet sought for.

When this note by Dr. Galle appeared (in June 1877) I was absent from home and did not see it for some time. As Dr. Galle has however quite recently again alluded to the subject, by correcting in the Astr. Nachr. No. 2414 an inaccuracy in the German edition of Newcomb's Astronomy (to the effect that Le Verrier had recommended the use of Bremiker's map). I have thought it not inappropriate briefly to describe, how the late Professor d'Arrest told me the story of the discovery of Neptune. As various conflicting accounts seem to have been circulated from time to time (without any having been printed except the one in Prof. Gyldén's book) this does not appear to me altogether useless.

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* Die Grundlehren der Astronomie nach ihrer geschichtlichen Entwickelung dargestellt von Hugo Gyldén (Leipzig, 1877). The Swedish original appeared in December, 1874, but it contains no particulars of the actual finding of Neptune.

† It should be remembered, that the Berlin 9-inch Refractor was at that time considered a very large instrument, to which in fact only the refractors at Pulkova, Harvard College, Munich, Cambridge and Markree were superior.

On the night of June 14, 1874, while observing Coggia's comet together, I reminded Professor d'Arrest, how he had once said in the course of a lecture, that he had been present at the finding of Neptune and that "he might say it would not have been found without him." He then told me (and I wrote it down the next day), how he had suggested the use of Bremiker's map (as first mentioned by Dr. Galle in 1877) and continued: "We then went back to the dome, where there was a kind of desk, at which I placed myself with the map, while Galle, looking through the refractor, described the configurations of the stars he saw. I followed them on the map one by one, until he said: and then there is a star of the 8th magnitude in such and such a position, whereupon I immediately exclaimed: that star is not on the map!"

That d'Arrest thus not only first thought of looking for a map (without which the search might have proceeded as slowly as the operations at the Cambridge Observatory did), but actually *took part in the observation*, does not appear to be without historical interest, and it seems only just to that afterwards distinguished astronomer to say that Neptune was found by Galle and him, observing together.

If Bremiker's map had been distributed early in 1846 instead of letting it wait for another map to be finished to keep it company,* Neptune would probably have been found at the Cambridge Observatory in the beginning of August, six weeks before Le Verrier wrote to Dr. Galle. Professor Adams would thus have had the pleasure of seeing the planet found from his own investigations, which were just as meritorious as those of Le Verrier, to whose work accident alone gave the priority.

J. L. E. DREYER.

Dunsink, February 1882.

OBITUARY NOTICE.

Astronomers will hear with regret of the death of the Rev. THOMAS ROMNEY ROBINSON, D.D., which took place at the Armagh Observatory on February 28. Dr. Robinson was born in Dublin on the 23rd April 1792, became a Fellow of Trinity College in 1814, and held for some years the post of Deputy Professor of Natural Philosophy in the University of Dublin, until he relinquished his Fellowship on obtaining a college living as Rector of Enniskillen. In 1824 he was transferred to another living in the diocese of Armagh with which he combined the post of Astronomer of the Armagh Observatory. This institution had been founded in 1793 by Primate Robinson, but possessed scarcely any instruments until 1827, when a Mural Circle by Jones (described in Vol. IX. of the Memoirs R.A.S.) and a transit instrument were mounted. The results of the work done with these instruments up to 1854 (to a very great extent by Dr. Robinson himself) form the well-known catalogue of 5345 Stars, published in 1859 at the expense of the Government. The Mural Circle was later furnished with a new seven-inch telescope, with which one thousand of Lalande's stars were reobserved in the years 1868–76 and published in the Transactions of the Royal Dublin Society in 1879.

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* After all, the map had to be distributed by itself in November 1846.

The great experience in the construction and management of reflecting telescopes which Dr. Robinson acquired at Birr Castle, where he and Sir James South were frequent guests after 1839, was utilized later on when he became an active member of the "Southern Telescope Committee" and afterwards of the Melbourne Telescope Committee. It was chiefly by his advice that a Cassegrain Reflector was decided on for the Melbourne Observatory.

In addition to his astronomical works (among which should also be mentioned his determination of the Constant of Nutation in 1840), Dr. Robinson has in the course of his long life published a considerable number of important papers on physics and meteorology. Notwithstanding his great age he retained his mental powers to the last, as may be seen by his two papers on the Constants of the Cup Anemometer recently published in the Philosophical Transactions.

RECENT LITERATURE.

"*Conférence internationale du Passage de Vénus. Procésverbaux.*" Paris 1881 (33 pp., large 8vo). The international conference in Paris on the transit of Venus held five sittings between October 5 and 13 under the presidency of M. Dumas. There were no delegates from the United States or Russia, otherwise most European and American governments were represented. Among the points discussed in the first sitting was the application of photography, several speakers alluding to the unfavourable results from 1874, and in particular Professor Förster declared that the German committee had decided not to employ photography during the coming transit, while Mr. Stone in a less pronounced manner stated that the English expeditions would chiefly devote their attention to contacts and not make any serious attempts to adopt photography. Against these speakers Prof. Backhuyzen and M. d'Abbadie maintained that photography might still yield good results, the latter referring to Mr. Todd's paper on the American photographs of 1874, in *Copernicus,* Vol. I.

In the course of the following sittings the various delegates announced which stations their respective Governments intended to occupy. The principal ones were:—

England: Madagascar, Cape of Good Hope (3), Jamaica, Barbadoes, Bermuda, New Zealand, Sydney, Melbourne, Brisbane.

France: Cuba, Martinique, Florida, Mexico, Chili, Argentine Republic (3).

Germany: United States (2), Straits of Magellan, Argentine Republic.

Netherlands: Curaçao or St. Martin.

Denmark: St. Thomas or St. Croix.

Spain: Cuba (2), Porto Rico.

Brazil: Rio Janeiro, Pernambuco, Itapeva (at an elevation of 1800 metres).

At the last meeting a sub-committee which had been appointed to consider the best methods and instruments to be employed, handed in a report. Messrs. Backhuyzen, Stone and Wolf had drawn up a series of instructions for contact observations based on the proposals of the British organizing committee. The unsatisfactory photographic results from 1874 decided the sub-committee not to recommend any extensive use of photography, but double-image micrometer and heliometer observations were specially recommended, while the desirability of arranging stations for the spectroscopic observation of external contact was pointed out. Concerning a proposal to create an international *Bureau des calculs* to discuss both the observations of the coming transit and those of the transit of 1874, it was resolved by a large majority, that the French Government be requested to submit to other Governments interested in the Transits of Venus, the project of convoking an international commission after the return of the expeditions of 1882, to bring about an understanding as to the means of utilizing the observations of 1874 and 1882 and to consider whether it would be desirable to establish for this purpose a temporary international Bureau.

"*Librorum in Bibliotheca Speculæ Pulkovensis contentorum Catalogus systematicus. Pars Secunda ab E. Lindemanno elaborata. Edendum curavit et præfatus est O. Struve. Petropoli* 1880" (XVII. and 640 pp. 8vo). This continuation of the well known Pulkova Catalogue of 1860 is in every respect arranged in the same way as the latter. Besides the titles of books and memoirs published since the end of 1858 it contains also a considerable number of titles of older publications acquired by the Pulkova Library since the latter date. The two catalogues together contain 5472 titles of "Libri majores" (11077 volumes), 168 celestial maps and 23208 titles of papers and pamphlets (of which however the library only has separate copies of 13541). No doubt it must have been very difficult to decide which papers (not actually existing in separate copies) deserved being mentioned in the catalogue, but we cannot refrain from the remark, that certain departments (*e.g.*, orbits of comets) ought to have been more fully represented. But even as a selection of titles, this part of the new Pulkova Catalogue will do as good service as the earlier catalogue has done so long, and even after the publication of the Brussels Bibliography it will remain a most useful guide to astronomical literature.

Sur l'enregistrement des battements de secondes d'une pendule du moyen du microphone. Par M. W. Meyer, astronome-adjoint à l'observatoire de Genève.* Since the end of 1880 Dr. Meyer had used a microphone, the carbon bar of which was on a small shelf nailed to the clock-case for transmitting the sound of the clock-beats from room to room of the Geneva observatory. In a special form devised by him for comparing clocks by coincident beats there is a microphone to each clock but both circuits pass through a single telephone. By regulating the currents the beats may be made exactly alike in character, thus greatly facilitating the comparisons. Dr. Meyer had noticed that during the actual production of the sound the current (as indicated by a galvanometer) is always much enfeebled, and had long had the idea of utilizing this change of intensity to perform some other functions more important than moving the plate of a telephone. Herr von Oppolzer, to whom Dr. Meyer mentioned this idea last spring, suggested a trial with an ordinary relay, and in fact on the very first trial they had the satisfaction of hearing the relay reproduce the beats of the clock as transmitted by the microphone. On the next day the microphonic current was made to actuate the seconds' pen of a chronograph and eventually the *audible* beats of the clock at Geneva were transmitted direct to Vienna. With four or five Meidinger elements (zinc and sulphate of copper with solution of sulphate of magnesium) and a primitive microphone any clock may be made to register its beats on a chronograph, working with a *break* current. In practice, however, it is better to use a relay and a secondary current to work the chronograph, as this permits of adjusting the microphone without altering the phase of the registered seconds. A variety of microphone made by the *Société Genevoise pour la construction d'instruments de physique* in which a plate of carbon is suspended by two slips of metal foil gives particularly good results.

"*Thirty-sixth Annual Report of the Director of Harvard College Observatory. By E. C. Pickering*" (Cambridge, 16 pp. 8vo). Three instruments, the equatoreal, meridian circle and meridian photometer, have been kept actively at work during the past year. Sixty-four eclipses of Jupiter's Satellites were observed photometrically. In this work an observer and two assistants are employed, the latter reading the photometer circle, recording and noting the time; thus the observer need not remove his eye from the eye-piece, and the satellite can be followed much nearer to the point of disappearance than if no assistants were employed. The search for objects having singular spectra has been continued with a four-inch telescope and the survey of the heavens has been completed from 20^h to 12^h RA. The most important result of this search was the spectrum of the star Ll. 13412 (see "Copernicus" Vol. I., p. 140), in which three bright lines were found at 545, 486 and 466 millionths of a millimeter. The second of these is probably the F line of hydrogen, the two others appear to have been seen by Konkoly in the spectrum of Comet *b* (III) 1881 besides the three usual cometary lines. The star L^2 Puppis has a banded spectrum; when observed it was less than 2° above the horizon. A list of about eighty stars with banded spectra was published in the Astr. Nachr. No. 2376. The photometric study of the stars β Persei and DM + 81°·25 (already described in this journal) is next alluded to. The work originally proposed for the meridian photometer of measuring on three nights the light of each of the naked-eye stars visible at Cambridge was completed in August last, but all the stars brighter than the fourth magnitude and a number of others are now being observed over again. All the stars in Heis' Atlas north of the Equator and brighter than the 6th mag. are being measured by the eye or by an opera-glass by three observers who compare them with stars near the pole. The results (together with the magnitudes of Southern Stars as given in the Uranometria Argentina) are to be compared with the photometric results. The Meridian Circle has been in constant use throughout the year for observations of the Sun and Fundamental Stars. In the Report a reference is also made to the part the Observatory has taken in the distribution of news about comets by defraying the cost of the telegrams sent from Boston to Dun Echt. The computations of orbits and ephemerides were generally made by Mr. Chandler, who is at present engaged in another important piece of work, collecting and sorting all references in astronomical literature relating to known or suspected variable stars.

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* Archives des sciences physiques et naturelles, troisième période, T. VI., 15 pp.

THE SPECTRUM OF THE NEBULA IN ORION PHOTOGRAPHED.

Dr. Huggins informs us that on Tuesday, March 7, he succeeded in obtaining a Photograph of the Spectrum of the Great Nebula in Orion with an exposure of 45 minutes. On the plate can be seen, though faintly, the four bright lines which he found some years ago to constitute the visible spectrum of the gaseous nebulæ, namely a strong line λ 5005 (coincident with the less refrangible line of the brightest double line of nitrogen), a second line λ 4957, and two lines coincident with the lines of hydrogen $H\beta$ (F) and $H\gamma$ near G. But on the plate besides these lines there is a strong line in the ultraviolet at λ 3730, or very nearly. This line may be coincident with ζ of the typical photographic spectrum of the brightest white stars, which he showed reasons for believing to be due to hydrogen.

He hopes, by a longer exposure and still more sensitive plates, to ascertain whether any fainter lines are present between $H\gamma$ and 3730, and also more refrangible than the new line.

EINE NEUE BEHANDLUNG DES PROBLEMS DER DREI KÖRPER

Von Dr. H. KOBOLD.

Die Berechnung absoluter Störungen erfordert bekanntlich die Darstellung derselben durch Ausdrücke, die die Zeit selbst oder einfache Funktionen derselben explicite enthalten und die daher durch Substitution der bestimmten Werthe dieser Funktion den Betrag der Störung zu einer beliebigen Zeit direkt ergeben. Es beruht die Auflösung dieser Aufgabe auf der Integration von Differentialgleichungen zweiter Ordnung, deren Aufstellung nicht die geringste Schwierigkeit darbietet. Trotzdem ist es bislang noch nicht gelungen eine Integration dieser Differentialgleichungen ohne eine Reihenentwicklung herbeizuführen. Für die Convergenz dieser letzteren ist nun natürlich die zweckmässigste Wahl der Grössen, nach welchen die Reihen fortschreiten, wesentliches Erforderniss. Solange es sich nur um Anwendungen auf die Bahnen der grösseren Planeten handelt, mögen Reihen, die nach Potenzen der Neigung und Excentricitat der Bahnen fortschreiten, genügen. Die hierauf beruhenden Methoden von Lagrange und Laplace würden dagegen bei einer Anwendung auf die kleinen Planeten mit wenigen Ausnahmen und bei einer solchen auf Kometen stets unausführbar sein. Zur Herbeiführung einer schnelleren Convergenz wandte daher schon Laplace zur Entwicklung der Störungsfunktion Reihen an, die nach den sin. und cos. der Vielfachen der mittleren Anomalien beider Körper fortschreiten. Hansen zeigte dann, dass die Einführung der excentrischen Anomalie an Stelle der mittleren eine noch grössere Convergenz herbeiführt. Schliesslich wird man die Entwicklung nach Vielfachen der wahren Anomalie ausführen müssen, wenn es sich um die allgemeinen Störungen eines Kometen handelt, da der Hülfswinkel der excentrischen Anomalie seine Bedeutung verliert, sobald die grosse Axe der Bahn unendlich gross wird. Es wird die Entwicklung nach der wahren Anomalie aber auch bei der Anwendung auf die kleinen Planeten die Convergenz bedeutend beschleunigen, wovon Verfasser bei der Berechnung der Parthenope Störungen schon Gelegenheit hatte sich zu überzeugen. Eine auf diese Entwicklung gegründete Methode gab derselbe in seiner im August 1880 der philosophischen Facultät zu Göttingen eingereichten Dissertation; der vorliegende Aufsatz bildet einen theilweise umgearbeiteten und erweiterten Auszug aus derselben.

Das Hauptmoment der folgenden Untersuchungen liegt nun weniger in der Entwicklung der Störungsfunktion, als vielmehr in einer Umformung der Differentialgleichungen der gestörten Bewegung. Weil die Entwicklung in unendliche Reihen nun einmal unvermeidlich ist, so muss man wenigstens auf möglichst grosse Convergenz bedacht sein. Die Nenner der einzelnen Ausdrücke bereiten nun dadurch eine grosse Schwierigkeit, dass sie Gliedern, die im allgemeinen nur von ganz verschwindendem Einflusse sind, durch ihre eigene Kleinheit grosse Bedeutung geben können. Diese Nenner enthalten aber neben der gegenseitigen Entfernung beider Körper noch die Radien ihrer Bahnen. Herr Professor Klinkerfues machte den Verfasser, seinen Schüler, nun darauf aufmerksam, dass durch eine Ersetzung der Integration nach der Zeit durch eine solche nach der wahren Anomalie des störenden Körpers die Entwicklung der negativen Potenzen des Radius der störenden Bahn ganz fortfällt, die analoge in Bezug auf die gestörte Bahn aber durch die Entwicklung des gegenseitigen Verhältnisses beider Radien ersetzt wird. Diese Entwicklung ausgeführt nach den cos. und sin. der Anomalien beider Himmelskörper, besitzt aber in allen Fällen eine starke Convergenz. Die Ausführung der angegebenen Vertauschung der unabhängigen Veränderlichen zeigt nun, dass aus den Finalgleichungen die Zeit als explicite Funktion völlig verschwindet. Wir erhalten vielmehr die Störungen dargestellt durch Ausdrücke, die explicite Funktionen der wahren Anomalie des störenden Körpers sind: eine Darstellung die nicht nur der Form, sondern auch dem Sinne nach der bislang üblichen Darstellung absoluter Störungen völlig gleichkommt.

1. *Entwicklung der Differentialgleichungen.*

Wir legen uns im folgenden die Beschränkung auf, nur den Fall eines störenden Körpers und ferner auch nur die Störungen erster Ordnung zu entwickeln. Im Wesentlichen wird dabei ein Gang befolgt werden, der dem von Encke in seinen “Untersuchungen über die allgemeinen Störungen der Planeten” (Berliner Jahrbuch, Bd. 82) eingeschlagenen analog ist.

Es seien x y z die auf drei feste rechtwinklige Axen bezogenen Coordinaten des gestörten Körpers, r sei sein Radius und m seine Masse. Die gleiche Bedeutung mögen x' y' und z', r' und m' hinsichtlich des störenden Körpers haben. Ist ferner—

$$\Delta^2 = (x' - x)^2 + (y' - y)^2 + (z' - z)^2$$

und—

$$\Omega = \frac{m'}{1 + m} \left\{ \frac{1}{\Delta} - \frac{xx' + yy' + zz'}{r'^3} \right\}$$

so besteht bekanntlich für die Bewegung bezüglich der X Axe die Gleichung—

$$(1) \quad . \; . \; . \; . \; . \; . \; . \quad \frac{d^2x}{dt^2} + k^2 (1 + m) \frac{x}{r^3} = k^2 (1 + m) \frac{d\Omega}{dx};$$

in derselben ist dem allgemeinen Gebrauche gemäss mit k die Gauss'sche Constante, das heisst die Intensität der Anziehungskraft für die Einheit der Geschwindigkeit, der Entfernung und der Masse bezeichnet. Die Ersetzung der unabhängig Veränderlichen dt durch die neue dv' kann aufgefasst werden als der Uebergang von der uns eigenthümlichen mittleren Sonnenzeit zu einer anderen dem störenden Körper angemessenen. Es kann durch diese Vertauschung in die obigen Bewegungsgleichung unmöglich eine Aenderung hervorgerufen werden; nur wird natürlich die

Constante k einen anderen Werth k_1 annehmen, da sich ja die Einheit der Geschwindigkeit änderte. Die beiden Constanten sind aber mit einander verbunden durch die Gleichung—

$$k : k_1 = \frac{1}{dt} : \frac{1}{dv'}$$

$$k_1 = k\,\frac{dt}{dv'}.$$

Wir erhalten demnach die Differentialgleichung—

$$(1a) \quad . \; . \; . \; \frac{d^2x}{dv'^2} + k^2\left(\frac{dt}{dv'}\right)^2 (1+m)\frac{x}{r^3} = k^2\,(1+m)\,\frac{d\Omega}{dx}\left(\frac{dt}{dv'}\right)^2.$$

Die Richtigkeit dieser Gleichung lehrt auch die folgende Betrachtung:—Der Differentialquotient $\frac{d^2x}{dt^2}$ ist doch offenbar nichts Anderes als der Grenzfall des Verhältnisses endlicher Grössen $\frac{\Delta^2 x}{\Delta t^2}$. Bekanntlich wächst aber die zweite Differenz dem Quadrate des Argument-Intervalles proportional. Wer getraut sich dann aber, zu leugnen, dass—

$$\frac{\Delta^2 x}{(\Delta t)^2} = \frac{\Delta^2 x}{(\Delta v')^2}\cdot\frac{(\Delta v')^2}{(\Delta t)^2}$$

und also auch im Grenzfalle $\frac{d^2x}{dt^2} = \frac{d^2x}{dv'^2}\cdot\frac{dv'^2}{dt^2}$ ist? Unter Benutzung dieser Relation geht aber Gleichung (1) ebenfalls in (1a) über.

Es ist nun bekanntlich—

$$\frac{dv'}{dt} = \frac{k\sqrt{1+m'}\,\sqrt{p'}}{r'^2}.$$

Demnach erhalten wir unter Hinzufügung der analog gebildeten Gleichungen für die beiden andern Coordinaten—

$$(2) \; . \; . \; . \; . \; . \; . \quad \left\{\begin{aligned} \frac{d^2x}{dv'^2} + \frac{1+m}{1+m'}\,\frac{r'}{p'}\,\frac{r'^3}{r^3}\,x &= \frac{1+m}{1+m'}\,\frac{r'^4}{p'}\,\frac{d\Omega}{dx} \\ \frac{d^2y}{dv'^2} + \frac{1+m}{1+m'}\,\frac{r'}{p'}\,\frac{r'^3}{r^3}\,y &= \frac{1+m}{1+m'}\,\frac{r'^4}{p'}\,\frac{d\Omega}{dy} \\ \frac{d^2z}{dv'^2} + \frac{1+m}{1+m'}\,\frac{r'}{p'}\,\frac{r'^3}{r^3}\,z &= \frac{1+m}{1+m'}\,\frac{r'^4}{p'}\,\frac{d\Omega}{dz}. \end{aligned}\right.$$

Verbinden wir die erste dieser Gleichungen durch Subtraction mit der entsprechenden der ungestörten Bewegung indem wir noch zur Unterscheidung die Coordinaten in der ungestörten Bahn durch $x_0\,y_0\,z_0$ bezeichnen, so ergiebt sich—

$$(3) \; . \; . \; . \; . \quad \frac{d^2\,(x - x_0)}{dv'^2} + \frac{1+m}{1+m'}\,\frac{r'^4}{p'}\left(\frac{x}{r^3} - \frac{x_0}{r_0^3}\right) = \frac{1+m}{1+m'}\,\frac{r'^4}{p'}\,\frac{d\Omega}{dx}.$$

Man setze nun—

$$x - x_0 = \xi \qquad r = r_0 + \delta r$$

dann wird—

$$\begin{aligned} \frac{x}{r^3} - \frac{x_0}{r_0^3} &= \frac{x_0 + \xi}{(r_0 + \delta_4)^3} - \frac{x_0}{r_0^3} \\ &= (x_0 + \xi)\left(\frac{1}{r_0^3} - 3\,\frac{\delta r}{r_0^4} + 6\,\frac{\delta r^2}{r_0^5} - 10\,\frac{\delta r^3}{r_0^6}\,.\,.\,.\,.\right) - \frac{x_0}{r_0^3} \\ &= \frac{\xi}{r_0^3} - \frac{3x_0\delta r}{r_0^4} - \frac{3\xi\delta r}{r_0^4} + \frac{6x_0\delta r^2}{r_0^5} + \frac{6\xi\delta r^2}{r_0^5} + \,.\,.\,.\,. \end{aligned}$$

In dieser Gleichung sind aber nur die beiden ersten Glieder von der ersten Ordnung, alle übrigen von höherer. Wir dürfen daher setzen—

$$\frac{x}{r^3} - \frac{x_0}{r_0^3} = \frac{\xi}{r_0^3} - \frac{3x_0\delta r}{r_0^4}.$$

Mit Hülfe dieser Beziehung verwandelt sich nun aber die Gleichung (3) in

$$(4) \quad . \; . \; . \quad \frac{d^2\xi}{dv'^2} + \frac{1+m}{1+m'}\frac{r'^4}{p'}\frac{\xi}{r_0^3} = \frac{1+m}{1+m'}\frac{r'^4}{p'}\frac{d\Omega}{dx} + \frac{1+m}{1+m'}\frac{r'^4}{p'}\frac{3x_0\delta r}{r_0^4}.$$

Die ungestörte Bewegung ergiebt aber—

$$\frac{1+m}{1+m'}\frac{r'^4}{p'} = -\frac{d^2x_0}{dv'^2}\frac{r_0^3}{x_0}.$$

Durch Einsetzen dieses Werthes folgt aber aus (4)—

$$(5) \quad . \; . \; . \; . \; . \quad x_0\frac{d^2\xi}{dv'^2} - \xi\frac{d^2x_0}{dv'^2} = \frac{1+m}{1+m'}\frac{r'^4}{p'}\left\{x_0\frac{d\Omega}{dx} + \frac{3x_0^2}{r_0^4}\delta r\right\}.$$

Wenden wir zur Integration dieser Gleichung das Encke'sche Verfahren an und setzen—

$$(6) \quad . \; . \; . \; . \; . \; . \quad \left\{\begin{aligned} x_0\mathrm{X} &= \frac{1+m}{1+m'}\frac{r'^4}{p'}\left\{x_0\frac{d\Omega}{dx} + \frac{3x_0^2}{r_0^4}\delta r\right\} \\ y_0\mathrm{Y} &= \frac{1+m}{1+m'}\frac{r'^4}{p'}\left\{y_0\frac{d\Omega}{dy} + \frac{3y_0^2}{r_0^4}\delta r\right\} \\ z_0\mathrm{Z} &= \frac{1+m}{1+m'}\frac{r'^4}{p'}\left\{z_0\frac{d\Omega}{dz} + \frac{3z_0^2}{r_0^4}\delta r\right\} \end{aligned}\right.$$

so erhalten wir als Auflösung der vorgelegten Differentialgleichungen—

$$(7) \; . \; . \quad \left\{\begin{aligned} \xi\left(x_0\frac{dy_0}{dv'} - y_0\frac{dx_0}{dv'}\right) &= y_0\int x_0\mathrm{X}dv' - x_0\int y_0\mathrm{X}dv' + y_0c_1 - x_0c'_1 \\ \eta\left(y_0\frac{dz_0}{dv'} - z_0\frac{dy_0}{dv'}\right) &= z_0\int y_0\mathrm{Y}dv' - y_0\int z_0\mathrm{Y}dv' + z_0c_2 - y_0c'_2 \\ \zeta\left(z_0\frac{dx_0}{dv'} - x_0\frac{dz_0}{dv'}\right) &= x_0\int z_0\mathrm{Z}dv' - z_0\int x_0\mathrm{Z}dv' + x_0c_3 - z_0c'_3. \end{aligned}\right.$$

Es ist nun bekanntlich $x_0\frac{dy_0}{dt} - y_0\frac{dx_0}{dt}$ die Projection der doppelten Flächengeschwindigkeit der ungestörten Bahn auf die xy-Ebene, und hat den constanten Werth $k\sqrt{1+m}\,\sqrt{p_0}$. Die Anwendung dieser Beziehung liefert uns noch die drei Gleichungen—

$$x_0\frac{dy_0}{dv'} - y_0\frac{dx_0}{dv'} = \frac{\sqrt{1+m}}{\sqrt{1+m'}}\frac{\sqrt{p_0}}{\sqrt{p'}}r'^2\cos i_0$$

$$y_0\frac{dz_0}{dv'} - z_0\frac{dy_0}{dv'} = \frac{\sqrt{1+m}}{\sqrt{1+m'}}\frac{\sqrt{p_0}}{\sqrt{p'}}r'^2\sin \text{☊}_0 \sin i_0$$

$$z_0\frac{dx_0}{dv'} - x_0\frac{dz_0}{dv'} = \frac{\sqrt{1+m}}{\sqrt{1+m'}}\frac{\sqrt{p_0}}{\sqrt{p'}}r'^2\cos \text{☊}_0 \sin i_0.$$

In diesen Gleichungen bezeichnet i_0 die Neigung der ungestörten Bahn gegen die xy-Ebene und ☊$_0$ die Länge ihres aufsteigenden Knotens.

Zur Bestimmung der Constanten der Gleichungen (7) benutzen wir das sich gleichzeitig ergebende System—

$$(8)\quad .\ .\left\{\begin{aligned}\frac{d\xi}{dv'}\left(x_0\frac{dy_0}{dv'}-y_0\frac{dx_0}{dv'}\right)&=\frac{dy_0}{dv'}\int x_0\mathrm{X}dv'-\frac{dx_0}{dv'}\int y_0\mathrm{X}dv'+\frac{dy_0}{dv'}c_1-\frac{dx_0}{dv'}c'_1\\ \frac{d\eta}{dv'}\left(y_0\frac{dz_0}{dv'}-z_0\frac{dy_0}{dv'}\right)&=\frac{dz_0}{dv'}\int y_0\mathrm{Y}dv'-\frac{dy_0}{dv'}\int z_0\mathrm{Y}dv'+\frac{dz_0}{dv'}c_2-\frac{dy_0}{dv'}c'_2\\ \frac{d\zeta}{dv'}\left(z_0\frac{dx_0}{dv'}-x_0\frac{dz_0}{dv'}\right)&=\frac{dx_0}{dv'}\int z_0\mathrm{Z}dv'-\frac{dz_0}{dv'}\int x_0\mathrm{Z}dv'+\frac{dx_0}{dv'}c_3-\frac{dz_0}{dv'}c'_3.\end{aligned}\right.$$

Da dann die der Rechnung zu Grunde liegenden osculierenden Elemente die Bewegung des gestörten Körpers zur Zeit $t = 0$ darstellen, so müssen für diesen Zeitmoment die Störungen der Coordinaten ξ, η und ζ sowohl, als auch die der Geschwindigkeiten $\frac{d\xi}{dv'}, \frac{d\eta}{dv'}, \frac{d\zeta}{dv'}$ verschwinden und wir erhalten so aus (7) und (8) sechs Gleichungen zur Bestimmung der sechs Constanten.

Die schliessliche Lösung erfordert jetzt nur noch die Bestimmung der in X, Y, Z enthaltenen Unbekannten δr. Auch diese verlangt nur eine einfache Umformung des von Encke gewählten von Laplace zuerst eingeschlagenen Verfahrens.

Die bekannte Gleichung—

$$\frac{d^2(r_0\delta r)}{dt^2}-\frac{k^2(1+m)}{r^3}(r_0\delta r)=\mathrm{Q}$$

geht für uns über in—

$$(9)\quad \frac{d^2(r_0\delta r)}{dv'^2}+\frac{1+m}{1+m'}\frac{r'^4}{p'}\frac{(r_0\delta r)}{r_0^3}=\mathrm{Q}=\frac{1+m}{1+m'}\left\{2\int\frac{r'^4}{p'}\frac{d\Omega}{dv'}+\frac{r'^4}{p'}\left[x\frac{d\Omega}{dx}+y\frac{d\Omega}{dy}+z\frac{d\Omega}{dz}\right]\right\}.$$

Aus dieser Gleichung ergeben sich aber in ganz derselben Weise wie vorhin die Auflösungen—

$$(10)\quad .\ .\left\{\begin{aligned}(r_0\delta r)\left(x_0\frac{dy_0}{dv'}-y_0\frac{dx_0}{dv'}\right)&=y_0\int x_0\mathrm{Q}dv'-x_0\int y_0\mathrm{Q}dv'+y_0c-x_0c'\\ \frac{d(r_0\delta r)}{dv'}\left(x_0\frac{dy_0}{dv'}-y_0\frac{dx_0}{dv'}\right)&=\frac{dy_0}{dv'}\int x_0\mathrm{Q}dv'-\frac{dx_0}{dv'}\int y_0\mathrm{Q}dv'+\frac{dy_0}{dv'}c-\frac{dx_0}{dv'}c'.\end{aligned}\right.$$

Die Bestimmung der Constanten c und c' geschieht einfach wieder dadurch, dass wir den Werth der Integrale zur Zeit $t = 0$ ermitteln, wo $(r_0\delta r)$ und $\frac{d(r_0\delta r)}{dv'}$ verschwinden.

2. *Entwicklung der störenden Kräfte.*

Der Ausdruck der Störungsfunktion war—

$$\Omega=\frac{m'}{1+m}\left(\frac{1}{\Delta}-\frac{xx'+yy'+zz'}{r'^3}\right).$$

Vermittels dieses Werthes ergeben sich nun, da man der Berechnung der Störungen die aus den osculierenden Elementen folgenden Coordinaten zu Grunde legen muss—

$$(1)\quad \left\{\begin{aligned}\frac{1+m}{1+m'}\frac{r'^4}{p'}\frac{d\Omega}{dx}&=\frac{m'}{1+m'}\frac{r'^4}{p'}\left(\frac{x_0-x'}{\Delta^3}-\frac{x'}{r'^3}\right)=\frac{m'}{1+m'}\frac{r'}{p'}\left[(x_0-x')\left(\frac{r'}{\Delta}\right)^3-x'\right]\\ \frac{1+m}{1+m'}\frac{r'^4}{p'}\frac{d\Omega}{dy}&=\frac{m'}{1+m'}\frac{r'^4}{p'}\left(\frac{y_0-y'}{\Delta^3}-\frac{y'}{r'^3}\right)=\frac{m'}{1+m'}\frac{r'}{p'}\left[(y_0-y')\left(\frac{r'}{\Delta}\right)^3-y'\right]\\ \frac{1+m}{1+m'}\frac{r'^4}{p'}\frac{d\Omega}{dz}&=\frac{m'}{1+m'}\frac{r'^4}{p'}\left(\frac{z_0-z'}{\Delta^3}-\frac{z'}{r'^3}\right)=\frac{m'}{1+m'}\frac{r'}{p'}\left[(z_0-z')\left(\frac{r'}{\Delta}\right)^3-z'\right].\end{aligned}\right.$$

Est ist hier noch zu erwähnen, dass man als Fundamentalebene gewöhnlich die Bahnebene der rein elliptischen Bewegung des gestörten Planeten benutzt, durch welche Annahme $z_0 = 0$ wird.

Wir bedürfen nun noch weiter der Ausdrücke, welche die Grösse Q, wie sie durch Gleichung (9) eingeführt ist, zusammensetzen. Man hat hier nun zunächst—

$$(2) \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad x\frac{d\Omega}{dx} + y\frac{d\Omega}{dy} + z\frac{d\Omega}{dz} = r\frac{d\Omega}{dr}$$

weil $\frac{x}{r}$, $\frac{y}{r}$, und $\frac{z}{r}$ die cos. der Winkel sind, welche der Radius mit den drei Axen bildet. Ferner verwandelt sich $\frac{d\Omega}{dv'}$ in $\left\{\frac{d\Omega}{dr}\frac{dr}{dv} + \frac{d\Omega}{dv}\right\}\frac{dv}{dv'}$. Die bekannte Gleichung—

$$r = \frac{p}{1 + e\cos v}$$

giebt nun—

$$\frac{dr}{dv} = \frac{r^2 e \sin v}{p};$$

folglich wird—

$$(3) \quad . \; . \; . \quad \frac{d\Omega}{dv'} = \left\{ r\frac{d\Omega}{dr}\frac{r}{p} e \sin v + \frac{d\Omega}{dv}\right\}\frac{dv}{dv'}.$$

Bezeichnet man nun noch den cos. des Winkels zwischen den beiden Radien r und r' mit H, wodurch—

$$\Delta^2 = r^2 + r'^2 - 2rr'\mathrm{H}$$

$$\Omega = \frac{m'}{1+m}\left\{\frac{1}{\Delta} - \frac{r}{r'^2}\mathrm{H}\right\}$$

wird, so erhält man noch—

$$(4) \quad . \; . \; . \quad \frac{1+m}{1+m'}\frac{r'^4}{p'}r\frac{d\Omega}{dr} = \frac{m'}{1+m'}\frac{r'}{p'}\left\{\left(\frac{r'}{\Delta}\right)^3 - 1\right\}rr'\mathrm{H} - \frac{m'}{1+m'}\frac{r'}{p'}\left(\frac{r'}{\Delta}\right)^3 r^2$$

$$(5) \quad . \; . \; . \quad \frac{1+m}{1+m'}\frac{r'^4}{p'}\frac{d\Omega}{dv'} = \frac{m'}{1+m'}\left[\frac{rr'}{pp'}\left\{\left(\frac{r'}{\Delta}\right)^3 - 1\right\}rr'\mathrm{H} - \frac{rr'}{pp'}\left(\frac{r'}{\Delta}\right)^3 r^2 e \sin v \right.$$
$$\left. - \frac{r'}{p'}\left\{\left(\frac{r'}{\Delta}\right)^3 - 1\right\}rr'\frac{d\mathrm{H}}{dv}\right]\frac{dv}{dv'}.$$

Wir bedürfen nun um X Y Z aus ihren Definitionsgleichungen zu bilden des Ausdrucks $\frac{1+m}{1+m'}\;\frac{r'^4}{p'}\;\frac{3\delta r}{r^4}$ multipliciert mit dem Quadrate einer der rechtwinkligen Coordinaten. Da diese letzteren nun aber r oder im speciellen Falle die halbe grosse Axe a selbst als Faktor enthalten und die einzelnen Glieder der soeben entwickelten Ausdrücke ebenfalls von rr' oder r^2 abhängen, so brauchen wir zur Bildung von X Y Z in der That keine negativen Potenzen von r zu entwickeln, sondern es reduciert sich alles auf $\frac{a}{r}$ und $\frac{r'}{r}$. Da nun aber auch in keinem der Ausdrücke ein von $\frac{1}{r}$ abhängiges Glied auftritt, so ist damit in der That das in der Einleitung versprochene geleistet.

Wir können nun sofort zur Entwicklung der einzelnen in den Gleichungen (1) (4) und (5) enthaltenen Glieder schreiten. Es sind dieses—

$$\left(\frac{r'}{\Delta}\right)^3, \; \frac{r'}{p'}, \; \frac{r}{p}, \; \frac{r'}{r}, \; \frac{a}{r} \;\; \mathrm{H}, \; \frac{d\mathrm{H}}{dv}$$

und ausserdem die Coordinaten beider Körper. Aus diesen Faktoren setzen sich die der Integration zu unterwerfenden Ausdrücke durch Multiplicationen und Additionen zusammen. Es bezeichne nun I die gegenseitige Neigung der beiden Bahnen, die mit der Ecliptic die Winkel i und i' bilden; ferner seien π und π' die Abstände der beiden Perihelien von der Durchschnittslinie beider Bahnen, für jeden Körper gezählt in der Ebene seiner Bahn, während Π und Π' die Perihellängen im Sinne der gewöhnlichen Bezeichnung darstellen. Man hat dann zunächst in dem von den beiden Bahnebenen und der Ecliptic gebildeten sphärischen Dreiecke—

$$-\cos\tfrac{1}{2}(\text{☊}-\text{☊}')\sin\tfrac{1}{2}(i'-i)=\sin\tfrac{1}{2}\,\mathrm{I}\cos\frac{b+c}{2}$$
$$\sin\tfrac{1}{2}(\text{☊}-\text{☊}')\sin\tfrac{1}{2}(i'+i)=\sin\tfrac{1}{2}\,\mathrm{I}\sin\frac{b+c}{2}$$

also—

$$(6a)\ \ldots\quad \operatorname{tang}\frac{b+c}{2}=-\operatorname{tang}\tfrac{1}{2}(\text{☊}-\text{☊}')\frac{\sin\frac{1}{2}(i'+i)}{\sin\frac{1}{2}(i'-i)}$$

$$(6b)\ \ldots\left\{\begin{aligned}\sin\tfrac{1}{2}\,\mathrm{I}&=-\cos\tfrac{1}{2}(\text{☊}-\text{☊}')\sin\tfrac{1}{2}(i'-i)\sec\tfrac{1}{2}(b+c)\\&=\sin\tfrac{1}{2}(\text{☊}-\text{☊}')\sin\tfrac{1}{2}(i+i')\operatorname{cosec}\tfrac{1}{2}(b+c).\end{aligned}\right.$$

Ferner ist—

$$(6c)\ \ldots\left\{\begin{aligned}\cos\tfrac{1}{2}(b-c)&=\cos\tfrac{1}{2}(\text{☊}-\text{☊}')\cos\tfrac{1}{2}(i'-i)\sec\tfrac{1}{2}\,\mathrm{I};\\\sin\tfrac{1}{2}(b-c)&=-\sin\tfrac{1}{2}(\text{☊}-\text{☊}')\cos\tfrac{1}{2}(i'+i)\sec\tfrac{1}{2}\,\mathrm{I}\end{aligned}\right.$$

und schliesslich—

$$(6d)\ \ldots\quad \pi=\Pi-\text{☊}-b\qquad\qquad \pi'=\Pi'-\text{☊}'=c.$$

In dem zwischen der Durchschnittslinie der beiden Bahnen und den Radien liegenden sphärischen Dreiecke, dessen Seiten $v+\pi$, $v'+\pi'$ und H sind hat man nun die Beziehung—

$$\mathrm{H}=\cos(v+\pi)\cos(v'+\pi')+\sin(v+\pi)\sin(v'+\pi')\cos\mathrm{I}.$$

Ordnen wir diesen Ausdruck nach v', so nimmt er die Gestalt an—

$$\begin{aligned}\mathrm{H}=&\cos v'\,\{\cos(\pi+v)\cos\pi'+\sin(\pi+v)\sin\pi'\cos\mathrm{I}\}\\&+\sin v'\,\{\sin(\pi+v)\cos\pi'\cos\mathrm{I}-\cos(\pi+v)\sin\pi'\}.\end{aligned}$$

Es sei nun—

$$(7)\ \ldots\left\{\begin{aligned}k\sin\mathrm{K}&=\cos\mathrm{I}\sin\pi'\qquad & k_1\sin\mathrm{K}_1&=\sin\pi'\\k\cos\mathrm{K}&=\cos\pi' & k_1\cos\mathrm{K}_1&=\cos\pi'\cos\mathrm{I}\end{aligned}\right.$$

dann wird—

$$\mathrm{H}=\cos v'\,\{\cos(\pi+v)\,k\cos\mathrm{K}+\sin(\pi+v)\,k\sin\mathrm{K}\}+\sin v'\,\{\sin(\pi+v)\,k_1\cos\mathrm{K}_1-\cos(\pi+v)\,k_1\sin\mathrm{K}_1\}$$

$$(8)\ \ldots\quad \mathrm{H}=k\cos v'\cos(\pi-\mathrm{K}+v)+k_1\sin v'\sin(\pi-\mathrm{K}_1+v).$$

In der Gleichung—

$$\left(\frac{\Delta}{r'}\right)^2=\left(\frac{r}{r'}\right)^2+1-2\frac{r}{r'}\mathrm{H}$$

die vermöge der Beziehungen—

$$r=\frac{p}{1+e\cos v}\qquad\qquad r'=\frac{p'}{1+e'\cos v'}$$

übergeht in—

$$\left(\frac{\Delta}{r'}\right)^2=1+\frac{p^2}{p'^2}\left(\frac{1+e'\cos v'}{1+e\cos v}\right)^2-2\frac{p}{p'}\,\frac{1+e'\cos v'}{1+e\cos v}\mathrm{H}\quad\ldots$$

setzen wir nun—

$$(9) \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad \frac{p}{p'\,(1 + e\cos v)} = \mathrm{K}$$

und erhalten dann mit Zuziehung von (8)—

$$\begin{aligned}
\left(\frac{\Delta}{r'}\right)^2 &= 1 + \mathrm{K}^2 + 2\mathrm{K}^2 e' \cos v' + \mathrm{K}^2 e'^2 \cos^2 v' - 2\mathrm{KH} - 2\mathrm{K}e' \cos v'\mathrm{H} \\
&= (1 + \mathrm{K}^2) + \cos v'[2e'\mathrm{K}^2 - 2\mathrm{K}k\cos(\pi - \mathrm{K} + v)] - \sin v'\, 2\mathrm{K}k_1 \sin(\pi - \mathrm{K}_1 + v) \\
&\qquad - \cos^2 v'[2\mathrm{K}e'k\cos(\pi - \mathrm{K} + v) - \mathrm{K}^2 e'^2] - \cos v' \sin v'\, 2\mathrm{K}e'k_1 \sin(\pi - \mathrm{K}_1 + v) \\
&= \left[1 + \mathrm{K}^2 - \mathrm{K}e'k\cos(\pi - \mathrm{K} + v) + \mathrm{K}^2\frac{e'^2}{2}\right] - 2\mathrm{K}\cos v'\,[-e'\mathrm{K} + k\cos(\pi - \mathrm{K} + v)] \\
&\qquad - 2\mathrm{K}k_1 \sin v' \sin(\pi - \mathrm{K}_1 + v) - \mathrm{K}e' \cos 2v' \left[k\cos(\pi - \mathrm{K} + v) - \frac{\mathrm{K}e'}{2}\right] \\
&\qquad - \sin 2v'\, \mathrm{K}e'k_1 \sin(\pi - \mathrm{K}_1 + v).
\end{aligned}$$

Setzt man nun—

$$(10) \quad . \; . \; . \quad \begin{cases}
\alpha_0 = 1 + \mathrm{K}^2 - \mathrm{K}e'k\cos(\pi - \mathrm{K} + v) + \mathrm{K}^2\dfrac{e'^2}{2} = 1 + \mathrm{K}^2 + \alpha_2 \\
\alpha_1 = 2\mathrm{K}[e'\mathrm{K} - k\cos(\pi - \mathrm{K} + v)] \\
\beta_1 = -2\mathrm{K}k_1 \sin(\pi - \mathrm{K}_1 + v) \\
\alpha_2 = \mathrm{K}e'\left[\mathrm{K}\dfrac{e'}{2} - k\cos(\pi - \mathrm{K} + v)\right] \\
\beta_2 = -\mathrm{K}e'k_1 \sin(\pi - \mathrm{K}_1 + v)
\end{cases}$$

so ist—

$$(11) \quad . \; . \; . \quad \left(\frac{\Delta}{r'}\right)^2 = \alpha_0 + \alpha_1 \cos v' + \beta_1 \sin v' + \alpha_2 \cos 2v' + \beta_2 \sin 2v'.$$

Man hat nun die Werthe der Coefficienten α und β für eine Anzahl specieller Werthe von v zu bestimmen und leitet aus diesen durch mechanische Quadratur die allgemeinen Ausdrücke derselben durch Reihen die nach den sin. und cos. der Vielfachen von v fortschreiten ab. Zur Controle jener numerischen Rechnung verwendet man ein zweites analog gebildetes Formelsystem, welches aus den Gleichungen (8), (9), (10) und (11) durch Vertauschung der auf den störenden und den gestörten Körper bezüglichen Buchstaben entsteht.

Der Vollständigkeit halber führen wir dieses System hier an—

$$(12) \quad \begin{cases}
\left(\dfrac{\Delta}{r'}\right)^2 = \left(\dfrac{\Delta}{r}\right)^2 \mathrm{K}^2 (1 + e' \cos v')^2 \\
\mathrm{K}^* = \dfrac{p'}{p(1 + e' \cos v')} \qquad k^* \sin \mathrm{K}^* = \cos \mathrm{I} \sin \pi \qquad k_1^* \sin \mathrm{K}_1^* = \sin \pi \\
\qquad\qquad\qquad\qquad\quad k^* \cos \mathrm{K}^* = \cos \pi \qquad\quad k_1^* \cos \mathrm{K}_1^* = \cos \pi \cos \mathrm{I} \\
\alpha_0^*\; 1 + \mathrm{K}^{*2} - \mathrm{K}^* e k^* \cos(\pi' - \mathrm{K}^* + v') + \mathrm{K}^{*2}\dfrac{e^2}{2} \\
\alpha_1^* = 2\mathrm{K}^*[e\mathrm{K}^* - k^* \cos(\pi' - \mathrm{K}^* + v')] \\
\beta_1^* = -2\mathrm{K}^* k_1^* \sin(\pi' - \mathrm{K}_1^* + v') \\
\alpha_2^* = \mathrm{K}^* e\left[\mathrm{K}^*\dfrac{e}{2} - k^* \cos(\pi' - \mathrm{K}^* + v')\right] \\
\beta_2^* = -\mathrm{K}^* e k_1^* \sin(\pi' - \mathrm{K}_1^* + v') \\
\left(\dfrac{\Delta}{r'}\right)^2 = \alpha_0^* + \alpha_1^* \cos v + \beta_1^* \sin v + \alpha_2^* \cos 2v + \beta_2^* \sin 2v.
\end{cases}$$

Die Gleichung (11) lässt sich nun stets in zwei Linearfaktoren zerlegen, die 5 Unbekannte enthalten müssen. Man setze also—

$$(13) \quad . \; . \; . \quad \left(\frac{\Delta}{r'}\right)^2 = [1 + \mathrm{B}\cos(v' - x)]\,[\mathrm{A}' + \mathrm{B}' \cos(v' - x')]$$

und erhält die Bedingungsgleichungen—

$$(14)\ .\ .\ .\ \begin{cases} \alpha_0 = \mathrm{A}' + \tfrac{1}{2}\mathrm{BB}' \cos(x - x') & \\ \alpha_1 = \mathrm{A}'\mathrm{B} \cos x + \mathrm{B}' \cos x' & \beta_1 = \mathrm{A}'\mathrm{B} \sin x + \mathrm{B}' \sin x' \\ \alpha_2 = \tfrac{1}{2}\mathrm{BB}' \cos(x + x') & \beta_2 = \tfrac{1}{2}\mathrm{BB}' \sin(x + x'). \end{cases}$$

Die beiden letzten dieser Gleichungen ergeben sofort—

$$(15)\ .\ .\ .\ .\ .\ .\ .\ .\ \begin{cases} \operatorname{tang}(x + x') = \dfrac{\beta_2}{\alpha_2} \\ \mathrm{BB}' = 2\sqrt{\alpha_2^2 + \beta_2^2}. \end{cases}$$

Aus den für α_1 und β_1 aufgestellten Bedingungen erhält man ferner—

$$\begin{aligned} \alpha_1^2 + \beta_1^2 &= (\mathrm{A}'\mathrm{B})^2 + \mathrm{B}'^2 + 2\mathrm{A}'\mathrm{BB}' \cos(x - x') \\ (a)\ .\ .\ .\ .\ .\ \qquad &= (\mathrm{A}'\mathrm{B})^2 + \mathrm{B}'^2 + 4\mathrm{A}'(\alpha_0 - \mathrm{A}'). \end{aligned}$$

Dieselben Gleichungen ergeben aber auch—

$$\begin{aligned} 2\alpha_1\beta_1 &= \mathrm{A}'^2\mathrm{B}^2 \sin 2x + \mathrm{B}'^2 \sin 2x' + 2\mathrm{A}'\mathrm{BB}' \sin (x + x') \\ &= \mathrm{A}'^2\mathrm{B}^2 \sin 2x + \mathrm{B}'^2 \sin 2x' + 4\mathrm{A}'\beta_2 \end{aligned}$$

$$(b)\ .\ \ 2\alpha_1\beta_1 = (\mathrm{A}'^2\mathrm{B}^2 + \mathrm{B}'^2) \sin(x + x') \cos(x - x') + \mathrm{A}'^2\mathrm{B}^2 - \mathrm{B}'^2) \cos(x + x') \sin(x - x') + 4\mathrm{A}'\beta$$

$$\begin{aligned} \alpha_1^2 - \beta_1^2 &= \mathrm{A}'^2\mathrm{B}^2 \cos 2x + \mathrm{B}'^2 \cos 2x' + 2\mathrm{A}'\mathrm{BB}' \cos(x + x') \\ &= \mathrm{A}'^2\mathrm{B}^2 \cos 2x + \mathrm{B}'^2 \cos 2x' + 4\mathrm{A}'\alpha_2 \end{aligned}$$

$$(c)\ \ \alpha_1^2 - \beta_1^2 = (\mathrm{A}'^2\mathrm{B}^2 + \mathrm{B}'^2) \cos(x + x') \cos(x - x') - (\mathrm{A}'^2\mathrm{B}^2 - \mathrm{B}'^2) \sin(x + x') \sin(x - x') + 4\mathrm{A}'\alpha_2.$$

Durch Combination der Gleichungen (*b*) und (*c*) kann man dann eine Gleichung zwischen A′B und B′ herleiten. Wir setzen der Einfachheit halber—

$$(d)\ .\ .\ .\ .\ .\ \begin{cases} 2\alpha_1\beta_1 \sin (x + x') + (\alpha_1^2 - \beta_1^2) \cos(x + x') = \mathrm{D} \\ \beta_2 \sin(x + x') + \alpha_2 \cos(x + x') = \mathrm{E} \end{cases}$$

und erhalten—

$$\begin{aligned} \mathrm{D} &= (\mathrm{A}'^2\mathrm{B}^2 + \mathrm{B}'^2) \cos(x - x') + 4\mathrm{A}'\mathrm{E} \\ &= 2(\mathrm{A}'^2\mathrm{B}^2 + \mathrm{B}'^2)\, \frac{\alpha_0 - \mathrm{A}'}{\mathrm{BB}'} + 4\mathrm{A}'\mathrm{E} \\ (e)\ .\ .\ .\ .\ .\ .\ .\ \qquad &= (\mathrm{A}'^2\mathrm{B}^2 + \mathrm{B}'^2)\, \frac{\alpha_0 - \mathrm{A}'}{\sqrt{\alpha_2^2 + \beta_2^2}} + 4\mathrm{A}'\mathrm{E}. \end{aligned}$$

Aus dieser Gleichung und der Gleichung (*a*) aber folgt—

$$(\alpha_1^2 + \beta_1^2)\, \frac{\alpha_0 - \mathrm{A}'}{\sqrt{\alpha_2^2 + \beta_2^2}} - \mathrm{D} = 4\mathrm{A}'\, \frac{(\alpha_0 - \mathrm{A}')^2}{\sqrt{\alpha_2^2 + \beta_2^2}} - 4\mathrm{A}'\mathrm{E}.$$

oder—

$$(f)\ \ \mathrm{D} - \frac{\alpha_1^2 + \beta_1^2}{\sqrt{\alpha_2^2 + \beta_2^2}}\, \alpha_0 + \mathrm{A}' \left\{ \frac{\alpha_1^2 + \beta_1^2 + 4\alpha_0^2}{\sqrt{\alpha_2^2 + \beta_2^2}} - 4\mathrm{E} \right\} - 8\mathrm{A}'^2 \frac{\alpha_0}{\sqrt{\alpha_2^2 + \beta_2^2}} + 4\mathrm{A}'^3 \frac{1}{\sqrt{\alpha_2^2 + \beta_2^2}} = 0.$$

Führt man nun die Substitution $\mathrm{A}' = \alpha_0 + \mathrm{A}$ ein, so vereinfacht sich diese Gleichung zu—

$$\left\{ \frac{2\alpha_1\beta_1\beta_2 + (\alpha_1^2 - \beta_1^2)\alpha_2}{4\alpha_0} - \beta_2^2 - \alpha_2^2 \right\} + \mathrm{A}\, \frac{\alpha_1^2 + \beta_1^2 - 4(\alpha_2^2 + \beta_2^2)}{4\alpha_0} + \mathrm{A}^2 + \mathrm{A}^3 = 0.$$

Setzt man ferner—

$$\alpha_1 = m \cos \psi \qquad \beta_1 = m \sin \psi$$

und benutzt die letzten der Gleichungen (14), so wird—

$$(g)\ .\ .\ \left\{ \frac{m^2\, \tfrac{1}{2}\mathrm{BB}' \cos(2\psi - x - x')}{4\alpha_0} - \left(\frac{\mathrm{BB}'}{2}\right)^2 \right\} + \mathrm{A}\, \frac{m^2 - (\mathrm{BB}')^2}{4\alpha_0} + \mathrm{A}^2 + \mathrm{A}^3 = 0.$$

Sei nun noch—

$$\mathrm{M} = \frac{m^2\, \tfrac{1}{2}\mathrm{BB}' \cos(2\psi - x - x')}{4\alpha_0} - \left(\frac{\mathrm{BB}'}{2}\right)^2 \qquad \mathrm{N} = \frac{m^2 - (\mathrm{BB}')^2}{4\alpha_0}$$

also—

$$M + NA + A^2 + A^3 = 0$$

so setze man—

$$(h) \quad . \; . \; . \; . \; . \; . \; . \; . \quad A_0 = -\frac{N}{2} \pm \sqrt{\left(\frac{N}{2}\right)^2 - M}$$

und—

$$\phi(A) = NA + A^2 + A^3.$$

Man hat dann—

$$\phi(A + \Delta A) = \phi(A) + \Delta A\,(N + 2A_0 + 3A_0^2) + \; . \; . \; .$$

Bestimmt man also der Reihe nach—

$$(i) \quad . \; . \; . \; . \; . \; \left\{ \begin{array}{ll} \Delta A_0 = \dfrac{-M - \phi(A_0)}{N + 2A_0 + 3A_0^2} & \qquad A_1 = A_0 + \Delta A_0 \\ \Delta A_1 = \dfrac{-M - \phi(A_1)}{N + 2A_1 + 3A_1^2} & \qquad A_2 = A_1 + \Delta A_1, \text{ etc.,} \end{array} \right.$$

so wird man schliesslich finden—

$$A_m = A_{m+1} = A_{m+2}$$

und hat dann—

$$\begin{aligned} A &= A_m \\ A' &= \alpha_0 + A_m. \end{aligned}$$

Die Gleichungen (h) und (i) convergieren sehr schnell; bei der Anwendung, die ich von diesen Formeln auf die Störungen der Parthenope machte, brauchte ich nie weiter als bis zur zweiten Näherung zu gehen, um die mit siebenstelligen Logarithmen zu erreichende Genauigkeit innehalten zu können.

Hat man so A' gefunden, so verfährt man zur Berechung der übrigen Coefficienten folgendermassen.

Der sich zunächst darbietende Weg mit Hülfe von A' aus der ersten der Gleichungen (14) $x - x'$ zu berechnen und aus der Verbindung desselben mit dem bekannten Werthe von $(x + x')$ die unbekannten Winkel selbst zu bestimmen ist desshalb ungenau, weil in Folge der Kleinheit von α_2 und β_2 auch das Produkt BB' immer nur sehr klein ist; da aber auch $\alpha_0 - A'$ nur unbedeutend ist, so erhält man $\cos(x - x')$ als Quotienten zweier kleiner Zahlen, also nur sehr ungenau. Man setze daher—

$$A'B = \mu \cos w \qquad\qquad B' = \mu \sin w$$

und hat dann sogleich—

$$(k) \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad BB' = \frac{\mu^2}{2A'} \sin 2w.$$

Ferner wird—

$$\alpha_1 = \mu \cos w \cos x + \mu \sin w \cos x' \qquad\qquad \beta_1 = \mu \cos w \sin x + \mu \sin w \sin x'$$

$$(\alpha_1 + \beta_1)^2 = \mu^2 + \mu^2 \cos^2 w \sin 2x + \mu^2 \sin^2 w \sin 2x' + \mu^2 \sin 2w \,\{\cos(x - x') + \sin(x + x')\}$$

$$(\alpha_1 - \beta_1)^2 = \mu^2 - \mu^2 \cos^2 w \sin 2x - \mu^2 \sin^2 w \sin 2x' + \mu^2 \sin 2w \,\{\cos(x - x') - \sin(x + x')\}$$

$$(\alpha_1 + \beta_1)^2 + (\alpha_1 - \beta_1)^2 = 2(\alpha_1^2 + \beta_1^2) = 2\mu^2 + 2\mu^2 \sin 2w \cos(x - x')$$

$$\mu^2 = \alpha_1^2 + \beta_1^2 - 2A'BB' \frac{\alpha_0 - A'}{\frac{1}{2}BB'}$$

$$(l) \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad \mu^2 = m^2 + 4AA'.$$

Endlich wird noch—

$$\alpha_1 - \mu \cos w \cos x = \mu \sin w \cos x'$$
$$\beta_1 - \mu \cos w \sin x = \mu \sin w \sin x'$$

folglich—

$$\alpha_1^2 + \beta_1^2 + \mu^2 \cos^2 w - 2\mu \cos w(\alpha_1 \cos x + \beta_1 \sin x) = \mu^2 \sin^2 w$$
$$2\mu m \cos w \cos(\psi - x) = \alpha_1^2 + \beta_1^2 + \mu^2 \cos 2w$$

$$(m) \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad \cos(\psi - x) = \frac{m^2 + \mu^2 \cos 2w}{2\mu m \cos w}.$$

Ebenso ergiebt sich aber auch—

$$(n) \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad \cos(\psi - x') = \frac{m^2 - \mu^2 \cos 2w}{2\mu m \sin w}.$$

Durch eine dieser Gleichungen wird man x oder x' scharf erhalten und findet dann den andern Winkel durch die entsprechende der Gleichungen—

$$(o) \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad \begin{cases} \dfrac{m \sin(x' - \psi)}{\mu \cos w} = \sin(x' - x) \\ \dfrac{m \sin(x - \psi)}{\mu \sin w} = \sin(x - x'). \end{cases}$$

Man bestimmt also aus der Gleichung (l) zunächst μ, findet dann aus (k) den Winkel w und hiermit die Grössen B und B′ aus den Definitionsgleichungen. Schliesslich berechnet man x und x' aus den Gleichungen (m) (n) (o). Zur Controle rechnet man mit diesen Werthen für ein bestimmtes v' den Werth von $\left(\frac{\Delta}{r'}\right)^2$ aus und vergleicht ihn mit dem direkt nach Gleichung (11) gefundenen.

Ist man so zur Kenntniss der Coefficienten der Gleichung (13) gelangt, so erhält man weiter—

$$\left(\frac{r'^3}{\Delta}\right) = \{1 + \mathrm{B} \cos(v' - x)\}^{-\frac{3}{2}} \{\mathrm{A}' + \mathrm{B}' \cos(v' - x')\}^{-\frac{3}{2}}$$

und es ist nun die Entwicklung der Faktoren der rechten Seite leicht auszuführen, da man hat—

$$[\mathrm{P} - \mathrm{Q} \cos \psi]^{-\frac{n}{2}} = \tfrac{1}{2}\alpha_0^n + \alpha_1^n \cos \psi + \alpha_2^n \cos 2\psi + \ldots . + \alpha_i^n \cos i\psi + \ldots .$$
$$\alpha_i^n = \frac{2\mathrm{P}^{-\frac{n}{2}}}{\pi} \int_0^\pi \left(1 - \frac{\mathrm{Q}}{\mathrm{P}} \cos \psi\right)^{-\frac{n}{2}} \cos i\psi \, d\psi\cdot$$

Die für eine zweckmässige Entwicklung der α_i^n Coefficienten nöthigen Formeln giebt Hansen in aller Ausführlichkeit in seiner "Auseinandersetzung einer zweckmässigen Methode . . . etc."; dieselben dürfen hier füglich als bekannt betrachtet werden. Man führe also die Entwicklung von $\left(\frac{r'}{\Delta}\right)^3$ für 16 oder, wenn nöthig für 24 oder 32 Werthe von v aus und erhält dann durch Verallgemeinerung für alle Werthe des v nach der Methode der Mechanischen Quadratur für $\left(\frac{r'}{\Delta}\right)^3$ eine nach den sin. und cos. der Summen und Differenzen der Vielfachen von v und v' fortschreitende Beihe.

Die Bildung der übrigen zur Entwicklung der Störungsfunktion noch nöthigen Ausdrücke erledigt sich schnell.

$\frac{r'}{p} = (1 - e^2)(1 + e' \cos v')^{-1}$ entwickelt man entweder nach bekannten Regeln in eine Reihe, deren Coefficienten analytische Funktionen der aufeinander folgenden Potenzen von e' sind oder man wendet, wenn dies nicht genügt die oben erwähnte Entwicklung negativer gebrochener Potenzen des Binoms an. Es wird hier—

$$\alpha_0{}^2 = \frac{1}{\pi}\int_{-\pi}^{+\pi} \frac{dv'}{1 + e' \cos v'} = \frac{1}{\pi}\,\frac{1}{\sqrt{1 - e'^2}} \left\{\text{arc cos}\,\frac{\cos v' + e'}{1 + e' \cos v'}\right\}_{-\pi}^{+\pi}$$

$$= \frac{1}{\pi\sqrt{1 - e'^2}} \{\text{arc cos}\,(\cos \epsilon')\}_{-\pi}^{+\pi} = \frac{2}{\sqrt{1 - e'^2}}$$

$$\alpha_0{}^2 = \frac{2}{\cos \phi'}$$

$$\sin \chi = -e' = -\sin \phi' \qquad \mathrm{F}_i = -\frac{\sin \phi'}{2} \qquad \lambda_i = \frac{\sin^2 \phi'}{4}$$

$$\gamma_i = \cfrac{1}{1 + \cfrac{\frac{1}{4}\sin^2 \phi'}{1 + \cfrac{\frac{1}{4}\sin^2 \phi'}{1 + \cfrac{\frac{1}{4}\sin^2 \phi'}{1 + \text{etc.}}}}} = \sec^2 \frac{\phi'}{2}$$

$$p_i = + \mathrm{F}_i\,\gamma_i = -\frac{\sin \frac{\phi'}{2} \cos \frac{\phi'}{2}}{\cos^2 \frac{\phi'}{2}} = -\text{tang}\,\frac{\phi'}{2}$$

$$(16) \quad \ldots\ldots\ldots \quad \alpha_i{}^2 = \alpha_0{}^2 \left(-\text{tang}\,\frac{\phi'}{2}\right)^i = \frac{2}{\cos \phi'} \left(-\text{tang}\,\frac{\phi'}{2}\right)^i.$$

Man hat dann—

$$\frac{r'}{p'} = (1 - e'^2)\,\{\tfrac{1}{2}\alpha_0{}^2 + \Sigma\,\alpha_i{}^n \cos iv'\}.$$

In dieser Weise muss jedenfalls—

$$\frac{r}{p} = (1 - e^2)(1 + e \cos v)^{-1}$$

entwickelt werden. $\frac{r'}{r}$ und $\frac{a}{r}$ können wir in Bezug auf die wahre Anomalie v des gestörten Körpers in geschlossener Form darstellen; denn es ist—

$$(17) \quad \ldots\ldots \quad \frac{r'}{r} = \frac{p'}{p}\,\frac{1 + e \cos v}{1 + e' \cos v'} \qquad \frac{a}{r} = \frac{1}{1 - e^2}(1 + e \cos v).$$

Endlich ist durch das soeben angegebene auch schon die Entwicklung der Coordinaten—

$$(18) \ldots\ldots \quad x = r \cos v = \frac{p \cos v}{1 + e \cos v} \qquad y = r \sin v = \frac{p \sin v}{1 + e \cos v}$$

erledigt, so dass uns nur noch die Behandlung von H und $\frac{d\mathrm{H}}{dv}$ übrig bleibt.

Die Entwicklung von H ergiebt sich direkt aus der Gleichung—

$$\mathrm{H} = \cos(v + \pi)\cos(v' + \pi') + \sin(v + \pi)\sin(v' + \pi')\cos \mathrm{I}.$$

Dieselbe lässt sich zunächst überführen in—

$$\mathrm{H} = \cos(v - v' + \pi - \pi') - 2\sin^2 \tfrac{1}{2}\mathrm{I}\,\sin(v + \pi)\sin(v' + \pi').$$

Nun ist aber—

$$\cos(v - v' + \pi - \pi') = \cos(v - v')\cos(\pi - \pi') - \sin(v - v')\sin(\pi - \pi')$$

$$\begin{aligned}2\sin(v + \pi)\sin(v' + \pi') = {} & \cos(v - v')\cos(\pi - \pi') - \cos(v + v')\cos(\pi + \pi') - \sin(v - v')\sin(\pi - \pi') \\ & + \sin(v + v')\sin(\pi + \pi').\end{aligned}$$

Es ergiebt sich so sehr einfach—

$$\begin{aligned}(19)\ .\ .\ \mathrm{H} = {} & \cos(v - v')\cos(\pi - \pi')\cos^2\tfrac{1}{2}\mathrm{I} + \cos(v + v')\cos(\pi + \pi')\sin^2\tfrac{1}{2}\mathrm{I} \\ & - \sin(v - v')\sin(\pi - \pi')\cos^2\tfrac{1}{2}\mathrm{I} - \sin(v + v')\sin(\pi + \pi')\sin^2\tfrac{1}{2}\mathrm{I}\end{aligned}$$

$$\begin{aligned}(20)\quad \frac{d\mathrm{H}}{dv} = {} & - \sin(v - v')\cos(\pi - \pi')\cos^2\tfrac{1}{2}\mathrm{I} - \sin(v + v')\cos(\pi + \pi')\sin^2\tfrac{1}{2}\mathrm{I} \\ & - \cos(v - v')\sin(\pi - \pi')\cos^2\tfrac{1}{2}\mathrm{I} - \cos(v + v')\sin(\pi + \pi')\sin^2\tfrac{1}{2}\mathrm{I}.\end{aligned}$$

Damit aber sind alle in der Störungsfunktion und ihren Differentialen vorkommenden Ausdrücke in Reihen entwickelt und der Integration ist ein Ausdruck von der Form—

$$\sum_{i=-\infty}^{i=\infty}\ \sum_{i'=0}^{i'=\infty} (ii'c)\cos(iv - i'v') + (ii's)\sin(iv - i'v')$$

zu Grunde zu legen.

3. *Die Integration der Reihen.*

Schon die erste Arbeit über das Problem der drei Körper, Euler's Untersuchungen über die Ungleichheiten des Jupiter und Saturn aus dem Jahre 1748 enthält Entwicklungen nach den Kreisfunktionen der Vielfachen gewisser Winkel; dort kommt in dieser Beziehung der Unterschied der heliocentrischen Elongationen beider Körper in Anwendung. So wichtig eine passende Wahl des Entwicklungselementes nun für die Convergenz der Reihen ist, ebenso einflussreich ist sie auf die grössere oder geringere Schwierigkeit der Integration. So lange nur eine Entwicklung nach den mittleren Anomalien, verbunden mit einer Differentiation nach der Zeit, in Anwendung kam, bot die Integration der einzelnen Ausdrücke irgend welche erhebliche Schwierigkeiten nicht. Aber schon zur Integration der nach der excentrischen Anomalie des gestörten und der mittleren des störenden fortschreitenden Reihen musste sich Hansen ein eigenes Verfahren entwickeln. Andere Reihen aber sind bislang noch nicht angewandt.

Um ein völlig allgemeines Integral für den hier vorliegenden Ausdruck zu erhalten, müsste man eine doppelt unendliche Reihe von Bedingungsgleichungen auflösen, wenn man den hier angewandten Weg der Entwicklung durch unbestimmte Coefficienten einschlägt. Auf eine einfach unendliche Reihe von Bedingungsgleichungen wird man aber geführt, wenn man die Entwicklung nach der Anomalie des einen Körpers durch mechanische Quadratur ausführt. Da nun bei den störenden Planeten wohl stets ein der Zeit fast proportionaler Gang in den Werthen der Anomalien auftreten wird, so ist die Integration hinsichtlich seiner im folgenden durch mechanische Quadratur ausgeführt. Es wird bei dieser Wahl stets ausreichend sein, sich bei der Bestimmung der Coefficienten auf höchstens 16 specielle Werthe der Anomalie des störenden Körpers zu beschränken.

Eine weitere wesentliche Erleichterung erhält man dadurch, dass man sich nur mit der Integration der Cosinusglieder zu beschäftigen braucht. Setzt man nämlich—

$$\int \cos \omega \, dw = \int \cos ix \, dy \sum_0^\infty a^i_\mu \cos(i+\mu)x$$

mit der Bedingung—

$$\frac{d\omega}{dw} = i \frac{dx}{dy}$$

so erhält man auch—

$$\int \sin ix \, dy = - \int \cos\left(\frac{\pi}{2} - ix\right) d\left(\frac{\pi}{2} - y\right) = - \int \cos \omega' \, dw'.$$

Da nun dieses Integral der Bedingung $\frac{d\omega}{dw} = i\frac{dx}{dy}$ genügt, so wird—

$$\int \sin ix \, dy = - \sum_0^\infty a^i_\mu \sin\left(\frac{\pi}{2} - (i+\mu)x\right) = \sum_0^\infty a^i_\mu \cos(i+\mu)\, x.$$

Es sind hiernach, abgesehen vom Vorzeichen, die Coefficienten der Entwicklung des Cosinus zugleich auch die der Entwicklung des Sinus.

Man hat nun—

$$\frac{dv}{dt} = \frac{k\sqrt{1+m}\sqrt{p}}{r^2} \qquad \frac{dv'}{dt} = \frac{k\sqrt{1+m'}\sqrt{p'}}{r'^2}$$

$$\frac{dv}{dv'} = \frac{\sqrt{1+m}}{\sqrt{1+m'}} \frac{\sqrt{p}}{\sqrt{p'}} \frac{r'^2}{r^2} = \frac{\sqrt{1+m}}{\sqrt{1+m'}} \frac{\sqrt{p}}{\sqrt{p'}} \frac{p'^2}{p^2} \frac{(1+e\cos v)^2}{(1+e'\cos v')^2}$$

$$= \frac{\sqrt{1+m}}{\sqrt{1+m'}} \frac{p'^{\frac{3}{2}}}{p^{\frac{3}{2}}} \frac{\left(1+\frac{e^2}{2}\right) + 2e\cos v + \frac{e^2}{2}\cos 2v}{\left(1+\frac{e'^2}{2}\right) + 2e'\cos v' + \frac{e'^2}{2}\cos 2v'}.$$

Der Einfachheit wegen werde nun eingeführt—

$$\psi(v') = \frac{\sqrt{1+m}}{\sqrt{1+m'}} \frac{p'^{\frac{3}{2}}}{p^{\frac{3}{2}}} \frac{1}{(1+e'\cos v')^2}.$$

Versteht man dann unter n und n' die mittlere siderische Bewegung des gestörten und des störenden Körpers, so hat man—

$$n = a^{-\frac{3}{2}} k \sqrt{1+m} = \frac{k\sqrt{1+m}\,(1+e^2)^{\frac{3}{2}}}{p^{\frac{3}{2}}}$$

$$n' = a'^{3} k \sqrt{1+m'} = \frac{k\sqrt{1+m'}\,(1+e'^2)^{\frac{3}{2}}}{p'^{\frac{3}{2}}}.$$

Damit erhält man dann einfacher—

(1) . . . . $\psi(v') = \frac{n}{n'} \frac{(1+e'^2)^{\frac{3}{2}}}{(1+e^2)^{\frac{3}{2}}} (1+e'\cos v')^{-2}$

(2) . . . . $\frac{dv}{dv'} = \psi(v') \left\{ \left(1+\frac{e^2}{2}\right) + 2e\cos v + \frac{e^2}{2}\cos 2v \right\}.$

Es besteht unsere Aufgabe nun in der Bestimmung der Coefficienten der Entwicklung—

$$\int \cos iv \, dv' = \sum_0^\infty a^i_\mu \sin(i+\mu)\, v.$$

Die Differentiation dieses Ausdruckes nach v' ergiebt uns die Bedingungsgleichungen zur Bestimmung der α Coefficienten in folgender Form—

$$(3) \quad \cos iv = \sum_0^\infty \alpha_\mu^i \cos\{(i+\mu)v\}(i+\mu)\frac{dv}{dv'}$$

$$= \sum_0^\infty \psi(v')(i+\mu)\alpha_\mu^i \cos[(i+\mu)v]\left\{\left(1+\frac{e^2}{2}\right)+2e\cos v+\frac{e^2}{2}\cos 2v\right\}$$

$$(4) \quad \cos iv = \sum_0^\infty \Big\{\psi(v')(i+\mu)\left(1+\frac{e^2}{2}\right)\alpha_\mu^i \cos[(i+\mu)v]$$
$$+\psi(v')(i+\mu)\,e\,\alpha_\mu^i[\cos(i+\mu+1)v+\cos(i+\mu-1)v]$$
$$+\psi(v')(i+\mu)\frac{e^2}{4}\alpha_\mu^i[\cos(i+\mu+2)v+\cos(i+\mu-2)v]\Big\}.$$

Diese Gleichung muss nun, weil sie für alle möglichen Werthe des v gelten soll, identisch erfüllt sein.

Man erhält daher die folgenden Bedingungsgleichungen—

$$\frac{1}{\psi(v')} = i\left(1+\frac{e^2}{2}\right)\alpha_0^i+(i-1)e\,\alpha_{-1}^i+(i+1)e\,\alpha_1^i+(i-2)\frac{e^2}{4}\alpha_{-2}^i+(i+2)\frac{e^2}{4}\alpha_2^i$$

$$\left\{\begin{aligned} 0 &= (i+1)\left(1+\frac{e^2}{2}\right)\alpha_1^i+i\,e\,\alpha_0^i+(i+2)e\,\alpha_2^i+(i-1)\frac{e^2}{4}\alpha_{-1}^i+(i+3)\frac{e^2}{4}\alpha_3^i \\ 0 &= (i-1)\left(1+\frac{e^2}{2}\right)\alpha_{-1}^i+(i-2)e\,\alpha_{-2}^i+i\,e\,\alpha_0^i+(i-3)\frac{e^2}{4}\alpha_{-3}^i+(i+1)\frac{e^2}{4}\alpha_1^i \end{aligned}\right.$$

$$\left\{\begin{aligned} 0 &= (i+2)\left(1+\frac{e^2}{2}\right)\alpha_2^i+(i+1)e\,\alpha_1^i+(i+3)e\,\alpha_3^i+i\frac{e^2}{4}\alpha_0^i+(i+4)\frac{e^2}{4}\alpha_4^i \\ 0 &= (i-2)\left(1+\frac{e^2}{2}\right)\alpha_{-2}^i+(i-3)e\,\alpha_{-3}^i+(i-1)e\,\alpha_{-1}^i+(i-4)\frac{e^2}{4}\alpha_{-4}^i+i\frac{e^2}{4}\alpha_0^i \end{aligned}\right.$$

etc., etc.

Es sind alle diese Gleichungen in der allgemeinen Form enthalten—

$$(5)\left\{\begin{aligned} 0 &= (i+\mu)\left(1+\frac{e^2}{2}\right)\alpha_\mu^i+(i+\mu-1)e\,\alpha_{\mu-1}^i+(i+\mu+1)e\,\alpha_{\mu+1}^i+(i+\mu-2)\frac{e^2}{4}\alpha_{\mu-2}^i \\ &\qquad +(i+\mu+2)\frac{e^2}{4}\alpha_{\mu+2}^i \\ 0 &= (i-\mu)\left(1+\frac{e^2}{2}\right)\alpha_{-\mu}^i+(i-\mu-1)e\,\alpha_{-\mu-1}^i+(i-\mu+1)e\,\alpha_{-\mu+1}^i+(i-\mu-2)\frac{e^2}{4}\alpha_{-\mu-2}^i \\ &\qquad +(i-\mu+2)\frac{e^2}{4}\alpha_{-\mu+2}^i. \end{aligned}\right.$$

Für μ darf man in diesen Gleichungen alle positiven Werthe mit Ausnahme der Null setzen. Man addire und subtrahire nun je zwei dieser Gleichungen, in denen μ denselben Werth hat. Dabei bediene man sich der einfacheren Bezeichnungen—

$$\sigma_r^i = (i+r)\alpha_r^i+(i-r)\alpha_{-r}^i$$
$$\delta_r^i = (i+r)\alpha_r^i-(i-r)\alpha_{-r}^i$$

Es entstehen so die beiden folgenden Systeme von Gleichungen—

$$(6) \quad \left\{\begin{aligned} 0 &= \left(1+\frac{e^2}{2}\right)\sigma_1^i+e\,\sigma_0^i+e\,\sigma_2^i+\frac{e^2}{4}\sigma_{-1}^i+\frac{e^2}{4}\sigma_3^i \\ 0 &= \left(1+\frac{e^2}{2}\right)\sigma_2^i+e\,\sigma_1^i+e\,\sigma_3^i+\frac{e^2}{4}\sigma_0^i+\frac{e^2}{4}\sigma_4^i \\ 0 &= \left(1+\frac{e^2}{2}\right)\sigma_3^i+e\,\sigma_2^i+e\,\sigma_4^i+\frac{e^2}{4}\sigma_1^i+\frac{e^2}{4}\sigma_5^i \\ &\text{etc.,} \qquad \text{etc.} \\ 0 &= \left(1+\frac{e^2}{2}\right)\sigma_\mu^i+e\,\sigma_{\mu-1}^i+e\,\sigma_{\mu+1}^i+\frac{e^2}{4}\sigma_{\mu-2}^i+\frac{e^2}{4}\sigma_{\mu+2}^i \end{aligned}\right.$$

$$(7)\ldots\quad\left\{\begin{array}{l} 0=\left(1+\frac{e^2}{2}\right)\delta_1^i \qquad\qquad + e\,\delta_2^i + \frac{e^2}{4}\delta_{-1}^i + \frac{e^2}{4}\delta_3^i \\ 0=\left(1+\frac{e^2}{2}\right)\delta_2^i + e\,\delta_1^i + e\,\delta_3^i \qquad\qquad + \frac{e^2}{4}\delta_4^i \\ 0=\left(1+\frac{e^2}{2}\right)\delta_3^i + e\,\delta_2^i + e\,\delta_4^i + \frac{e^2}{4}\delta_1^i + \frac{e^2}{4}\delta_5^i \\ \qquad\qquad\qquad \text{etc.,} \qquad\qquad \text{etc.} \\ 0=\left(1+\frac{e^2}{2}\right)\delta_\mu^i + e\,\delta_{\mu-1}^i + e\,\delta_{\mu+1}^i + \frac{e^2}{4}\delta_{\mu-2}^i + \frac{e^2}{4}\delta_{\mu+2}^i. \end{array}\right.$$

Die Grössen δ lassen sich nun aus ihren Bestimmungsgleichungen in Kettenbrüche entwickeln. Bedenkt man nämlich, dass zufolge der Definition—

$$\delta_\mu^i = -\,\delta_{-\mu}^i$$

ist, so vereinfacht sich die erste der Gleichungen (8) zu—

$$(9)\ \ldots\ldots\ldots\quad 0=\frac{4+e^2}{4}\,\delta_1^i + e\delta_2^i + \frac{e^2}{4}\,\delta_3^i.$$

Man dividire diese Gleichung durch $\frac{4+e^2}{4}\,\delta_1^i$ und setze—

$$(10)\ \ldots\ldots\ldots\ldots\quad c_1=\frac{e^2}{2(2+e^2)}$$

$$(11)\ \ldots\ldots\ldots\ldots\quad n_1=\frac{4e}{4+e^2}$$

so erhält man—

$$0=1+\frac{4e}{4+e^2}\,\frac{\delta_2^i}{\delta_1^i}+\frac{e^2}{4+e^2}\,\frac{\delta_3^i}{\delta_2^i}\,\frac{\delta_2^i}{\delta_1^i}$$

folglich wird—

$$(12)\ \ldots\ldots\ldots\ldots\quad \frac{\delta_2^i}{\delta_1^i}=\frac{-1}{n_1+\frac{e}{4}\,n_1}\,\frac{\delta_3^i}{\delta_2^i}$$

$$(13)\ \ldots\ldots\ldots\quad \frac{\delta_1^i}{\delta_2^i}=-\,n_1-\frac{e}{4}\,n_1\,\frac{\delta_3^i}{\delta_2^i}$$

$$(14)\ \ldots\ldots\ldots\ldots\quad \frac{\delta_1^i}{\delta_3^i}=-\,n_1\,\frac{\delta_2^i}{\delta_3^i}-\frac{e}{4}\,n_1.$$

Die zweite der Gleichungen (8) ergiebt weiter durch Division mit $\frac{2+e^2}{2}\,\delta_2^i$

$$\begin{aligned} 0 &= 1+\frac{2e}{2+e^2}\,\frac{\delta_1^i}{\delta_2^i}+\frac{2e}{2+e^2}\,\frac{\delta_3^i}{\delta_2^i}+\frac{e^2}{2(2+e^2)}\,\frac{\delta_4^i}{\delta_3^i}\,\frac{\delta_3^i}{\delta_2^i} \\ &= 1-\frac{4}{e}\,c_1\,n_1+\left(\frac{4}{e}\,e_1-c_1\,n_1\right)\frac{\delta_3^i}{\delta_2^i}+c_1\,\frac{\delta_4^i}{\delta_3^i}\,\frac{\delta_3^i}{\delta_2^i}. \end{aligned}$$

Mit Einführung der Bezeichnungen—

$$(15)\ \ldots\ldots\ldots\ldots\quad z_2=1-\frac{4}{e}\,c_1\,n_1$$

$$(16)\ \ldots\ldots\ldots\ldots\quad n_2=\frac{4}{e}\,c_1-c_1\,n_1$$

wird also—

$$(17)\ \ldots\ldots\ldots\ldots\quad \frac{\delta_3^i}{\delta_2^i}=\frac{-\,z_2}{n_2+c_1\,\frac{\delta_4^i}{\delta_3^i}}$$

(18) . . . . $\dfrac{\delta_2^i}{\delta_3^i} = -\dfrac{n_2}{z_2} - \dfrac{c_1}{z_2}\dfrac{\delta_4^i}{\delta_3^i}$

(19) . . . . $\dfrac{\delta_2^i}{\delta_4^i} = -\dfrac{n_2}{z_2}\dfrac{\delta_3^i}{\delta_4^i} - \dfrac{c_1}{n_1}.$

Behandelt man in derselben Weise die dritte der Gleichungen (8), so bestimmt sich aus derselben $\frac{\delta_4^i}{\delta_3^i}$ unter Benutzung der Gleichungen (14) und (18) in folgender Weise—

$$0 = 1 + \frac{2e}{2+e^2}\frac{\delta_2^i}{\delta_3^i} + \frac{2e}{2+e^2}\frac{\delta_4^i}{\delta_3^i} + \frac{e^2}{2(2+e^2)}\frac{\delta_1^i}{\delta_2^i}\frac{\delta_2^i}{\delta_3^i} + \frac{e^2}{2(2+e^2)}\frac{\delta_5^i}{\delta_4^i}\frac{\delta_4^i}{\delta_3^i}$$
$$= \left(1 - \frac{4}{e}\frac{n_2}{z_2}c_1 - \frac{e}{4}n_1 c_1 + n_1 c_1 \frac{n_2}{z_2}\right) + \frac{\delta_4^i}{\delta_3^i}\left(\frac{4}{e}c_1 - \frac{4}{e}\frac{c_1^2}{n_2} + n_1\frac{c_1^2}{z_2}\right) + c_1\frac{\delta_5^i}{\delta_4^i}\frac{\delta_4^i}{\delta_3^i}$$

(20) . . . . $z_3 = 1 - \dfrac{4}{e}\dfrac{n_2}{z_2}c_1 - \dfrac{e}{4}n_1 c_1 + n_1 c_1\dfrac{n_2}{z_2}$

(21) . . . . $n_3 = \dfrac{4}{e}c_1\left(1 - \dfrac{c_1}{z_2} + \dfrac{e}{4}\dfrac{n_1 c_1}{z_2}\right)$

(22) . . . . $\dfrac{\delta_4^i}{\delta_3^i} = \dfrac{-z_3}{n_3 + c_1\dfrac{\delta_5^i}{\delta_4^i}}$

(23) . . . . $\dfrac{\delta_3^i}{\delta_4^i} = -\dfrac{n_3}{z_3} - \dfrac{c_1}{z_3}\dfrac{\delta_5^i}{\delta_4^i}$

(24) . . . . $\dfrac{\delta_3^i}{\delta_5^i} = -\dfrac{n_3}{z_3}\dfrac{\delta_4^i}{\delta_5^i} - \dfrac{c_1}{z_3}.$

Führen wir endlich mit zu Hülfenahme der Gleichungen (19) und (23) dieselbe Betrachtung noch an der vierten der Gleichungen (8) durch, so stellt sich die Entwicklung so—

$$0 = 1 + \frac{2e}{2+e^2}\frac{\delta_3^i}{\delta_4^i} + \frac{2e}{2+e^2}\frac{\delta_5^i}{\delta_4^i} + \frac{e^2}{2(2+e^2)}\frac{\delta_2^i}{\delta_4^i} + \frac{e^2}{2(2+e^2)}\frac{\delta_6^i}{\delta_5^i}\frac{\delta_5^i}{\delta_4^i}$$
$$= 1 - \frac{4}{e}c_1\left(\frac{n_3}{z_3} + \frac{c_1}{z_3}\frac{\delta_5^i}{\delta_4^i}\right) + \frac{4}{e}c_1\frac{\delta_5^i}{\delta_4^i} + c_1\frac{n_2}{z_2}\left(\frac{n_3}{z_3} + \frac{c_1}{z_3}\frac{\delta_5^i}{\delta_4^i}\right) - \frac{c_1^2}{z_2} + c_1\frac{\delta_6^i}{\delta_5^i}\frac{\delta_5^i}{\delta_4^i}$$

(25) . . . . $z_4 = 1 - \dfrac{4}{e}c_1\dfrac{n_3}{z_3} + c_1\dfrac{n_2}{z_2}\dfrac{n_3}{z_3} - \dfrac{c_1^2}{z_2}$

(26) . . . . $n_4 = \dfrac{4}{e}c_1\left(1 - \dfrac{c_1}{z_3} + \dfrac{e}{4}\dfrac{n_2}{z_2}\dfrac{c_1}{z_3}\right)$

(27) . . . . $\dfrac{\delta_5^i}{\delta_4^i} = \dfrac{-z_4}{n_4 + c_1\dfrac{\delta_6^i}{\delta_5^i}}$

(28) . . . . $\dfrac{\delta_4^i}{\delta_5^i} = -\dfrac{n_4}{z_4} - \dfrac{c_1}{z_1}\dfrac{\delta_6^i}{\delta_5^i}$

(29) . . . . $\dfrac{\delta_4^i}{\delta_6^i} = -\dfrac{n_4}{z_4}\dfrac{\delta_5^i}{\delta_6^i} - \dfrac{c_1}{z_4}$

Von hier ab greift nun in der Bildung der Grössen z und n ein leicht zu fassendes Gesetz ob. Da nämlich die folgenden der Gleichungen (8), wie auch die Hülfsgrössen zu ihrer Entwicklung aus einander durch Vertauschung der Indices der veränderlichen Grössen mit den nächst höheren entstehen, so kann man für $\mu \geqq 4$ allgemein setzen—

$$(30) \ldots\ldots \begin{cases} z_\mu = 1 - \frac{4}{e} c_1 \frac{n_{\mu-1}}{z_{\mu-1}} + c_1 \frac{n_{\mu-2}}{z_{\mu-2}} - \frac{n_{\mu-1}}{z_{\mu-1}} - \frac{c_1^2}{z_{\mu-2}} \\ n_\mu = \frac{4}{e} c_1 \left(1 - \frac{c_1}{z_{\mu-1}} + \frac{e}{4} \frac{n_{\mu-2}}{z_{\mu-2}} \frac{c_1}{z_{\mu-1}}\right). \end{cases}$$

Man erhält dann—

$$(31) \ldots\ldots \frac{\delta^i_{\mu+1}}{\delta^i_\mu} = \frac{-z_\mu}{n_\mu + c_1 \frac{\delta^i_{\mu+2}}{\delta^i_{\mu+1}}}.$$

Verbindet man die Gleichungen (12) (17) (22) (27) etc. mit einander, so entstehen die folgenden Kettenbrücke—

$$(32) \quad p_1 = \frac{\delta_2^i}{\delta_1^i} = \cfrac{-1}{n_1 - \cfrac{\frac{e}{4} n_1 z_2}{n_2 - \cfrac{c_1 z}{n_3 - \cfrac{c_1 z_4}{n_4 - \text{etc.}}}}}$$

$$(33) \quad p_2 = \frac{\delta_3^i}{\delta_2^i} = \cfrac{-z_2}{n_2 - \cfrac{c_1 z_3}{n_3 - \cfrac{c_1 z_4}{n_4 - \cfrac{c_1 z_5}{n_5 - \text{etc.}}}}}$$

$$(34) \ldots p_\mu = \frac{\delta^i_{\mu+1}}{\delta^i_\mu} = \cfrac{-z_\mu}{n_\mu - \cfrac{c_1 z_{\mu+1}}{n_{\mu+1} - \cfrac{c_1 z_{\mu+2}}{n_{\mu+2} - \cfrac{c_1 z_{\mu+3}}{n_{\mu+3} - \text{etc.}}}}}.$$

Es ist nun nach der Definition in (6)—

$$p_m = \frac{\delta^i_{m+1}}{\delta^i_m} = \frac{(i + m + 1)\, a^i_{m+1} - (i - m - 1)\, a^i_{-m-1}}{(i + m)\, a^i_m - (i - m)\, a^i_{-m}}.$$

Der Werth des Kettenbruches p_m ist aber völlig unabhängig von i. Nimmt man also m hinreichend gross, so dass i dagegen verschwindet, so wird sein—

$$\lim \frac{i + m + 1}{i + m} = 1 \qquad \lim \frac{i - m - 1}{i + m} = -1 \qquad \lim \frac{i - m}{i + m} = -1$$

folglich—

$$\lim p_m = \frac{a^i_{m+1} + a^i_{-m-1}}{a^i_m + a^i_{-m}}.$$

Ebenso kann man aber schliessen—

$$\lim p_{m+1} = \frac{a^i_{m+2} + a^i_{-m-2}}{a^i_{m+1} + a^i_{-m-1}}$$

$$\lim p_{m+2} = \frac{a^i_{m+3} + a^i_{-m-3}}{a^i_{m+2} + a^i_{-m-2}}.$$

Durch Multiplication erhält man demnach—

$$\lim p_m\, p_{m+1} \ldots\ldots p_{m+\infty} = \frac{a^i_{m+\infty} + a^i_{-m-\infty}}{a^i_m + a^i_{-m}}.$$

Nun nehmen aber die a^i_μ Coefficienten mit wachsendem Index fortwährend ab; man kann daher den Zähler jenes Bruches im Vergleich zum Nenner unter jede angebbare Grenze hinabbringen. Folglich erhält man—

$$\lim p_m - p_{m+1} - p_{m+2} \ldots\ldots p_{m+\infty} = 0.$$

Für einen hinreichend grossen Werth von z erhält man mithin—

$$(35) \quad . \quad . \quad . \quad . \quad \delta_s^i = \delta_1^i p_1 p_2 p_3 \cdot \cdot \cdot \cdot p_{m-1} (p_m p_{m+1} \cdot \cdot \cdot \cdot p_s) \doteq 0$$

womit bewiesen ist, dass der Werth der Grössen δ^i gegen Null convergiert.

Um nun ein Urtheil über die Schnelligkeit der Convergenz des Kettenbruches p zu erhalten, ist der Werth von p_1 für zwei verschiedene Excentricitäten berechnet. Es sind dabei als Beispiele die mittlere Excentricität $e = 0{\cdot}2$ der Bahnen der kleinen Planeten und der extreme Fall $e = 1$ gewählt.

Im ersten Falle führte die Rechnung zu folgendem Resultate—

$e = 0{\cdot}2$	$c_1 = 0{\cdot}009804$	
$z_1 = 1$	$n_1 = 0{\cdot}19802$	$p_1 = 5{\cdot}0499$
$z_2 = 0{\cdot}96117$	$n_2 = 0{\cdot}19414$	$5{\cdot}0977$
$z_3 = 0{\cdot}96069$	$n_3 = 0{\cdot}19410$	$5{\cdot}0971$
$z_4 = 0{\cdot}96068$	$n_4 = 0{\cdot}19410$	$5{\cdot}0971$

Die für p_1 gegebenen Werthe sind die aufeinander folgenden Näherungswerthe des Kettenbruchs. Für den zweiten extremen Fall ergaben sich folgende Ziffern—

$z_1 = 1$	$n_1 = \frac{4}{5}$	$p_1 = -1{\cdot}25000$
$z_2 = \frac{7}{15}$	$n_2 = \frac{8}{15}$	$-1{\cdot}60000$
$z_3 = \frac{5}{14}$	$n_3 = \frac{10}{21}$	$-1{\cdot}75000$
$z_4 = \frac{11}{36}$	$n_4 = \frac{4}{9}$	$-1{\cdot}82857$
$z_5 = \frac{91}{330}$	$n_5 = \frac{14}{33}$	$-1{\cdot}87500$
$z_6 = \frac{10}{39}$	$n_6 = \frac{16}{39}$	$-1{\cdot}90482$
$z_7 = \frac{17}{70}$	$n_7 = \frac{2}{5}$	$-1{\cdot}91295$
$z_8 = \frac{95}{408}$	$n_8 = \frac{20}{51}$	$-1{\cdot}91400$
$z_9 = \frac{77}{342}$	$n_9 = \frac{22}{57}$	$-1{\cdot}91413$
$z_{10} = \frac{23}{105}$	$n_{10} = \frac{8}{21}$	$-1{\cdot}91413$

Man erkennt hieraus, dass man im äussersten Falle nie weiter als bis zur Berechnung der ersten 10 Näherungswerthe zu gehen braucht, um den Kettenbruch bis auf die fünfte Decimale genau zu erhalten. Weil nun aber bei den Grössen p mit höherem Index die ersten am meisten abweichenden Näherungswerthe nicht in Betracht kommen, so wird die Convergenz eine noch weit schnellere sein.

Es ist nun schon gezeigt, dass für einen genügend grossen Werth von n sich das Produkt $p_n p_{n+1} \cdot \cdot \cdot \cdot \cdot \cdot p_\infty$ dem Werthe Null beliebig nähert. Dies ist natürlich nur möglich, wenn jede der Grössen p_{n+v} von Null nur sehr wenig verschieden ist. Berechnet man also die Grössen p, bis man zu der Null hinreichend genäherten Werthen gelangt, so sei etwa—

$$0 = p_{q-1} \doteq \frac{\delta_q^i}{\delta_{q-1}^i}$$

Es kann nun ein Bruch nur dadurch Null werden, dass sein Zähler unter jede angebbare Grenze sinkt, oder sein Nenner beliebig gross wird. Wir haben schon gezeigt dass hier das erstere eintritt.

Sobald also p_q dem Werthe Null beliebig nahegekommen ist, weiss man dass $\delta_q^i = 0$ gesetzt werden darf. Man erhält dann aber nach der Definition—

$$(36) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \delta_q^i = (i + q)\, a_q^i - (i - q)\, a_{-q}^i = 0$$

$$\frac{a_q^i}{a_{-q}^i} = \frac{i - q}{i + q}$$

Durch die Berechung der Grössen p kann man also den Werth des q finden, von wo ab man setzen darf—

$$(37) \quad \ldots\ldots\ldots \quad a_q^i = k(i-q) \qquad a_{-q}^i = k(i+q).$$

Die Grössen δ_i erhalten dadurch folgende einfache Bedeutung—

$$(38) \quad \begin{cases} \sigma_q^i = 2k(i+q)(i-q) & \sigma_{q+1}^i = 2k(i+q+1)(i-q-1) \\ \sigma_{q+2}^i = 2k(i+q+2)(i-q-2) & \sigma_{q+3}^i = 2k(i+q+3)(i-q-3). \end{cases}$$

Setzt man diese Werthe nun in die $(q+1)^{\text{ste}}$ der Gleichungen (7) ein, welche lautet—

$$0 = \left(1 + \frac{e^2}{2}\right)\sigma_{q+1}^i + e\sigma_q^i + e\sigma_{q+2}^i + \frac{e^2}{4}\sigma_{q+3}^i + \frac{e^2}{4}\sigma_{q-1}^i$$

so kann man aus derselben σ_{q-1}^i durch k ausdrücken. Mit diesem Werthe geht man dann in die 9^{te} Gleichung ein und erhält aus ihr σ_{q-2}^i ausgedrückt durch k. So fortfahrend erhält man schliesslich σ_0^i aus der Gleichung—

$$0 = \left(1 + \frac{e^2}{2}\right)\sigma_2^i + e\sigma_1^i + e\sigma_3^i + \frac{e^2}{4}\sigma_0^i + \frac{e^2}{4}\sigma_4^i$$

in welcher alle Grössen σ bis auf σ_0^i schon durch k ausgedrückt sind. Durch die erste Gleichung, die sich schreiben lässt in der Form—

$$\frac{1}{\psi(v')} - \frac{2+e^2}{4}\sigma_0^i + e\sigma_1^i + \frac{e^2}{4}\sigma_2^i$$

bestimmt man dann k und kann schliesslich die Gleichung—

$$0 = \left(1 + \frac{e^2}{2}\right)\sigma_1^i + e\sigma_0^i + \frac{e^2}{4}\sigma_1^i + \frac{e^2}{4}\sigma_0^i$$

zur Controle benutzen.

Es sind damit also zunächst alle Grössen σ^i bekannt. Weil nun aber—

$$(39) \quad \ldots\ldots\ldots \quad \sigma_i^i = 2i\, a_i^i = \delta_i^i$$

ist, und das Verhältniss aller Grössen δ_μ^i zu einander ebenfalls gegeben ist, so werden dadurch auch die Grössen δ_μ^i bekannt.

Aus den Systemen der Grössen σ und δ erhält man dann auf einfachste Weise die a^i Grössen durch die Gleichungen—

$$(40) \quad \ldots\ldots\ldots \quad \begin{cases} 2(i+r)\, a_r^i = \sigma_r^i + \delta_r^i \\ 2(i-r)\, a_{-r}^i = \sigma_r^i - \delta_r^i. \end{cases}$$

Hervorzuheben ist nun noch, dass die Grössen p, wie schon früher erwähnt wurde, von dem speciellen Werthe des i völlig unabhängig sind. Dieser Theil der Rechnung braucht also nur einmal ausgeführt zu werden, da auch v' nirgends vorkommt. Letzteres gilt schliesslich auch für die Beziehungsgleichungen zwischen den verschiedenen σ Grössen, da nur der allgemeine Faktor k von $\psi(v')$ abhängig ist.

Die Integration der Ausdrücke cos $iv\ dv'$ ist damit für specielle Werthe von v' erledigt. Ihre Verallgemeinerung für alle Werthe des v' geschieht durch mechanische Quadratur. . . . . . .

O'Gyalla, 1882, März.

BEOBACHTUNGEN VON SONNENFLECKEN AM ASTROPHYSICALISCHEN OBSERVATORIUM ZU O'GYALLA.

1882. Rotations periode 282.

Gruppe.	Mittlere Zeit O'Gyalla.	p	ρ	l	L	b	Bemerkungen.
1	Januar 3·4783	339°·46	305″·1	123°·71	31°·73	+13°·72	Fleck an der Westgrenze der Gruppe.
	„ „	347·64	313·1	122·28	30·30	+ 14·95	Grösster, mehrkerniger Fleck.
	„ „	0·13	339·2	118·18	26·20	+ 17·04	Östlichster Fleck.
	„ 8·4879	288·68	873·7	184·09	20·64	+ 16·45	Hauptfleck der Gruppe.
	„ 9·5579	285·65	949·6	188·89	20·17	+ 16·53	
2	Januar 3·4783	128·81	561·9	94·90	2·92	— 24·51	Flecke aus einer sehr unbeständigen Gruppe.
	„ 9·5579	238·36	674·5	182·72	3·00	— 27·23	

Rotations periode 283.

Gruppe.	Mittlere Zeit O'Gyalla.	p	ρ	l	L	b	Bemerkungen.
5	Januar 11·4790	293·27	607·8	160·87	314·75	+ 12·74	Südwestlichster Fleck einer schwachen Gruppe.
	„ 12·4770	286·79	772·6	176·66	316·30	+ 13·38	
	„ 13·4803	299·61	738·7	170·65	295·98	+ 21·71	Fleck derselben Gruppe.
	„ 14·4801	293·06	853·4	185·42	296·49	+ 21·60	
	„ 15·4868	292·07	898·7	192·84	289·54	+ 22·89	Letzte Spur der Gruppe.
4	Januar 9·5579	55·44	890·3	64·18	245·46	+ 27·11	Einzelner kleiner Fleck.
	„ 11·4790	42·52	716·8	89·89	243·77	+ 27·41	
	„ 12·4770	30·57	616·2	103·57	243·21	+ 27·47	
	„ 13·4803	13·12	546·7	117·35	242·68	+ 27·97	
	„ 14·4801	352·28	525·4	131·04	242·11	+ 28·01	

$L_0 = 236^\circ{\cdot}78 \pm 0^\circ{\cdot}167$ $b_0 = + 27^\circ{\cdot}59 \pm 0^\circ{\cdot}387$ $\xi = 13^\circ{\cdot}5975$ $\xi_0 = 13^\circ{\cdot}687.$

Gruppe.	Mittlere Zeit O'Gyalla.	p	ρ	l	L	b	Bemerkungen.
	Januar 11·4790	58·06	624·9	90·80	244·68	+ 14·48	Fleck im S. W. der Gruppe. Der Kern theilte sich zwischen Januar 15 und Januar 18; es wurde die Mitte beider Stücke beobachtet.
	„ 12·4770	45·95	471·0	104·91	244·55	+ 14·23	
	„ 13·4803	17·71	341·0	120·46	245·79	+ 14·55	
	„ 14·4801	338·86	331·5	134·89	245·96	+ 14·55	
	„ 15·4868	310·85	441·0	149·08	245·78	+ 14·65	
	„ 18·4822	282·56	855·6	191·90	245·87	+ 14·22	
	„ 19·4845	279·31	940·7	207·02	246·69	+ 14·31	

$L_0 = 246^\circ{\cdot}98 \pm 0^\circ{\cdot}417$ $b_0 = + 14^\circ{\cdot}43 \pm 0^\circ{\cdot}172$ $\xi = 14^\circ{\cdot}2976$ $\xi_0 = 14^\circ{\cdot}163.$

Gruppe.	Mittlere Zeit O'Gyalla.	p	ρ	l	L	b	Bemerkungen.
6	Januar 12·4770	69·81	934·3	55·81	195·45	+ 14·39	Grösserer schön entwickelter Fleck. Bis Januar 18 zeigte er einen mehrfachen Kern, dessen grösstes Stück beobachtet wurde. Später stellte er sich als einfacher runder behofter Fleck dar.
	„ 13·4803	66·17	856·4	69·51	194·84	+ 14·88	
	„ 14·4801	60·71	732·7	84·36	195·43	+ 15·13	
	„ 15·4868	51·80	587·5	98·77	195·47	+ 15·27	
	„ 18·4822	331·65	352·1	141·23	195·20	+ 14·67	
	„ 19·4845	306·70	473·6	155·52	195·19	+ 15·03	
	„ 21·5886	285·44	778·7	185·68	195·32	+ 15·19	

$L_0 = 195^\circ{\cdot}23 \pm 0^\circ{\cdot}222$ $b_0 = + 14^\circ{\cdot}94 \pm 0^\circ{\cdot}291$ $\xi = 14^\circ{\cdot}2607$ $\xi_0 = 14^\circ{\cdot}150.$

Gruppe.	Mittlere Zeit O'Gyalla.	p	ρ	l	L	b	Bemerkungen.
9	Januar 19·4845	183·28	254·6	137·15	176·82	— 19·69	Einzelnes Fleckchen.
7	Januar 18·4822	19·64	511·9	118·71	172·68	+23·44	Kleiner sich auflösender Fleck.
	„ 19·4845	355·28	467·6	133·11	172·78	+ 23·64	
	„ 21·5886	312·34	602·7	161·88	171·52	+ 23·76	

Gruppe.	Mittlere Zeit O'Gyalla.	p	ρ	l	L	b	Bemerkungen.
	$L_0 = 171^\circ{\cdot}34 \pm 0^\circ 270$		$b_0 = 23^\circ{\cdot}61 \pm 0{\cdot}162$		$\xi = 13^\circ{\cdot}8583$		$\xi_0 = 13^\circ{\cdot}861.$
8	Januar 18·4822	99°·27	961″·8	51°·93	105°·90	— 16°·38	
	„ 19·4845	98·26	907·3	65·27	105·21	— 15·98	
	„ 21·5886	100·88	658·0	94·74	104·38	— 16·39	
	„ 26·4774	232·01	410·5	164·14	104·05	— 16·36	
	$L_0 = 104^\circ{\cdot}68 \pm 0^\circ{\cdot}439$		$b_0 = -\,16^\circ{\cdot}28 \pm 0^\circ{\cdot}199$		$\xi = 13^\circ{\cdot}9968$		$\xi_0 = 14^\circ{\cdot}114.$
12	Januar 31·5059	273·57	818·0	201·50	69·46	+ 9·88	Einzelner kleiner Fleck.
	Februar 1·5085	270·11	911·9	215·34	69·20	+ 9·73	
11	Januar 26·4774	111·71	617·7	116·37	56·28	— 25·43	Vorangehender eines Doppelfleckes.
13	Januar 31·5059	239·94	540·6	179·48	47·64	— 14·67	Einzelner kleiner Fleck.
10	Januar 26·4774	57·91	841·2	85·24	25·15	+ 15·83	Grosser behofter Fleck. Neben dem grossen Hauptkerne, der beobachtet wurde, zeigten sich stets noch 1 oder 2 kleinere. Auch die Penumbra erlitt mannigfaltige Aenderungen.
	„ 31·5059	323·03	400·6	156·81	24·97	+ 16·14	
	Februar 1·5085	301·11	517·5	171·04	24·90	+ 16·10	
	„ 2·5328	288·62	666·1	185·81	25·06	+ 16·27	
	„ 3·4850	281·65	787·1	199·15	24·79	+ 16·15	
	„ 4·4850	276·79	889·9	213·37	24·77	+ 16·03	
	„ 5·5275	273·58	953·7	227·82	24·34	+ 16·05	
	$L_0 = 25^\circ{\cdot}48 \pm 0{\cdot}161$		$b_0 = +\,16^\circ{\cdot}06 \pm 0^\circ{\cdot}138$		$\xi = 14^\circ{\cdot}2080$		$\xi_0 = 14^\circ{\cdot}120.$
15	Februar 1·5085	241·39	183·7	158·10	11·96	— 8·76	Mitte einer aus drei Fleckchen bestehenden Gruppe.
	„ 2·5328	250·85	437·6	160·75	14·57	— 8·05	Kleiner Fleck.

Die benutzten Bezeichnungen sind durchgehends den Spörer'schen Publicationen angepasst. Zur Reduction wurden ebenfalls die Spörer'schen Constanten benutzt Die angegebene Beobachtungszeit ist die bürgerliche vermindert um den Betrag der Aberration. L_0 ist der nach der Methode der kleinsten Quadrate abgeleitete wahrscheinlichste Werth der heliocentrischen Länge, ξ der entsprechende Rotationswinkel; b_0 ist das arithmetische Mittel aller beobachteten heliocentrischen Breiten, ξ_0 endlich der aus der Spörer'schen Formel—

$$\xi = 8^\circ{\cdot}548 + 5{\cdot}798 \cos b,$$

folgende Werth des Rotationswinkels. L_0 und b_0 sind die übrigbleibenden *mittleren* Fehler der einzelnen Beobachtung hinzugefügt, aus deren Grösse man einen Schluss darauf machen kann, ob der betreffende Fleck während der Beobachtungen eine merkliche eigne Bewegung besass oder nicht. Freilich reicht das vorliegende Material noch nicht aus um eine obere Grenze dieses Fehlers bei unzweifelhafter Unveränderlichkeit des Ortes zu geben, da ich erst seit Anfang des Jahres die Beobachtungen und Rechnungen ausführe.

Dr. H. Kobold, Observator.

O'Gyalla, 1882, Februar 10.

UEBER DAS ZODIAKALLICHT.

VON PROFESSOR W. KLINKERFUES.

Was die Entstehung des Zodiakallichts betrifft, so lässt sich beweisen, dass die Cometen, darunter vorzugsweise die von Kurzer Umlaufszeit um die Sonne, indem sie sich in Sternschnuppenringe auflösen, in der Gesammtheit des von diesen Ringen reflectirten Sonnenlichts gerade eine solche Erscheinung, wie das Zodiakallicht mit allen seinen oft auffallenden Eigenthümlichkeiten, hervorrufen müssen. Da nun dieses zu Erwartende auch wirklich geschehen wird, so rechtfertigt sich durch einen einfachen Schluss die Annahme, dass das Zodiakallicht sich nach und nach aus Cometentrümmern aufgebaut hat und noch weiter ausbaut.

Göttingen, 1882 März.

NOTE ON THE ASPECT OF MARS IN 1881–82.

BY C. E. BURTON.

Professor Schiaparelli has continued his researches on the configurations and changes of the several classes of markings on this planet by the aid of the fine refractor of 218$^{\mathrm{mm.}}$ (8·7 inches) aperture which was employed by him in studying them during the apparitions of 1877–8 and 1879–80. The most remarkable result of his latest researches* has been the confirmation and utterly unexpected extention of a discovery first made in 1879; that under certain circumstances (one of which seems to be the neighbourhood of the planet to the vernal equinox of its northern hemisphere) the dusky streaks, provisionally named "canals," become double in whole or part of their length—not by the central division of a previously existing streak and the increase of the distance between the parts—but by the production of a new streak to the right or left of that originally existing, which retains its position unchanged. In some cases a slight difference of aspect between the original streak and its companion can be perceived and occasionally they are slightly divergent in direction. The premonitory symptoms of duplication have been detected in more than one instance, notably those of the Ganges, Euphrates, Titanum, and Pyriphlegethon, in the appearance of a nebulous shading on one side which speedily presents itself under the aspect of an independent dusky streak.

It would also seem, that a tendency to division lengthways is occasionally manifested by some of the coarser markings to which a certain degree of permanence has hitherto been attributed. This is markedly displayed by the large dark spaces denominated by Prof. Schiaparelli the Mare Cimmerium (the eastern half of the Maraldi Sea of Mr. Green) and the Sinus Sabæus (Herschel II Strait). It is also remarkable that all the principal projections southwards from the dark girdle of the north polar region, part of which bears the name of the Mare Boreum and the Sinus Alcyonius, and may be identical with the Campani, Schröter and Delambre Seas of Mr. Green's map, are divided in the direction of a parallel or nearly so, by a narrow bright line. Part of this bright line seems to be indicated on Mr. Green's map as Laplace Land and as a division between Knobel and Airy Seas and Campani Sea to the north of them.

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* Published in March last in a short preliminary note in the Transunti della R. Academia dei Lincei (6 pp.).

Professor Schiaparelli further notes that there is reason to believe that some of the principal markings undergo changes of form of very large amount. This was very evident in the case of the Syrtis Major (Kaiser Sea) which was seen to have increased considerably in breadth on its western and north-western side as well as to have extended itself as a broad dark ribbon of double curvature far to the northward of its apparent termination in that direction in 1879. The Nepenthes and Lacus Moeris (Main Sea) were much broader and darker than in 1879 while there remained scarcely a trace of the Coloe Palus (part of Nasmyth Inlet), which had been so conspicuous. These changes render it demonstrable that the dark markings are due to some obscuring cause which is capable of being transferred from place to place, and it is suggested that this may be some liquid or vegetation, though it is difficult to see how the former could be instrumental in bringing about the duplication of the "canals" in the manner described.

The westward encroachment of the Syrtis Major (Kaiser Sea) on Libya seems to have begun between the termination of Prof. Schiaparelli's observations in 1878 and their re-commencement in 1879, having been detected at the first examination of this region under sufficiently favourable circumstances on October 24–29, 1879. The amount of this encroachment may be judged from the fact that in 1879 the distance from the Triton Lacus to the Lacus Moeris was twice the distance from the latter to the Syrtis Major, while in 1877 the reverse had been the case. In 1882 the Triton Lacus seems to have become much less conspicuous than at the two previous oppositions and is not traceable on a provisional chart issued from Milan, while the Lacus Moeris has become much larger and darker and is partially immersed in the dusky nebulosity spreading westward from the Syrtis Major. This state of things appears to have become much more marked in March last when I looked in vain for the Lacus Tritonis, though the principal canal in its neighbourhood, probably the Lethes, and the brilliant point Nix Atlantica were very distinct. At this latter epoch the darkest part of the Syrtis Major inclined south-eastward and appeared as if about to effect a junction with the Sinus Sabæus (Herschel II Strait), the whole of the darker shadings in this region resembling more closely those shown on the sketches of Mr. Knobel (Month. Not. XXXIII) and my own given in the Transactions of the R. Irish Academy Vol. XXVI (Nos. 5, 6, 15, 16) than the appearances seen at Milan during the last oppositions.

Using a nine-inch Newtonian (silvered glass) I have during the past opposition seen the following "canals" which can be identified with Schiaparelli's: Lethes, Astapus, Astaboras, part of Nilus, the northern half of Phison, Indus south of the elbow, with its continuation the Oxus. Chiefly owing to bad weather I have had no opportunity of studying the regions between 90° and 250° longitude.

The general impression left after reading Prof. Schiaparelli's monographs is, that the larger and more conspicuous features possess a high degree of permanence, while there are continual and possibly periodical changes affecting the minuter details which will require prolonged study under the most favourable conditions to give an insight into the law of their action. But it is to be hoped that observers in good climates will in future follow Mars longer than has hitherto been customary until Prof. Schiaparelli led the way. The least unsteadiness of the image is far more destructive to the advance of enquiry than a considerable increase of distance.

A FEW ADDENDA TO ADMIRAL SMYTH'S "CYCLE."

In the library at Dun Echt are two copies of Baily's "New Tables for facilitating the computation of Precession, &c." (London 1827), which formerly belonged to Dr. Lee, of Hartwell. At the beginning and end of the volumes a number of leaves have been bound, on which are entered various instrumental constants and notes on the methods of observing practised at Hartwell and doubtless also at Bedford. On the margins of both volumes are many manuscript notes of colour and duplicity of stars, generally without dates or references, but evidently for the greater part taken from Herschel's and South's observations. Many of these notes agree with the "Cycle," but many others refer to objects not contained in that work. On one of the fly-leaves are given the results of the single observations made by Admiral Smyth with Troughton's reflecting circle to find the latitude of Hartwell Observatory. As these are the observations on which the latitude so long given in the Nautical Almanac are based, it seems worth while to repeat the results here:

1829, April 5.	By Sun's lower limb	51° 48′ 34″·4
	Procyon	37·3
	Regulus	32·6
April 8.	Sun's lower limb	38·2
		51° 48′ 35″·62

The mean value is to be found on p. 169 of the "Speculum Hartwellianum."

On another fly-leaf are tables for reducing measures made with the bi-filar and rock-crystal micrometers used at Bedford and Hartwell, based on the values given in the Cycle, Vol. I. p. 339, viz.: $1^r = 19''{\cdot}845$ for the filar and 1 division $= 0''{\cdot}27$ for the chrystal micrometer. The table for the filar micrometer is arranged in columns of twenties with revolutions or divisions as argument, so that the most probable errors from using the wrong column of the table are $20^{r} = 6' \, 36''{\cdot}9$ or $20^{p} = 3''{\cdot}969$ in defect. On the opposite page there is a diagram of the position circle (similar to the one on p. 342 of the Cycle); the divisions go from 0° to 360° in the same direction as position angles are counted, but as the circle rotates, the readings are in the opposite sense. The same page contains the following list of the eye-pieces (including that of the finder) with their magnifying powers, which will be useful to readers of the Cycle. We see that the crystal had but one power, while the filar micrometer had seven—

No.	Quality.	Times.	No.	Quality.	Times.
1	The Finder	$5\frac{3}{4}$	13	Single lens (revolver)	185
2	Negative	22	14	Positive (micrometer)	228
3	Negative	50	15	Negative	240
4	Positive (micrometer)	62	16	Positive (micrometer)	340
5	Negative	66	17	Single lens (revolver)	350
6	Terrestrial	90	18	Negative	416
7	Negative	93	19	Single lens (revolver)	510
8	Positive (micrometer)	110	20	Positive (micrometer)	600
9	Negative	118	21	Single lens (revolver)	650
10	Positive (micrometer)	142	22	Positive (micrometer)	850
11	Spherical Crystal	145	23	Single lens (revolver)	950
12	Negative	147	24	Single lens (revolver)	1200

It is scarcely necessary to remark that Nos. 13, 17, 19, 21, 23, and 24 are the single lenses of the "Polycratic Wheel" of the Celestial Cycle.

COMET WELLS 1882.

This comet was discovered by Mr. S. C. Wells, Assistant at the Dudley Observatory, Albany, on March 17, 16^{h}, in RA $17^{h} \, 52^{m}$, Decl. + 32° 30′. The Dun Echt Circulars, Nos. 46–50, contain elements computed by Messrs. Chandler, Boss, Holetchek, Hind (two sets), Meyer, and Oppenheim, all of which are founded on observations made within eleven days of the discovery. The systems as yet published, which are based on the largest geocentric arcs, are those given by Mr. Hind in *Nature*, April 20, founded on observations made at Harvard College, March 19, Vienna, March 28, and Rome, April 6, and by Dr. Lamp (A.N. 2428) from observations of March 19, 30, April 9 and 27 :—

$T =$	1882, June, 9·96974, Gr. M. T.		1882, June, 10·56380, Berlin M. T.	
$\pi =$	54° 25′ 17″·2	App. Eq. April 0.	53° 54′ 39″·7	M. Eq. 1882·0.
☊ $=$	204 37 42·1		204 54 49·5	
$i =$	73 35 39·4		73 47 29·2	
$\log q =$	8·748238		8·783674	
	Direct Motion.			

The comet will become rapidly brighter in the course of May and may possibly become visible in daylight about June 8, when it will pass very close to the Sun.

BEOBACHTUNGEN DES COMETEN WELLS.

Angestellt von Herrn B. von Engelhardt am Fadenmicrometer des Aequatoreals (von 12 Zoll) seiner Privatsternwarte zu Dresden.

1882.	Mittlere Zeit Dresden.	$a' - a$	$\delta' - \delta$	Zahl der Vergl.	Æ app.	l.f.p.	Decl. app.	l.f.p.	Vergl. Stern.
April, 5,	9^{h} 13^{m} 40^{s}	$+ 1^{m}$ $58^{s}\cdot27$	$+ 6'$ $28''\cdot0$	16:8	18^{h} 26^{m} $38^{s}\cdot23$	9·630*n*	$+ 43°$ $57'$ $15''\cdot6$	0·852	1
„ 5,	10 44 11	+ 2 7·35	+ 9 21·9	16:8	18 26 47·32	9·704*n*	+ 44 0 9·5	0·756	1
„ 6,	8 59 35	+ 1 16·81	+ 4 9·5	16:8	18 28 58·54	9·619*n*	+ 44 42 50·1	0·860	2
„ 7,	8 36 51	− 2 30·37	− 4 55·2	16:8	18 31 21·69	9·591*n*	+ 45 29 2·5	0·875	3
„ 7,	11 47 21	− 2 10·28	+ 1 25·9	16:8	18 31 41·78	9·723*n*	+ 45 35 23·6	0·648	3
„ 10,	12 11 16	− 1 16·88	− 3 54·1	16:8	18 39 36·61	9·575*n*	+ 48 3 33·9	0·570	4
„ 12,	13 36 31	+ 0 26·71	+ 8 3·4	12:12	18 45 34·68	9·680*n*	+ 49 50 22·7	0·311	5
„ 15,	11 25 59	+ 3 10·58	− 4 12·2	12:6	18 54 59·87	9·785*n*	+ 52 28 30·8	0·619	6
„ 15,	12 15 31	+ 3 17·83	− 2 17·0	12:6	18 55 7·12	9·772*n*	+ 52 30 26·0	0·490	6

Mittlere Oerter der Vergleichsterne für 1882·0.

	a (Wolfers).	δ (Auwers).	Autorität.
1	18^{h} 24^{m} $38^{s}\cdot59$	$+ 43°$ $51'$ $1''\cdot1$	Groombridge, Rümker, Radcliffe.
2	18 27 40·37	+ 44 38 54·0	Fundamental Catalog, Auwers' Publ. XV.
3	18 33 50·71	+ 45 34 10·9	„ „ „ „
4	18 38 43·54	+ 48 1 53·5	Anschluss an Bonner Beob. VI. + 48°, 2749.
5	18 45 6·58	+ 49 42 33·2	Argelander-Oeltzen, 18^{h}, 18654.
6	18 51 47·88	+ 52 32 55·7	Rümker 18^{h}, 6903.

OSSERVAZIONI DELLA COMETA WELLS FATTE AL OSSERVATORIO DEL COLLEGIO ROMANO.

	T. M. Roma	α app.	log $p\Delta$	δ app.	log $p\Delta$
1882, Marzo 25	$13^h\ 34^m\ 46^s$	$18^h\ 4^m\ 41^s\!\cdot\!49$	9·701*n*	+ 36° 39′ 58″·7	0·484
„ 29	11 13 21	11 59·41	9·757*n*	39 5 38·0	0·715
Aprile 2	11 24 16	20 9·46	9·778*n*	41 49 6·6	0·670
„ 6	14 38 7	29 33·62	9·632*n*	44 54 34·9	9·874
„ 11	10 35 18	42 14·84	9·819*n*	48 56 32·9	0·691

P. Tacchini.

Roma, 12 Apr., 1882.

UEBER DIE ERSTE AUFFINDUNG DES PLANETEN NEPTUN.

Aus den in dem März-Hefte des "Copernicus" veröffentlichten Notizen über die Theilnahme und Hülfsleistungen des verewigten d'Arrest bei der ersten Aufsuchung des Planeten Neptun scheint hervorzugehen, dass es Aeusserungen von d'Arrest selbst gewesen sein dürften, die durch Missverständnisse successive umgestaltet den in Band 89 der Astr. Nachrichten erwähnten irrthümlichen Bericht über diesen Gegenstand erzeugt haben und über dessen Ursprung mir bisher nichts bekannt geworden war. Nachdem ich von der in dieser Hinsicht dankenswerthen Veröffentlichung Kenntniss genommen, muss ich nun aber doch um die Gestattung der nachfolgenden Bemerkungen darüber bitten, die ich theils dem Andenken Encke's in Betreff seiner diese Details nicht speciell erwähnenden Publication, theils auch mir selbst dabei schuldig zu sein glaube.

Le Verrier's ausführlicher, auf eine frühere Correspondenz von mir Bezug nehmender und durch den Ausdruck eines besondern persönlichen Vertrauens mir werthvoller Brief vom 18 Sept. 1846, den ich am 23 Sept. erhielt, gab zu der Nachsuchung in Berlin bekanntlich den ersten Anlass. Unmittelbar nach dem Empfang zeigte ich diesen Brief Encke, welcher einer Berücksichtigung dieser Angelegenheit vorher sehr abgeneigt war, nunmehr aber zustimmte, mir jedoch die Ausführung dieser mir persönlich zugegangenen Aufforderung allein überliess. Inzwischen sprach d'Arrest, davon vernehmend, den Wunsch aus, an der Beobachtung theilnehmen zu dürfen. Obgleich es nun nicht meine Absicht sein konnte, diese Nachsuchung und die Möglichkeit eines etwa sich ergebenden Erfolges in wesentlichen Puncten an einen andern Beobachter abzutreten, so lagen doch dergleichen Gedanken hierbei fern und es würde mir unfreundlich erschienen sein, den Wunsch dieses jungen eifrigen Astronomen irgendwie abzulehnen, so dass ich zu der Begleitung gern meine Zustimmung gab. Derselbe half daher bei dem Aufschreiben, dem Nachsehen auf der Karte und vielleicht noch einigen andern

Operationen, die ich im einzelnen nicht mehr in der Erinnerung habe: welche ich jedoch, wie nicht näher erörtert zu werden braucht, ohne erheblichen Zeitverlust durchaus auch selbst hätte ausführen können, und die daher zwar persönlich dankenswerth, sachlich aber irrelevant waren. Dagegen habe ich stets als einen wesentlich werthvollen Beitrag für eine schnellere Aufsuchung des Planeten das baldige Erinnern d'Arrest's an die akademischen Karten betrachtet, obgleich eben auch hierbei die Meinungen verschieden sein können und sein konnten, in wie weit es schliesslich für mich des Erinnerns an die akademischen Karten unumgänglich und nothwendig bedurft habe. Immerhin thut es mir leid, dass sich für mich nicht früher schon eine genügende Veranlassung gefunden hat oder ich eine solche nicht früher gesucht habe, die in Bd. 89 der Astron. Nachrichten gegebenen, jene freiwilligen Hülfsleistungen d'Arrest's sehr verspätet würdigenden, Details mitzutheilen: auf welche ich hier, um Wiederholungen zu vermeiden, und in Verbindung mit den vorstehenden Bemerkungen, in Bezug auf alle einzelnen Puncte zurückverweisen muss.

Breslau 1882 April 14.

J. G. Galle.

RECENT LITERATURE.

Sur le compagnon de l'étoile γ d'Andromède et sur un nouveau mode de réglage d'un équatorial (Comp. Rend. T. XCIV. pp. 410 and 411) is the title of a note by M. Ch. André on the performance of a new equatorial of 6 Paris inches aperture, made by Messrs. Brunner for the Lyons Observatory. With a power of 200 the companion of γ Andromedæ is neatly divided into two stars of the 8th and 11–12th magnitudes, the distance being measurable with considerable difficulty. On employment, however, of a special screen in front of the object-glass (most probably of the kind frequently used by M. André in his experiments on diffraction and consisting of a number of concentric rings*) the distance could be appreciated with certainty and was found to be $0''\cdot65$. This is slightly below the limit of separation, $0''\cdot713$, assigned by Dawes for an object-glass of the size used. On account of its admirable optical qualities it is intended to use the telescope for the measurement of double stars.

The method of adjustment presupposes the use of a spirit-level applied to a cylindrical and adjustable rod attached to the telescope parallel to the declination axis, and also of a trough of quicksilver and a nadir eye-piece. The latitude being known, all the adjustments, excepting that for azimuth, can evidently be made with the apparatus mentioned, without reference to the heavens. The cylindrical rod would indeed not be required for many English instruments of medium size, in which the declination axis is provided with a striding level.

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* Previous to 1835 Dawes, at the suggestion of Sir J. Herschel, had used a central stop "for the purpose of diminishing the images of the stars." See Mem. Roy. Ast. Soc., Vol. VIII., p. 63.

"*G. V. Schiaparelli : Osservazioni . . . . del Pianeta Marte, Memoria seconda, osservazioni dell' opposizione* 1879–80" (Roma 1881, 109 pp. 4to with 6 plates). In our December number an abstract of this memoir, which appeared last autumn, was reviewed by Mr. Burton. The memoir itself has now been published, and is fully as interesting as its predecessor of 1878, but differs from it in the appearance of the Map of Mars from the observations of 1879–80, on which the surface markings are shaded according to their different degrees of intensity. This is a very great improvement on the map from the observations of 1877, on which only sharp outlines were given.

"*Notes on the Physical Appearance of the Planet Jupiter during the Season* 1880-1. *By Dr. O. Boeddicker*" (Trans. R. Dublin Soc. Vol. I., 4 pp. and one plate). Twenty-five drawings made with Lord Rosse's 3 feet telescope accompany these notes ; they are reproduced by the Woodbury type-process and look extremely well, but probably the original drawings contained much more detail. The notes refer to the time of observation, definition, colour of the markings, &c. ; they contain no measures or transits of spots across the central meridian.

Publications of the Cincinnati Observatory, 6 ; *Micrometrical Measurements of* 455 *Double Stars observed during the year ending September* 1, 1880 (Cincinnati 1882, 69 pp.). These measures were made by Professor Ormond Stone and Messrs. Howe and Egbert, whose personal equations were determined. The double stars observed are partly those found by Mr. Burnham which seemed to need re-observing, partly objects situated between 0° and − 30° Declination, for which zone a general catalogue of all known doubles is in preparation. The observations are given with all details.

Reports of Astronomical Observatories for 1880 (Washington 1881, 126 pp. 8vo, from the Report of the Smithsonian Institution for 1880). We have already, (Vol. I. p. 241) noticed the corresponding Series of Reports for 1879. The one for 1880 has, owing to the absence of Professor Holden at Madison, been prepared at the Smithsonian Institution, and a circular is affixed, requesting readers to furnish any corrections or omissions they might notice. It is to be hoped that this invitation will be responded to, as there is a very considerable number of errors in the Reports, not only misprinted names, but also erroneous lists of directors of Observatories. If cleared of these errors the collection of Reports would furnish most important materials for a future historian of astronomical science, and directors of European and American observatories have it in their own power to increase the value of the collection by furnishing the Smithsonian Institution with the necessary annual reports. Some of the communications might be shortened with advantage.

Publicationen des astrophysicalischen Observatoriums zu Potsdam. Zweiter Band, 1881. This volume contains besides meteorological observations the following three memoirs: Beobachtungen der Sonnenflecken vom Januar 1874 bis December 1879, von G. Spörer; Darstellung des Sonnenspectrums bei mittlerer und schwacher Dispersion, von G. Müller; Beobachtungen des grossen Cometen von 1881 (Comet III. 1881), von H. C. Vogel.

Professor Spörer's Observations of Sun Spots are arranged exactly like those from 1861–78 previously published by the same observer and are accompanied by 32 plates. From the observations the angle of rotation is deduced in 78 cases and compared with the formula $\xi = 8^{\circ}{\cdot}548 + 5^{\circ}{\cdot}798 \cos b$ (where b is the latitude), and here as formerly it is found, that very great deviations from the formula always indicate a motion towards the West (*i.e.* in the direction of the rotation). The Author ascribes this phenomenon to atmospheric currents directed vertically downwards, which transfer the greater linear velocity of rotation of the higher strata to the surface. The amount of the forward motion would thus depend on the excess of the actual height over the normal height in which the currents originate. The formula of rotation is explained by assuming, that currents from a certain depth within the body of the Sun reach the surface and transfer the lesser linear velocity from a lower level to the surface, while for the spot thus produced a decrease of the eastward drift is brought about by the downward atmospheric currents. In the case of spots of long duration the currents from the interior as also the atmospheric ones must be supposed to continue steadily without any considerable disturbance and only strong enough to sustain the spot. When such spots reach the solar limb, flaming protuberances are never seen. The observations show also, that such lasting spots in all heliographic latitudes are never subject to violent motions nor to great changes of shape. Isolated penumbrated spots originate generally from large groups, of which the other members disappear, while the causes to which the formation of the group was due, now have decreased so much that they only suffice for the preservation of the one spot. When these causes are again intensified, the spot is often torn asunder, and the group thus produced may then a second time give place to a regularly shaped spot. These revolutions, when they occur during cloudy weather or while the spot is on the far side of the Sun, often make it impossible to reconcile the observations with any formula of rotation, and even from one day to another disturbances may take place which reveal themselves less through the shape and appearance of the spot than through a sudden change of its heliographic longitude. Of the various tabulated results of Professor Spörer's observations we shall only mention two, which express the mean heliographic latitude as a function of the time elapsed since a maximum. If t and t_1 be expressed in fractions of a year and counted respectively from 1860·46 and 1870·84, the following results are found from the observations made in 1856–68 and 1866–1878 :

$$b = 16^\circ{\cdot}31 - 2^\circ{\cdot}36\, t + 0^\circ{\cdot}184\, t^2$$
$$b_1 = 17{\cdot}50 - 2{\cdot}21\, t_1 + 0{\cdot}139\, t_1^2.$$

The second memoir, by Dr. G. Müller (15 pp. and 2 plates) gives representations of the solar spectrum as seen with instruments of mean and low dispersion. The former extends from B to H and is drawn on a scale proportional to the wave-lengths, the latter extends from C to H and is drawn on a scale which increases from the red to the violet in order to approach nearer to the actual appearance of the prismatic spectrum. No measures were taken, but the lines visible in the two instruments employed were identified on Ångström's and Vogel's plates. Dr. Müller's two maps will no doubt be found very useful in observations of stellar spectra or in observing the Sun Spectrum with smaller instruments.

Dr. Vogel's Observations of Comet III. 1881 (14 pp. and one plate) relate to the physical appearance of the head and tail and to the spectrum. The mean results for the wave-lengths of the three spectral bands were—

First	band, beginning	. . . 562·9	Mill. Mm.
Second	,, ,,	. . . 517·4	,, ,,
Third	,, middle or brightest part	. . . 469·8	,, ,,

in very good accordance with Dr. Hasselberg's mean results from 18 comets.* The relative intensities of the various parts of the bands agreed better with those of the hydro-carbon spectrum than was the case in several other comets. Some laboratory experiments made by Dr. Vogel seem to indicate, that the Comet-Spectrum is composed chiefly of the spectrum of a hydro-carbon, but modified by the addition of the carbon monoxide spectrum ; also, that the greater part of a comet's light probably arises from electric discharges.

Astronomical Observations and Researches made at Dunsink, Part IV., Dublin 1882 (85 pp. 4to). This part contains the results of about 1140 meridian observations of 321 Red Stars (nearly all from Schjellerup's Catalogue), made in 1875–76 by Copeland and in 1878–80 by Dreyer. The separate mean results are given for 1875·0 and are accompanied by notes on the magnitude and colour of the stars. The introduction contains a full description of the Pistor and Martins Meridian circle with which the observations were taken and determinations of the errors, constants, &c.

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* Copernicus I., p. 83.

SPECTROSCOPIC AND OTHER OBSERVATIONS OF SCHMIDT'S NOVA CYGNI MADE AT DUN ECHT OBSERVATORY.

(Communicated by the Earl of CRAWFORD and BALCARRES.)

It will be remembered that this remarkable object was discovered by Dr. J. F. J. Schmidt at Athens on November 24, 1876, at $5^h\ 45^m$ P.M. as a star between the 3rd and 4th magnitude shining with an intense golden-yellow almost reddish light.* Although Dr. Schmidt that evening telegraphed the news of his remarkable discovery to Vienna, it was only by very slow degrees that any particulars found their way into the scientific periodicals. Not until late in December did any notice of the new star reach Dun Echt, and then only through the daily newspapers. Eventually a succession of bad weather deferred the first observation until January 2, 1877, by which time the star had already declined to the 7th magnitude. From that date until the middle of the following month spectroscopic and other observations were made whenever the weather permitted. From an impression that the spectrum of a star invisible to the naked eye would not bear a great amount of dispersion along with the amplification of the viewing telescope of a regular spectroscope, one of Professor Vogel's spectroscopes was used throughout this first series of observations and there is little reason to doubt that the instrument employed was well calculated to reveal the fainter details of the spectrum although it might not furnish the best attainable measures of the wave-lengths of the more obvious lines. Decreasing brightness of the star combined with increasing hour-angle and the evening twilight to put a stop to the observations on the 16th of February and the star was not again looked at until September 2, 1877, when the startling fact was at once apparent that the spectrum was restricted practically to a single line; the estimated magnitude being then 10½. Spectroscopic measures were made of this line on September 2 and 3 and October 10, besides which the object was frequently tested prismatically in a less regular way. From the position of the star's light in the spectrum its rays did not accord in focus with the mean of the brighter rays of other stars and it is possibly from this cause that the new star at this stage of its history generally seemed to have a small disk, one or two seconds in diameter. Towards the end of October 1877 Nova Cygni had decreased still further in brightness and so pointed to the possibility that it might eventually fade altogether from view. In the event of such a contingency it seemed very desirable to possess a map of the small stars in the immediate neighbourhood; accordingly measures in position and distance from the new star to surrounding objects were commenced on October 29, and added to as occasion offered until the whole series of measures extended to 38 nights, the last being 1882, March 24. The resulting map includes every star fairly measurable with the Dun Echt refractor of 15·06 inches aperture within 7½′ of Nova Cygni as well as 12 stars, generally brighter ones, beyond that limit. In this place it may be well to particularize a very small star probably about the 15th magnitude at a distance of 19″·1, in position angle 314°·2 from Nova, as in a superficial examination it might possibly be confounded with the new star in the event of the total disapperance of the latter.

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* Ast. Nach., No. 2113; at the end of this paper will be found a list of the principal memoirs on Nova Cygni.

Observations.

1877.

Jan. 2.—Nova about the 7th magnitude and of a decided red colour. In the Vogel spectroscope the spectrum is of surprising brilliancy, consisting of a faint continuous spectrum intensified into five bright lines. The positions of the lines are given in parts of the scale of the instrument as actually recorded; the wave-lengths have been derived from a comparison of the spectra of moonlight and various Geissler tubes, checked by an examination of the solar spectrum. The method of reduction will be given further on.

					W. L. Mill. mm.		
Line.	Scale.	Red.	2nd series.	Red.	1st ser.	2nd ser.	
1	1^{p}·0	— ^{p}·02	1^{p}·2	— ^{p}·02	(657·8)	(654·7)	Intense bright line.
2	7·6	— ·08	7·7	— ·08	579·9	579·0	Pretty bright band in the yellow, fading off rapidly on both sides.
3	18·7	— ·14	18·2	— ·14	502·4	505·1	Bright well-defined line.
4	22·0	— ·15	21·5	— ·15	486·1	488·4	Do. do. do.
5	27·5	— ·17	26·6	— ·16	463·4	466·8*	Faint band.

Adopted readings for C and F for this day: 1^{p}·1 and 22^{p}·0, the latter from the lunar spectrum; reductions to curve — 0^{p}·02 and — 0^{p}·15.

Spectroscope directed to the Moon.

	Scale.	Red.	W.L. mmm.
D	6^{p}·5	— 0^{p}·07	590·1
E	14·6	— 0·12	526·3
b (mean)	16·0	— 0·13	517·7
F	22·0	— 0·15	(486·1)

Jan. 6. Spectrum of a Hydrogen tube.

	Scale.	Red.	W.L.
C = H*a*	1^{p}·0	+ 0^{p}·08	656·2
	5·0	+ 0·07	604·1
	7·2	+ 0·07	582·2
	9·8	+ 0·06	559·8
	15·8	+ 0·06	517·8
F = H*β*	21·8	+ 0·05	486·1
	28·7	+ 0·05	458·2

+ 0^{p}·08 and + 0^{p}·05 adopted as reductions for C and F. The adjustments of the Spectroscope were then slightly changed.

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* The wave-length of this line published in Astr. Nach., Bd. 89, p. 63, and also that of line 7 on p. 79 of the same volume, are largely in error owing to mistaken identifications of the lines used in constructing the upper, or violet, part of a provisional reducing curve.

1877.
Jan. 6.

Spectrum of Solar Light.

Line.	Scale.	Red.	Wave-length. mmm.	C — O. mmm.	C — O.
C	$1^{p}\cdot6$	$-\,0^{p}\cdot52$	656·2	0·0	$0^{p}\cdot00$
D	6·9	— 0·45	589·9	— 0·7	+ 0·07
E	14·8	— 0·39	526·8	+ 0·1	— 0·01
b (mean)	16·4	— 0·38	516·8	+ 0·5	— 0·08
F	22·2	— 0·35	486·1	0·0	0·00
?	32·2	— 0·31			
W.L. 438·3	35·8	— 0·30	437·0	+ 1·3	— 0·50
W.L. 434·0	37·2	— 0·30	433·2	+ 0·8	— 0·32
G	37·9	— 0·30	431·3	— 0·6	+ 0·20

Adopted reductions: — $0^{p}\cdot52$ for C, and for F — $0^{p}\cdot35$.

On this day a number of other vacuum-tubes containing traces of various gases were also observed, but the results are of little value, the most prominent lines being those of Hydrogen as an impurity, the readings for which agree closely with those just given. These observations are given chiefly for the purpose of establishing the identity of lines Nos. 1 and 4 in the Spectrum of Nova with Hα and Hβ.

Jan. 8.—11^{h} G. M. T., Nova red with a tinge of purple. Estimated at 3 grades brighter than *a*, D.M. + 41° 4243, which is given as $7^{m}\cdot0$ in the D.M., but the new star does not seem to be brighter than the 7th magnitude. The Spectrum is more continuous than on January 2nd.

Lines without Slit.

Line.		Red.	W.L. mmm.	
1	$0^{p}\cdot0$	$+\,1^{p}\cdot08$	(656·2)	. Deep glowing red.
3	18·5	— 0·03	502·9	. Bright blue.
4	22·0	— 0·15	(486·1)	„ „
5	(27·25)	— 0·30	464·8	. An exceedingly faint line at 1½ interval of the blue lines towards the violet, say at $27^{p}\cdot25$.

The reductions are based on the assumed identity of the 1st and 3rd lines with C and F.

Jan. 9.—8^{h} G. M. T., Nova is now of the same brightness as *a*.

Spectroscope Readings.

Line.	Scale.	Red.	W.L. mmm.	
1	$0^{p}\cdot5$	$+\,0^{p}\cdot58$	(656·2)	. Red line.
1*a*	6·0	+ 0·49	589·5	. Very narrow line.
2	7·5	+ 0·47	575·9	. Pretty bright line.
3	18·5	+ 0·37	500·7	
4	21·5	+ 0·35	(486·1)	
5	27·5	+ 0·32	461·6	
7	35·0	+ 0·29	437·6	. Exceedingly faint line, but still certainly seen.

The readings of C and F have been taken as the basis of the reductions.

The space between the red line and the next one is certainly "columnar" and probably contains two maxima. See intensity-curve (plate IV.), which was sketched at the telescope.

The nebula in Orion gives a bright line at $18^{p}\cdot4$ of the Spectroscope scale.

Jan. 17.—10^{h} G. M. T., Nova 2 grades fainter than *a* and a full magnitude brighter than *c*, D.M. + 42°, 4188, which is $8^{m}\cdot1$ ±. No change in the spectrum.

Jan. 19.—Nova, which is much less red than when last seen, is 4 grades fainter than *a* and 10 brighter than *c*.

In the spectroscope the C line is fainter and the line at W.L. 500 is now the brightest in the spectrum. The lines near D cannot be made out although the one at 438 ± is still visible when exactly in focus.

The colour is scarcely deeper than "orange" and can hardly be called "full orange."

1877.

Jan. 24.—Nova one grade brighter than *b*, D.M. + 41°, 4237.

Jan. 25.—When viewed within 50 minutes of the lower meridian, altitude 10°·0, the new star was seen to scintillate *red* and *blue*, while the intermediate tints exhibited by other stars in the neighbourhood were completely wanting.

Nova 4 grades brighter than *b* and 6 fainter than *a*.

Jan. 27.—Nova 3 grades brighter than *c*, 4 brighter than *b*, and 5 fainter than *a*.

Line.			Scale.	Red.	W.L.
1	C line seen as also traces of the yellow lines.				
3	A very bright line,	at	17^{p}	$+\ 1^{p}{\cdot}85$	500·8
4	Bright line, F	,,	20	+ 1·85	(486·1)
5	Pretty bright line,	,,	26	+ 1·85	461·5
	Faint line towards the violet.				...

The correction for F has been adopted for the other lines.

The line at 17 is by far the brightest in the spectrum, while C is only just certainly visible.

The scintillation of the star shows green, yellow, and red; the red being comparatively faint.

Jan. 30.—Nova 2 grades brighter than *b*, 4 brighter than *c*, and 5 fainter than *a*.

With the spectroscope: Line C, well seen.
3, very bright.
F, bright.
5, well seen.

The moonlight very troublesome.

Feb. 3.—Nova equal to *b* and 3 grades brighter than *c*.

Spectrum Line 1, red line, fairly seen.
2, yellow line, fairly seen.
3, ,, very bright.
4, ,, pretty bright.
5, ,, seen.

There is also a good deal of continuous spectrum from one end to the other with a maximum about *b*.

Feb. 7.—Nova 1 grade fainter than *b*, also 1 grade fainter than *c*, and 6 fainter than *a*. *b* specially estimated equal to *c*.

Nova is deep orange but not so red as the star *d* (D.M. + 42°, 4184 = No. 95 of the list) immediately following, which may be classed as "light red."

The intensity curve, plate IV., gives an idea of the relative brightness of various parts of the spectrum on this day.

Feb. 13.—Nova ½ grade brighter than *c*, 5 fainter than *b*, and 12 brighter than *d*.

The lines in the spectrum are still remarkably bright. The relative intensities are:—

No. 1. A mere reddish glow visible after long attention, but afterwards distinctly seen.
No. 2. Excessively faint.
No. 3. By far the brightest.
No. 4. Fairly bright.
No. 5. Faint.
No. 7. Very faint.

The spectrum is continuous from D to F. Line No. 2 is 10·5, 11·0, 10·4 parts of the spectroscope scale by 3 independent estimates below No. 3. No. 1 is 19·0, 18·4 below No. 3.

Adopting the curve reading $18^{p}{\cdot}58$ corresponding to the wave-length $502^{mmm}{\cdot}3$, for line No. 3 we have for the estimations of the other two lines—

		W.L.
Line No. 2,	$8^{p}{\cdot}08$	574·9
,, ,,	7·58	579·4
,, ,,	8·18	574·0
Line No. 1,	− 0·42	681·5
,, ,,	+ 0·18	670·8

Feb. 16.—8^{h} G. M. T., *b* 1 grade brighter than Nova. Nova equal to *c*.

This closes the series of measures with the Vogel Spectroscope and the regular estimates of brightness.

Reduction of the Observations with the Vogel Spectroscope.

In the *Berichte über die Verhandlungen der Kön. Sächsischen Gesellschaft der Wissenschaften zu Leipzig. Math. phys. Classe* 1873, p. 538 *et seq.* Prof. H. C. Vogel has given a description of a spectroscope which he specially designed for observing faint stars. One of these instruments is at Dun Echt; it was made by H. Heustreu, Kiel, and bears the number 70. It has a slit which can be removed at pleasure. One jaw of the slit is adjustable by a fine screw. The collimating lens of 0·44 in. aperture to a focal length of about 1·1 in., transmits the light to a small direct-vision prism of which the outer surfaces are inclined 55° to the axis. The dispersion is 6° 50′ from C to G. The spectrum is viewed through one of two plano-convex lenses—plano-cylindrical and plano-spherical—of 5½ in. focus. Making an angle of 70° with the eye-tube is a sliding tube carrying a scale of 60 parts ruled on a silvered glass plate. This scale is seen by reflexion from the end of the prism. In practice the scale can only be used with the spherical lens or with the cylindrical lens set with its axis parallel to the slit and the divisions. For normal vision the eye lens is then about 4·4 in. from the scale, and the collimating lens must then be adjusted to a distance of 0·34 in. less than its focal length from the slit to give the rays of the spectrum the same degree of divergence as those from the scale. The spherical lens then serves to render the rays parallel in general while the cylindrical lens makes them approximately parallel only in sections at right angles to the slit. In either case the lines of both the spectrum and the scale are distinctly visible but with the cylindrical lens those of the scale appear pointed at the ends. To make the scale available when the slit is in the focus of the collimating lens would have required the insertion of a convex lens of about 2·8 in. focus between the scale and the face of the prism. Such a lens was in fact used by Prof. Vogel in his earlier instruments and it is not easy to see why it was discarded. The slit was used in the observations made under favourable circumstances and with it the readings for any given line are fairly constant. When, however, owing to haze or the inherent faintness of the star it was impracticable to employ the slit, estimates were made of the relative intervals of the lines, or one of the brighter lines of the spectrum was kept in coincidence with one of the longer scale lines while readings were taken of the fainter lines or bands. This class of measures are naturally not so accurate as those made with the slit. The readings in either case were reduced to wave-lengths by means of a curve derived from measures of the solar spectrum made early in January 1877, see p. 103, checked by later estimates.

The subsequent table shows the scale readings taken from the curve for every ten millionths of a millimeter of wave-length as well as for a few of the chief lines of the spectrum.

Table for reducing the corrected Scale-readings of the Vogel Spectroscope to Wave-lengths.

Wave-length. mmm.	Reading of Scale.	Wave-length. mmm.	Reading of Scale.	Line.	Wave-length.	Reading of Scale.
660	0·84	540	12·46	C	656·2	1·08
650	1·49	530	13·92	D	589·2	6·52
640	2·18	520	15·49	E	526·9	14·40
630	2·92	510	17·18	*b*	517·3	15·94
620	3·70	500	19·01	F	486·1	21·85
610	4·54	490	21·02	Hγ	434·0	36·58
600	5·45	480	23·21	G	430·7	37·30
590	6·44	470	25·61			
580	7·51	460	28·26			
570	8·64	450	31·19			
560	9·83	440	34·46			
550	11·10	430	38·10			

For slight alterations of the position of the scale-tube it is found that the changes of the readings laid down as ordinates with the wave-lengths as abscissæ are best represented by a straight line. Therefore to reduce the readings taken with a slightly altered scale to the adopted curve and thus to wave-lengths, it is only necessary to have the readings for two known lines sufficiently far apart. As the scale-tube was set as nearly as might be to the same position on each night and as the readings for the two lines most frequently observed coincided with those of C and F in the spectra of the light both of the sun and moon, and also of a hydrogen tube, it has been considered permissible to adopt for these lines in the spectrum of Nova Cygni the wave-lengths of C and F, and thus to utilize them in reducing the other readings to the curve. In this way, for example, on January 9th the readings of the spectroscope for C and F are 0·5 and 21·5 ; the corrections to obtain the curve readings are therefore + 0·58 and + 0·35 from which + 0·49, + 0·47, + 0·37 + 0·32 and + 0·29 have been taken *in proportion to the wave-lengths* as corrections to the readings of the remaining lines. The table then gives the wave-lengths for the corrected readings.

The new star was not further examined at Dun Echt until it was observed on September 2nd, 1877, at 14^h G. M. T. by Ralph Copeland. It was then of the 10½ magnitude and of a decided blue tint, especially when viewed in the same field with the reddish star D. M. + 42°, 4184 (No. 95 of the list on p. 119), which it precedes by about 25^s. Viewed through an eye-piece of low power and a highly dispersive direct-vision prism held between the eye and the eye-piece it showed scarcely a trace of elongation. A single prism Browning spectroscope with a slit, but without a cylindrical lens gave an image exactly like a star in an ordinary micrometer, which bore a bright illumination of the measuring wire astonishingly well. A few hurried measures were all that could be obtained, they indicated a wave-length of 512^{mmm} ; but great uncertainty attaches to this determination as a slight derangement of the spectroscope prevented the immediate introduction of a comparison spectrum. This observation, therefore, can only be taken as very roughly indicating the region of the spectrum to which the star's light belongs.

The comparative brightness of the sole line of the star's spectrum raised the hope that it might be measurable with the Grubb spectroscope, which from the general solidity of its construction promised to afford results of relatively high accuracy. Accordingly on September 3rd the compound prism belonging to that instrument was replaced by a single prism of Steinheil's giving a dispersion of about 3½° from B to *h* when adjusted to minimal deviation for D. The night proving fine Lord Lindsay succeeded in obtaining readings of the stellar spectrum and then measured the bright lines of burning magnesium as well as the D-line in the flame of a spirit-lamp. A single measure of the star and a few measures of magnesium were also made by Ralph Copeland. In effecting the comparison with the magnesium flame the telescope and spectroscope were kept in the same position as when the star was observed; the slit too was left unchanged, that is to say it was sufficiently wide open to admit the whole of the star's light. Owing to this the three specially characteristic lines of the Mg spectrum, b_1, b_2 and b_4—merged as they were by overlapping into a single band—were not immediately recognised. This might the more easily happen as their combined light is far inferior in brightness to that of a great band or series of lines further towards the violet. This circumstance combined with a considerable discordance in the measures of the lines led to their being excluded in the immediate determination of the wave-length, which was therefore based solely on the readings for the D-lines and a reducing curve got from observations of the solar spectrum.

Up to this time this spectroscope had been very little used and it was not until some time afterwards that it became evident, that although a continued set of measures generally agreed *inter se* in a most satisfactory manner, yet they differed from equally accordant series of readings for the same line by as much as twice the interval of the D-lines, as shown by the prism supplied with the instrument. At first this was attributed to shakiness in the bearings of the large tangent screw by which the viewing telescope is moved and the measures are made. A careful adjustment of the screw slightly but not materially lessened the discrepancies—at last it became evident that the evil lay in weakness of the attachments between the collimator and the prism box, and between the latter and the micrometer sector. The main source of weakness lay in the fact that while the collimator was attached by radial screws to a short arm on the side of the cylindrical prism-box, the sector was clamped to the bottom of the same box, which in its turn was held by 8 small radial screws. Now the measures are made by rotating the arm carrying the telescope by means of the tangent micrometer-screw gearing into the sector which is clamped to the bottom of the box; hence any friction retarding the motion of the telescope arm reacts on the bottom of the box through the sector and tended to move it within the limits permitted by the looseness of the eight radial screws. In general it was found that although these displacements occurred only when the telescope was moved through a considerable arc, they then usually took place *per saltum*, owing to the elasticity of the arm and the frame of the sector. ± $0^{r}{\cdot}120$ may be assigned as the *extreme* possible limit of these changes when extraordinary force was applied. A comparison between two extended sets of measures shows the amount of accidental change to have been ± $0^{r}{\cdot}083$. These changes naturally told most seriously on measures made with prisms of small dispersive power.

Sufficient rigidity not being attained by tightening the binding screws or adding to their number, eventually in the spring of 1879 the box, the bottom of the box—lined with an additional plate to add to its strength—and the short arm carrying the collimator were all soldered together. This has completely remedied the difficulty. With the experience thus obtained it seemed advisable to recur once more to the spectroscopic observations of Nova Cygni made in September 1877, with a view to utilize all the comparison observations and thus to eliminate as far as practicable any serious error that might have arisen through changes of the zero-point during the measures. To this end we give the measures *in extenso*.

Measures of the Spectrum of Nova Cygni and of comparison spectra—

1877, September 3.

Nova: centre of bright line.		Extreme limit of light towards the red.	Extreme limit of light towards the violet.
Lord Lindsay.	R. Copeland.		
$56^{r}{\cdot}839$	$56^{r}{\cdot}446$	$56^{r}{\cdot}732$	$56^{r}{\cdot}077$
·640		Measured by Lord Lindsay.	
·560			
·657			
·596			
·511			
·510			
·490			
·495			
·590			
·490			
·509			
·489			
56·567 ± 0·028 (mean error).			

D-line.—Lord Lindsay.	
$62^{r}{\cdot}381$	$62^{r}{\cdot}419$
·420	·422
·420	·418
·435	·405
·423	·420
Mean, . . 62·416 ± 0·028	

Measures of the Spectrum of burning Magnesium wire—

Great band: edge towards the red.

	Lord Lindsay.		
	1st Series.	2nd Series.*	R.C.
	$56^{r}{\cdot}684$	$56^{r}{\cdot}528$	$56^{r}{\cdot}586$
	·654	·512	·646
	·641	·515	
	·653	·525	
		·531	
		·522	
		·518	
Means,	56·658	56·522	56·616

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* The difference between these sets is an extreme instance of disagreement probably caused by the looseness of the sector.

The *b* Group.

Lord Lindsay. Mean of the group.	R.C. b_1	 b_2	 b_4
58^{r}·111	58^{r}·270	58^{r}·150	58^{r}·025
·096	·245	·107	·057
·093	·230	·160	
·101	·238		
·112	·246		
·117			
·121			
·101			
·121			
Means, 58·108	58·246	58·139	58·041

The following is a sufficiently extended table of the reducing curve for the prism used—

Wave-length. mmm.	Micrometer. rev.	Wave-length. mmm.	Micrometer. rev.	Wave-length. mmm.	Micrometer. rev.
400	43·35	520	58·20	640	64·53
410	45·30	530	58·92	650	64·89
420	47·03	540	59·60	660	65·24
430	48·62	550	60·23	670	65·57
440	50·07	560	60·82	680	65·89
450	51·39	570	61·37	690	66·20
460	52·61	580	61·90	700	66·49
470	53·73	590	62·40	710	66·76
480	54·77	600	62·88	720	67·03
490	55·72	610	63·34	D	62·360
500	56·60	620	63·76	b_1	58·071
510	57·43	630	64·15	b_2	57·984
				b_4	57·949

To this table must be added 500·6^{mmm} as the wave-length* of the first line in the great band of Magnesium as determined by M. Lecoq de Boisbaudran from the spark-spectrum of the chloride of that metal which evidently agrees with the flame-spectrum, in this region at least.

Experience has shown that the zero-correction of this spectroscope, now that the looseness of the binding-screws has been got rid of, is constant throughout the whole length of the spectrum, so that a reading of a single known line serves to determine this correction. Keeping, therefore, Lord Lindsay's two sets of readings of the great Mg band distinct, there result four values of the zero-correction for the readings of each observer. By using each of these to find the wave-lengths the widest possible limits are obtained which the observations can fairly yield and near the mean of which the true value probably lies.

In this way we have the following reductions of all the readings to wave-lengths :—

Lord Lindsay's Observations.

Centre of bright line.	Reduction to Curve.		Wave-length. mmm.
56^{r}·567 (13 measures.)	From 1st series of measures of band 500·6 of Mg Spectrum.	$-$ 0^{r}·007	499·5
	From 2nd series of ditto,	$+$ 0·129	501·1
	From *b* band $= \frac{b_1 + b_4}{2}$	$-$ 0·098	498·5
	From D-line, . .	$-$ 0·056	499·0
			499·5 ± 0·6

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* It is worthy of note that this line almost absolutely coincides with the brightest line in the spectra of planetary nebulæ.

Lord Lindsay's limiting measures of $56^{r}{\cdot}732$ and $56^{r}{\cdot}077$ reduced by the aid of the same four corrections give wave-lengths of 501·5, 503·1, 500·4 and 500·9 for the lower and 493·9, 495·4, 492·9 and 493·4 for the upper end of the spectrum, with mean values of $501{\cdot}5^{mmm}$ and $493{\cdot}9^{mmm}$.

For the single observation by Ralph Copeland the available corrections from the Mg band 500·6, b_1, b_2 and b_4 are respectively $+\,0^{r}{\cdot}035$, $-\,0^{r}{\cdot}175$, $-\,0^{r}{\cdot}155$ and $-\,0^{r}{\cdot}092$ with 498·6, 496·2, 496·4 and 497·2 as separate results or $497{\cdot}1^{mmm}$ as the value of the wave-length of the star's light. Combining this number with Lord Lindsay's figures we have:—Wave-length of the brightest part of the Spectrum of Nova Cygni $499{\cdot}3^{mmm} \pm 0{\cdot}6$.

September 6.—Viewed with the highest power of the Grubb spectroscope the spectrum was extended somewhat in length, showing traces of one or two faint lines on the violet side of the chief line, but very close to it. With a power of 980 on the micrometer it was impossible to obtain quite as sharp an image as that given by other stars in the neighbourhood, still the extreme diameter could not be above $2''$ or $1\frac{1}{2}''$. Since the light of this star is monochromatic it ought, one would think, to give a sharper image than an ordinary star.—R. C.

October 1.—Nova examined with the Vogel spectroscope with a view to detecting any very faint continuous spectrum: nothing of the kind was visible.—Lord L., R. C. and J. G. Lohse.

October 2.—Nova 12 grades less than No. 110; 95 4 grades brighter than 110. The focus for Nova is about 2 millimeters further from the object-glass than for 95 which is of a decided red tint.

October 5.—Nova 4 gr. less than 110; 95 12 gr. brighter than 110.

On October 10, 1877, with the same spectroscope Lord Lindsay noted: "Since the last measures a decided change is to be seen, as the light is now more spread out. The main brightness is still as before, but is divisible into two lines very close together with a dark gap and then another very faint line."

The following measures to which the wave-lengths are appended, were made by Ralph Copeland. They were reduced to the curve by a common correction of $-\,0^{r}{\cdot}485$:—

Microm. reading.		No. of Meas.	Wave-length. mmm. mmm.
1st band,	$56^{r}{\cdot}555 \pm 0{\cdot}038$	6	$499{\cdot}5 \pm 1{\cdot}1$
2nd „	$55{\cdot}922 \pm 0{\cdot}077$	2	$492{\cdot}2 \pm 1{\cdot}3$
3rd „	$55{\cdot}880 \pm 0{\cdot}038$	4	$491{\cdot}8 \pm 1{\cdot}0$
D-line,	$61{\cdot}360 \pm 0{\cdot}009$		

The mean errors of the wave-lengths have been formed from those of the readings combined with $\pm\,0{\cdot}083$ as the error due to possible displacement of the sector. It is evident that the 2nd and 3rd bands were too feeble and too near together to be effectively measured separately, hence their combined light must be taken as marking the violet end of the spectrum. $14^{h}{\cdot}5$ G. M. T., Nova 12 gr. fainter than 110; 110 = 3 while 95 is 8 gr. brighter than 110 or 3.

October 18.—Nova seen through the Vogel spectroscope looks exactly like a star in an ordinary eye-piece.

1878, June 1.—Nova, which seems to have a small disk with a soft margin, is fully one magnitude brighter than the nearest star of the trapezium. Nova = 10^{m}.—R. C. and J. G. L. August 17, Nova estimated of 10·8 mag.; it appears as a very small disk gradually decreasing in brightness towards the limb.—J. G. L.

1879, January 27.—Nova estimated 11^{m}·7; Nos. 48 and 54, 11^{m}·8; Nos. 94 and 104, 11^{m}·5. Nova = 11^{m}·7.—R. C.

1880, February 5.—Nova estimated 12^{m}·2; Nos. 18 and 54, 12^{m}·1; No. 2, 12·2; Nos. 74 and 49, 12·3. Nova = 12^{m}·3.

1881, January 20, 7^{h} to 8^{h} G. M. T., Nova estimated 12^{m}·2; Nova 3 grades fainter than 48 and 2 grades fainter than 87. It is the faintest star in the trapezium formed by itself and Nos. 48, 54 and 60. Nova = 12^{m}·5.

January 25, Nova 5 grades less than the faintest of the 3 other stars in the trapezium, from which it differs in focus. Tested the spectrum on this day and also on January 20 with various simple prisms, flint and crown, but could not be sure whether the light was monochromatic or not. Nova 12^{m}·4.

February 1.—No spectrum is visible with a Steinheil prism of 47° 8′ refracting angle and 2° 32′ dispersion from B to *h* in the Grubb spectroscope. The 11^{m}·2 star near (No. 60, 11^{m}·0 in the list) showed a faint continuous spectrum and as Nova is about 12^{m}·2, had its light been monochromatic it must have been visible in the spectroscope.*

1882, March 24.—Nova of 14·0 magnitude.

Summary of Spectroscopic Results.

a. Measures with the Vogel Spectroscope.

No. of Line.†	1.	1*a*.	2.	3.	4.	5.	7.	
Date.	W.L. mmm.	W.L. mmm.	W.L. mmm.	W.L. mmm.	W.L. mmm.	W.L. mmm.	W.L. mmm.	Weight.
1877, Jan. 2, .	(657·8)‡	—	579·9	502·4	486·1	463·4	—	1
„ 2, .	(654·7)	—	579·0	505·1	488·4	466·8	—	1
„ 8, .	(656·2)	—	—	502·9	(486·1)	464·8	—	½
„ 9, .	(656·2)	589·5	575·9	500·7	(486·1)	461·6	437·6	1
„ 27, .	—	—	—	500·8	(486·1)	461·5	—	1
Feb. 13, .	681·5	—	574·9	(502·3)	—	—	—	½
„ 13, .	670·8	—	579·4	—	—	—	—	½
„ 13, .	—	—	574·0	—	—	—	—	½
Adopted means, .	[656·2]§	589·5	577·5	502·3	[486·1]§	463·5	437·6	
Mean error of result,			± 1·1	± 0·8		± 1·0		

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* In the 35th annual report of the Observatory of Harvard College, Cambridge: 1881, p. 7, Professor Pickering, writing of the new star in Cygnus, says—"This object has now diminished greatly in light and appears to have lost this peculiarity [of a monochromatic spectrum] and to give a continuous spectrum like that of an ordinary star."

† To avoid the inconvenience of renumbering the lines the numbers used in Ast. Nach., Nos. 2116 and 2117, have been retained.

‡ The numbers in round brackets ( ) have been taken as data for the reductions.

§ Ångström's values have been assumed for these lines, see p. 106.

b. Measures with the Grubb Spectroscope.

	Spectrum begins.	Bright line.	Line.	Very faint line.	Spectrum ends.
	W.L. mmm.	W.L. mmm.	W.L. mmm.	W.L. mmm.	W.L. mmm.
1877, Sep. 3,	501·5	499·3			493·9
Oct. 10,		499·5	492·2	491·8	
Means, . .		499·4 ± 0·5	492·0		

Assuming the bright line of September and October to be identical with line 3 of the beginning of the year we have as a final value for its wave-length $500{\cdot}2^{mmm}$ with a mean error of $\pm\ 0{\cdot}4^{mmm}$. The discordance of the results with the two instruments, however, points to an error of not less than $\pm\ 1{\cdot}0^{mmm}$.

Respecting the origin of these lines, Nos. 1 and 4 are certainly, and No. 7 probably to be attributed to Hydrogen: 1*a* may safely be set down as identical with D, the more so as M. Cornu (Compt. Rend. T. 83, p. 1173) has measured the wave-length at 588^{mmm}. The mean of the D-lines on Ångström's chart is 589·2. Line 3 coincides almost exactly with the brightest line of gaseous nebulæ. The complete reduction of the measures of line 2 strengthens the opinion already expressed in 1877 that it corresponds very nearly with one of the bright lines in the spectra of the three remarkable stars in Cygnus pointed out by Messrs. Wolf and Rayet, and subsequently observed by Professor Vogel (see Berichte d. Kön. Sächs. Ges. der Wiss., Math. Phys. cl. 1873, p. 556). This view is much strengthened by the fact, already pointed out by Prof. Vogel (Monatsberichte d. Berlin. Akad. 1877, p. 254) that line 5 also agrees with a bright band in the spectra of the same stars; this agreement is very close when Prof. Vogel's own measures of this line or band are compared. These are—for the line in the spectrum of Nova Cygni, 467^{mmm}, and for the three stars in question (for which we also quote the line agreeing with No. 2) :—

	mmm.	mmm.
D.M. + 35°, 4001	579·5	468
35, 4013	580·5	467
34, 3956	580·5	467·5

It is especially worthy of note that a line of about the same degree of refrangibility is to be found in the spectra of several planetary nebulæ, see this Journal, Vol I., p. 3, as bearing on the autumn observations of Nova Cygni it is also to be noted that line 7 occurs in the Spectrum of the Great Nebula in Orion. The remarkable fact that the whole of the complicated spectrum which this star exhibited at the end of 1876 and beginning of 1877 should in the space of eight or nine months be reduced to a single line accompanied by the merest traces of a continuous spectrum led at the time to the expression of an opinion that was shared by all who observed the star at Dun Echt. This opinion was thus expressed in number 2158 of the Astronomische Nachrichten :—"Bearing in mind the history of this star from the time of its discovery by Schmidt it would seem certain that we have an instance before us in which a star has changed into a planetary nebula of small angular diameter. At least it may be safely affirmed that no astronomer discovering the object in its present state would, after viewing it through a prism, hesitate to pronounce as to its present nebulous character."

In a communication to the Royal Academy of Berlin (Berichte, 1878, p. 302) Prof. Vogel, while fully agreeing with the second sentence of the above quotation, differs from the conclusion in the first sentence mainly on the ground that the surviving line of the spectrum is identical with the first of the nebular lines, while the second of those lines is the most characteristic of gaseous nebulæ, and that while in the earlier part of 1877, the first line gradually became the brightest line of the star's spectrum the second was not seen. To this it may be replied that instances are not wanting of gaseous Nebulæ the spectra of which are very nearly reduced to a single line. Taking Professor Vogel's own observations on p. 59 of the Beobachtungen . . . zu Bothkamp, Heft 1, we find an observation of the nebula Gen. Cat. 4510 "die erste Linie ist sehr hell, die zweite recht schwach und die dritte nur mit Mühe zu sehen . . . . Die dritte Linie war zu schwach zur Messung." And further on, respecting the fine planetary nebula G. C. 4572 . . . "im Spectrum desselben waren nur die beiden ersten hellen Linien zu erkennen. Die zweite Linie war sehr schwach." It seems therefore not unreasonable to assume that in the case of a planetary nebula the whole light of which did not equal the light of a 10th magnitude star, the spectrum might be reduced to a single line and that this line would then be the one of which the wave-length may be taken at $500{\cdot}4^{\mathrm{mmm}}$. The faint extension of the spectrum of the new star towards the violet measured on Oct. 10, 1877, was seen with so much difficulty that it would be unwise to regard it as in any certain degree indicating the presence of other nebular lines. In the same way it is desirable to receive with some degree of caution the negative evidence with regard to the star's spectrum obtained on February 1, 1881. We must therefore look to Professor Pickering for conclusive evidence that this truly wonderful star had again completely changed the character of its spectrum. Perhaps even now it may not be too late to test this very interesting point with one of the larger instruments in existence.

Estimated magnitude of Nova Cygni.

To reduce the comparisons of magnitude made in January and February 1877, all the observations in which more than one star was compared with Nova, were combined so as to give the relative intensities of the comparison-stars in grades; these again were assimilated as closely as might be to the magnitudes of the

Durchmusterung. Excluding No. 95, which was used on one occasion only, we have :—

	Grades.	D.M..	m.	Adopted magnitude.
a =	10·5 =	+ 41°, 4243,	7·0	7·00
b =	2·6 =	+ 41, 4237,	7·6	7·69
c =	0·0 =	+ 42, 4188,	8·0	7·91

11·5 grades = a difference of one magnitude.

The magnitude of Nova Cygni as it results immediately from the observations is given in the ensuing table, in which are also included the results of occasional estimations made when measuring the position of Nova from the neighbouring stars.

	Grades.	Mag.		Mag.
1877, Jan. 8	13·5	6·7	1877, Sep. 2	10½
„ 9	10·5	7·0	Oct. 2	9·7
„ 17	9·7	7·1	„ 5	10·1
„ 19	8·3	7·2	„ 10	10·6
„ 24	3·6	7·6	1878, June 1	10
„ 25	5·5	7·4	Aug. 17	10·8
„ 27	5·0	7·5	1879, Jan. 27	11·7
„ 30	4·7	7·5	1880, Feb. 5	12·3
Feb. 3	2·8	7·7	1881, Jan. 20	12·5
„ 7	+ 1·7	7·7	„ 25	12·4
„ 13	− 0·9	8·0	Feb. 1	12·2
„ 16	+ 0·8	7·8	1882, Mar. 24	14·0

Micrometric measures of stars in the neighbourhood of Nova Cygni.

The great decrease in the brightness of the new star between February and September 1877, concurred with a still further diminution in October to induce the opinion that it might eventually disappear altogether. It seemed therefore desirable before such an event took place to obtain a map of the neighbouring stars somewhat similar to that prepared by the late Professor D'Arrest of the region round about the place of the great temporary star of 1572, (Oversigt over det Kgl. danske Videnskabernes Selskabs Forhandlinger, 1864 pp. 1–10 and 1 plate). The micrometric measures for this purpose as well as the spectroscopic observations already given were made with the Dun Echt refractor of 15·06 in. aperture and 15 ft. 3·2 in. focal length. The filar micrometer has two measuring screws, both of which are of great excellence as is shown by their practical agreement throughout their range. Their common value was found in April 1877 to be $1^{\mathrm{r}} = 22''{\cdot}5189$, and 22″·5216 ± 0″·0044 in December 1880 ; a mean value of 22″·5207 ± 0″·0036 has been used in all the more recent reductions including those for this paper. Each wire frame carried originally a pair of wires 7″ asunder and a single wire 171″·3 from the mean of the pair. These were arranged so that the corresponding wires on both frames were in coincidence at the same time. Through accident the inner double wire of frame A was broken in October 1878 while the single wire of frame B was lost in like manner in December 1880. Two parallel position wires about 7″ apart mark the centre of the field. The position circle forms part of the butt end of the tube, it has four verniers reading to 2′ but 1′ is easily estimated; in practice only one vernier is read off, the one chosen being next to a small aperture through which a transparent position circle is visible. This latter circle is particularly intended for double-star measures as it is illuminated from within and can be read at a glance to 0°·1. Owing to the proportions of the forks of the wire-frames the wires cannot pass each other to a greater extent than 4½′, nor is it desirable that this limit should be too closely approached as the slightest attempt to pass it is certain to derange the springs which retain the micrometer screws in their places. This structural peculiarity necessitates a knowledge of the screw readings when the wires are in coincidence. Fortunately these coincidences are remarkably constant, not varying appreciably beyond the probable error of a determination in the course of many months; as a rule, however, coincidences for four equidistant points in a revolution were taken whenever the micrometer was used. The agreement of these shows that the screws are also remarkably free from periodic error. The earlier measures were made chiefly with a power of 134, but the bulk of the work was done with one of 229 although a power of 307 was occasionally used. The illumination of the wires is very satisfactory and can be varied in tint or intensity at pleasure. But in the wider measures when a wire is followed far towards the edge of the field, light from the illuminating prisms enters the eye-piece and renders the measurement of faint objects impossible. As we have already said, the focus for Nova was different from that for other stars. This probably affected the measures comparatively little so long as Nova was kept exactly in the centre of the eye-piece, but in the case of wide measures this could not be done, as the range of the "slipping piece" or slide carrying the eye-piece was restricted by the necessity of excluding the direct rays of the illumination from the field of view. This defect it is proposed to remedy, and at the same time to introduce screens by which the light may at pleasure be cut off from the position wires or distance wires leaving the other set visible. By this arrangement it is believed that faint objects may be more conveniently observed.

The measures have been corrected for differential refraction and in the case of the earlier ones the position angles have been slightly altered to allow for want of exact adjustment of the polar-axis at the time when they were taken. The almost nominal reductions for precession, &c., have also been applied. Nearly all the measures were made direct from Nova; the few exceptional ones were taken to fix more exactly the configurations of stars lying close together. All the measures, however, have been referred to Nova the place of which for 1878·0 has been derived from the following materials:—

Meridian Observations of Nova Cygni.

	α 1878·0	δ 1878·0	
Becker—Berlin. (Ast. Nach. Vol. 89, p. 13) 1876, Dec. 5, 6,	21^h 36^m 55^s·103[*]	+ 42° 17′ 9″·47	(2)
Holetschek—Vienna. (Ibid. p. 61) 1876, Dec. 5, 7, 8, 12,	55·085	9·66	(4)
Greenwich. (Greenw. Obs. 1877), 1877 Sept. 26, Oct. 3, 6, 8, 23,	55·227	8·86	(5)
Dun Echt, 1877 Oct. 28,	55·081	10·86	(1)
Adopted mean place:	21 36 55·15	+ 42 17 9·40	

[*] In forming the mean for Berlin, the second observation has received half weight, as it was made under unfavourable circumstances.

In vol. 89 of the Astronomische Nachrichten p. 42, Prof. Schmidt gives a place based on comparisons with four stars, which on reduction to 1878·0 becomes $21^{h}\ 36^{m}\ 55^{s}{\cdot}29 + 42^{\circ}\ 17'\ 7''{\cdot}64$.

Meridian Observations of D.M. + 42°, 4184.

		α 1878·0	δ 1878·0
Greenwich,	(Greenwich Obs. 1877), .	$21^{h}\ 37^{m}\ 20^{s}{\cdot}074$ (3)	$+42^{\circ}\ 15'\ 55''{\cdot}00$ (3)
Dun Echt,	. . . .	20·006 (5)	55·38 (4)
		21 37 20·03	+ 42 15 55·22

The place of this star, which is No. 95 of our list, is given as a check on the differential result.

The table at the end of this paper exhibits the mean results of all the measures reduced to 1878·0. The 1st column contains a current number; the 2nd gives the mean magnitude; the measured position angle and distance are shown in the 3rd and 4th columns, for which the mean of the individual differences of right ascension and declination are given in the 5th and 6th columns; the right ascensions and declinations for 1878·0 occupy the 7th and 8th columns, while the nights on which each star was observed are entered in the last column in accordance with the following list:—

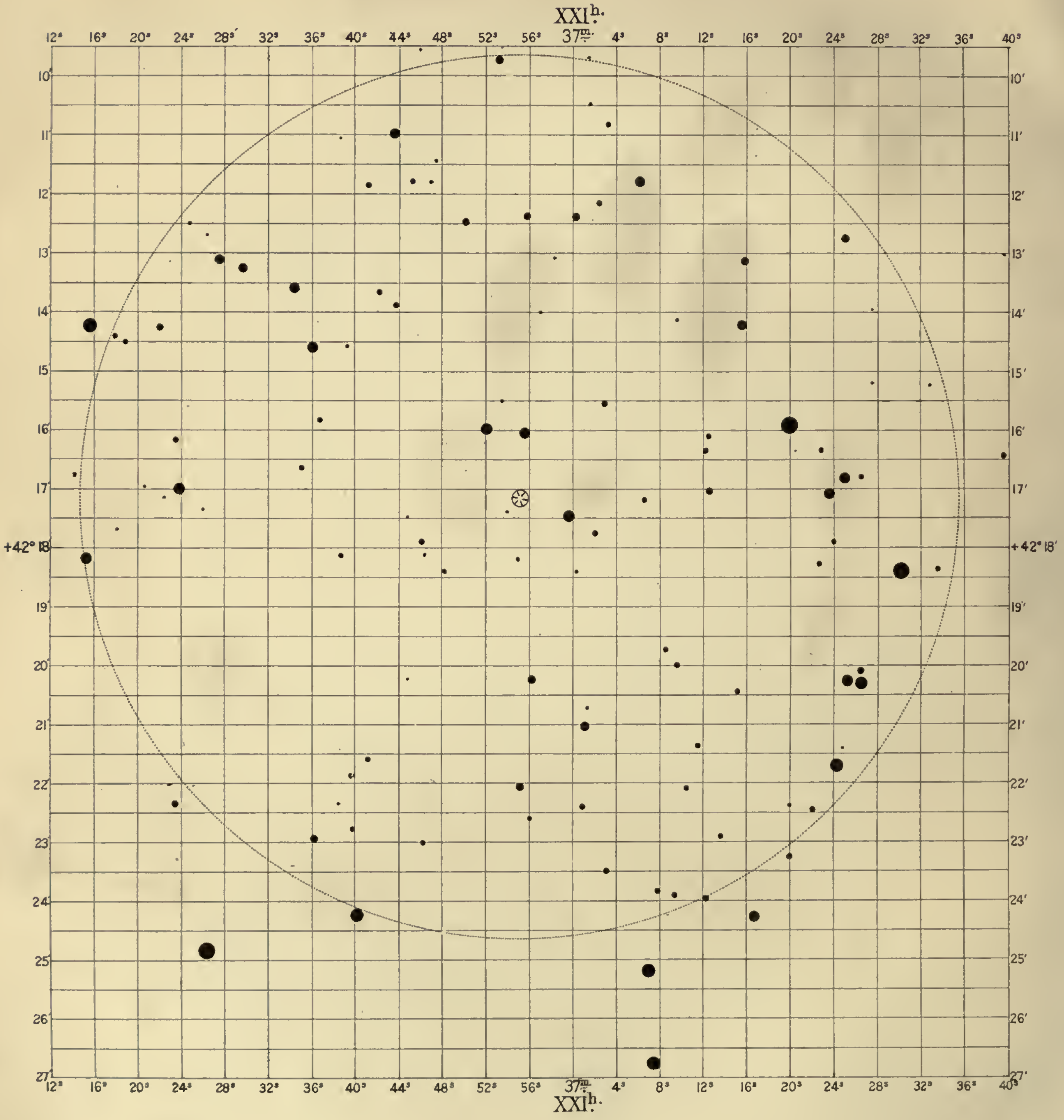
Index of Nights on which micrometric measures were made.

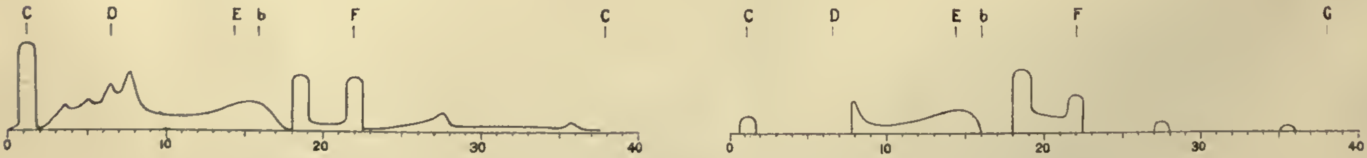
Night.	Date.	Night.	Date.	Night.	Date.
1	1877, . Oct. 29	14	1877, . Dec. 27	27	1878, . Oct. 1
2	„ . „ 31	15	„ . „ 31	28	„ . „ 11
3	„ . Nov. 9	16	1878, . Jan. 6	29	„ . „ 22
4	„ . „ 13	17	„ . „ 29	30	„ . „ 23
5	„ . „ 16	18	„ . „ 30	31	„ . „ 24
6	„ . „ 18	19	„ . Sep. 7	32	„ . „ 26
7	„ . „ 24	20	„ . „ 16	33	„ . Nov. 17
8	„ . „ 27	21	„ . „ 18	34	„ . Dec. 24
9	„ . „ 28	22	„ . „ 20	35	1879, . Jan. 13
10	„ . Dec. 12	23	„ . „ 25	36	„ . „ 16
11	„ . „ 13	24	„ . „ 26	37	„ . Feb. 8
12	„ . „ 14	25	„ . „ 27	38	1882, . Mar. 24
13	„ . „ 17	26	„ . „ 30		

An observation consisted of two single measures of distance and two settings of the position circle. The magnitudes are based partly on estimates made when the stars were measured, but chiefly on systematic comparisons made on three nights for which we give the numbers of the intercompared stars, as it may possibly be useful to know the exact dates on which any star was seen at least approximately of the recorded magnitude.

These were:—On January 27, 1879, Nos. 1–8, 11–13, 16–19, 21–24, 28–35, 38, 41, 42, 44–48, 51–57, 60, 61, 63–65, 68, 69, 71–75, 78, 79, 82–93, 95, 96, 99, 101, 103–107 and 110; on February 5, 1880, Nos. 2, 3, 8, 11, 12, 17–19, 21, 22, 30, 34, 41, 42, 45–48, 52–55, 57, 60, 61, 64, 65, 68, 69, 72–74, 86, 87, 90–93, 95–97, 99–101, 103–107, 110 and 112; on February 8, 1880, Nos. 1, 4–8, 13, 15, 20, 22–29, 31–33, 35–38, 40, 43, 44, 49–51, 56, 58, 59, 62 *not seen*, 65–67, 70–72, 77–85, 88, 89, 94, 98, 102, 108, 109, 111 and 112.

Chart of Stars in the neighbourhood of Schmidt's Nova Cygni. 1878·0


XXI^h.
37^m.
12^s 16^s 20^s 24^s 28^s 32^s 36^s 40^s 44^s 48^s 52^s 56^s
4^s 8^s 12^s 16^s 20^s 24^s 28^s 32^s 36^s 40^s
10′ 11′ 12′ 13′ 14′ 15′ 16′ 17′
+42° 18′
19′ 20′ 21′ 22′ 23′ 24′ 25′ 26′ 27′



1877 January 9.

1877 February 7.

Intensity Curves.

In the lower part of the scale we endeavoured to accommodate the magnitudes as closely as possible to the Durchmusterung while we chose the 15th magnitude for the faintest stars certainly shown by the instrument under specially favourable circumstances of zenith distance and transparency of the atmosphere. It is scarcely necessary to add that on exceptionally clear nights when Nova was high in the heavens, by long attention a number of points of light not laid down on our map might doubtless have been detected but which it would have been exceedingly difficult to measure; on this point see the notes to the list of stars.

The probable error of one of the places is about ± 1″·1 in either co-ordinate: this quantity is disappointingly large but is probably to be ascribed in some measure to two special causes—the peculiar light of Nova, which threw its image out of focus and the fact that the settings could not all be made in the centre of the apparent field of view. Discordances in the results for some of the faintest stars are probably owing to the fact that two or even three very small stars within a few seconds of each other have been seen by us as a single star without the same point having been fixed upon in each of the measures.

List of Stars near Nova Cygni.

No.	Mag.	Position Angle.	Distance.	$\Delta\alpha$	$\Delta\delta$	α (1878·0).	δ (1878·0).	Date.
1	14·4	267° 0′	459″·0	− 41^s·30	− 24″·0	21^h 36^m 13^s·85	+ 42° 16′ 45″·4	30
2	12·2	277 52	446·2	− 39·83	+ 61·1	15·32	18 10·5	27
3	9·8	248 0	469·9	− 39·25	− 176·1	15·90	14 13·3	13, 37
4	14·0	248 18	445·6	− 37·29	− 164·8	17·86	14 24·6	29
5	14·8	274 33	412·4	− 37·05	+ 32·7	18·10	17 42·1	30
6	14·0	248 24	434·2	− 36·37	− 159·8	18·78	14 29·6	17, 29
7	14·9	268 16	384·0	− 34·58	− 11·6	20·57	16 57·8	30
8	13·4	244 42	407·8	− 33·21	− 174·2	21·94	14 15·2	17, 20
9	14·5	269 46	360·5	− 32·49	− 1·4	22·66	17 8·0	20
10	13·5	311 28	469·0	− 31·69	+ 310·6	23·46	22 20·0	36
11	14·2	260 15	356·4	− 31·65	− 60·4	23·50	16 9·0	17
12	11·8	268 32	347·2	− 31·27	− 8·9	23·88	17 0·5	13
13	15·0	230 23	438·3	− 30·41	− 279·5	24·74	12 29·9	25
14	15	272 0	323·9	− 29·17	+ 11·3	25·98	17 20·7	37
15	15	229 58	417·4	− 28·78	− 268·4	26·37	12 41·0	29
16	9·2	325 30	562·8	− 28·76	+ 463·8	26·39	24 53·2	13
17	12·2	231 40	392·6	− 27·73	− 243·5	27·42	13 5·9	17, 19
18	12·6	230 14	365·6	− 25·31	− 233·8	29·84	13 15·6	18, 19
19	12·2	226 52	316·4	− 20·80	− 216·3	34·35	13 33·1	3
*20	14·5	262 8	224·3	− 20·02	− 30·7	35·13	16 38·7	33
21	11·8	233 49	263·1	− 19·13	− 155·3	36·02	14 34·1	3
22	13·2	328 57	404·8	− 18·83	+ 346·8	36·32	22 56·2	26
23	14·5	248 48	218·4	− 18·35	− 79·0	36·80	15 50·4	32, 33
24	14·8	329 36	361·1	− 16·48	+ 311·5	38·67	22 20·9	26
*25	14·0	288 17	192·2	− 16·45	+ 60·3	38·70	18 9·7	2, 33
26	15	206 26	409·1	− 16·40	− 366·3	38·75	11 3·1	23
27	15	228 10	234·4	− 15·74	− 156·3	39·41	14 33·1	31
28	14·0	328 56	332·6	− 15·47	+ 284·9	39·68	21 54·3	26
29	14·6	333 26	378·5	− 15·27	+ 338·5	39·88	22 47·9	26
30	11·4	338 37	454·4	− 14·94	+ 423·1	40·21	24 12·5	13

List of Stars—*continued.*

No.	Mag.	Position Angle.	Distance.	Δα	Δδ	α (1878·0).	δ (1878·0).	Date.
31	13·9	206° 9′	354″·0	$-14^s{\cdot}04$	− 317″·7	$21^h\ 36^m\ 41^s{\cdot}11$	+ 42° 11′ 51″·7	21
32	14·4	329 53	307·1	− 13·90	+ 265·7	41·25	21 35·1	26
33	13·8	214 27	254·3	− 12·96	− 209·7	42·19	13 39·7	18, 21
34	11·6	199 7	392·1	− 11·56	− 370·5	43·59	10 58·9	21
35	13·6	212 36	234·9	− 11·39	− 197·9	43·76	13 51·5	18, 21
36	15	327 48	216·9	− 10·42	+ 183·5	44·73	20 12·9	26
37	15	279 34	116·9	− 10·38	+ 19·4	44·77	17 28·8	32
38	14·0	198 25	340·7	− 9·69	− 323·2	45·46	11 46·2	21
39	14·2	192 38	468·7	− 9·22	− 457·4	45·93	9 32·0	21
*40	14·5	344 10	365·6	− 9·00	+ 351·7	46·15	23 1·1	26
41	13·4	293 23	108·6	− 8·98	+ 43·1	46·17	17 52·5	2,8,19,24
42	15	299 28	111·8	− 8·77	+ 55·0	46·38	18 4·4	24
43	14·5	195 47	333·8	− 8·17	− 321·2	46·98	11 48·2	22
44	14·8	194 15	352·0	− 7·81	− 341·2	47·34	11 28·2	23
45	14·4	315 2	107·9	− 6·87	+ 76·3	48·28	18 25·7	2,8,23,24
46	13·0	191 34	286·0	− 5·16	− 280·2	49·99	12 29·2	3
47	11·9	206 10	78·2	− 3·11	− 70·2	52·04	15 59·2	1, 19
48	12·7	182 59	445·5	− 2·08	− 444·9	53·07	9 44·5	21
49	14·7	190 31	101·0	− 1·66	− 99·3	53·49	15 30·1	14
*50	15	314 12	19·1	− 1·23	+ 13·0	53·92	17 22·4	23
51	14·3	358 28	63·7	− 0·15	+ 63·7	55·00	18 13·1	9
Nova						55·15	17 9·4	
53	13·4	0 7	292·4	+ 0·05	+ 292·4	55·20	22 1·8	15
54	12·0	175 25	68·1	+ 0·49	− 67·8	55·64	16 1·6	1, 19
55	13·0	178 34	287·9	+ 0·65	− 287·8	55·80	12 21·6	3, 6
56	14·6	1 56	325·5	+ 0·99	+ 325·4	56·14	22 34·8	31
57	13·4	3 35	184·8	+ 1·04	+ 184·5	56·19	20 13·9	2
58	14·8	174 31	190·6	+ 1·64	− 189·8	56·79	13 59·6	27
59	14·8	172 3	246·6	+ 3·07	− 244·2	58·22	13 5·2	24
60	11·0	69 44	55·2	+ 4·67	+ 19·1	59·82	17 28·5	1, 19
61	12·7	168 59	293·9	+ 5·06	− 288·5	37 0·21	12 20·9	3, 6
62	15	37 0	94·8	+ 5·14	+ 75·7	0·29	18 25·1	9
63	14·5	11 14	322·3	+ 5·66	+ 316·1	0·81	22 25·5	34
64	12·6	16 13	241·4	+ 6·08	+ 231·8	1·23	21 1·2	10, 11
65	14·6	169 54	407·9	+ 6·44	− 401·6	1·59	10 27·8	23
66	14·7	18 36	225·6	+ 6·49	+ 213·8	1·64	20 43·2	34
67	15	170 47	453·4	+ 6·54	− 447·6	1·69	9 41·8	23
68	14·0	65 8	83·5	+ 6·82	+ 35·1	1·97	17 44·5	7, 8
69	14·0	165 15	311·0	+ 7·13	− 300·8	2·28	12 8·6	3
70	13·8	138 39	129·3	+ 7·70	− 97·0	2·85	15 32·4	4
71	14·2	13 6	390·4	+ 7·98	+ 380·2	3·13	23 29·6	34
72	14·1	166 55	392·1	+ 7·99	− 381·9	3·14	10 47·5	21
73	13·4	89 4	122·3	+ 11·02	+ 2·0	6·17	17 11·4	1
74	12·0	159 18	346·5	+ 11·03	− 324·2	6·18	11 45·2	3, 5
75	10·3	15 25	499·5	+ 11·97	+ 481·5	7·12	25 10·9	13
76	10·8	13 22	593·6	+ 12·39	+ 577·5	7·54	26 46·9	13
77	14·5	19 30	424·4	+ 12·78	+ 400·0	7·93	23 49·4	35
78	14·6	43 56	215·1	+ 13·46	+ 154·9	8·61	19 44·3	15
79	14·6	42 49	231·9	+ 14·20	+ 170·1	9·35	19 59·5	15
80	14·5	21 32	436·0	+ 14·43	+ 405·5	9·58	23 54·9	35
81	15	138 25	241·9	+ 14·46	− 180·9	9·61	14 8·5	27
*82	13·9	29 42	338·7	+ 15·13	+ 294·3	10·28	22 3·7	15
83	14·4	35 57	307·9	+ 16·30	+ 249·2	11·45	21 18·6	15, 16
84	14·4	104 2	195·2	+ 17·07	− 47·3	12·22	16 22·1	14
85	14·4	24 54	450·0	+ 17·10	+ 408·2	12·25	23 57·6	35
86	13·5	108 14	200·9	+ 17·19	− 62·9	12·34	16 6·5	4
87	12·9	92 8	192·4	+ 17·32	− 7·2	12·47	17 2·2	1

List of Stars—*continued.*

No.	Mag.	Position Angle.	Distance.	Δα	Δδ	α (1878·0).	δ (1878·0).	Date.
*88	13·8	30° 46′	402″·0	+ 18$^{\mathrm{s}}$·54	+345″·4	21$^{\mathrm{h}}$ 37$^{\mathrm{m}}$ 13$^{\mathrm{s}}$·69	+ 42° 22′ 54″·8	16
89	14·2	48 42	297·6	+ 20·16	+ 196·5	15·31	20 25·9	15
90	12·0	128 6	287·9	+ 20·41	— 177·6	15·56	14 11·8	4
91	13·0	136 17	332·5	+ 20·70	— 240·3	15·85	13 9·1	4
92	11·6	29 10	489·7	+ 21·53	+ 427·6	16·68	24 17·0	13
93	13·2	36 51	457·2	+ 24·73	+ 365·8	19·88	23 15·2	16
94	14·8	41 16	416·7	+ 24·78	+ 313·2	19·93	22 22·6	16
95	9·0	105 18	286·1	+ 24·87	— 75·5	20·02	15 53·9	1, 28, 35
96	14·2	43 12	437·9	+ 27·03	+ 319·2	22·18	22 28·6	15
97	14·0	77 51	311·9	+ 27·48	+ 65·7	22·63	18 15·1	14
98	13·5	99 2	312·0	+ 27·77	— 49·0	22·92	16 20·4	5
99	11·4	90 53	314·0	+ 28·29	— 4·9	23·44	17 4·5	1, 35
100	14·0	82 14	321·8	+ 28·74	+ 43·4	23·89	17 52·8	14
101	11·4	50 0	422·3	+ 29·17	+ 271·4	24·32	21 40·8	12, 13
102	14·8	51 56	415·6	+ 29·51	+ 256·2	24·66	21 25·6	16
103	12·6	128 38	424·1	+ 29·84	— 264·8	24·99	12 44·6	5
104	11·4	93 22	332·1	+ 29·87	— 19·5	25·02	16 49·9	1, 2, 35
105	11·3	60 40	385·4	+ 30·29	+ 188·9	25·44	20 18·3	9, 35, 38
106	12·7	62 44	388·6	+ 31·14	+ 178·0	26·29	20 7·4	38
107	11·2	61 16	396·6	+ 31·35	+ 190·7	26·50	20 20·1	9, 12
108	14·5	93 30	349·4	+ 31·43	— 21·4	26·58	16 48·0	24
109	14·8	107 46	378·2	+ 32·45	— 115·4	27·60	15 14·0	27
110	9·4	78 48	396·6	+ 35·07	+ 77·1	30·22	18 26·5	1, 35
111	14·8	105 21	433·6	+ 37·67	— 114·8	32·82	15 14·6	27
112	14·6	80 22	434·2	+ 38·58	+ 72·7	33·73	18 22·1	24
113	14·0	95 2	496·1	+ 44·53	— 43·5	39·68	16 25·9	27

NOTES.

No. 20. 1878, Nov. 17, several companions to this star seen: they were too faint for satisfactory measurement. 1880, Feb. 8. Perhaps double.

„ 25. One or more very faint companions to this star repeatedly seen.

„ 40. This star is probably double or multiple, 1878, Nov. 30.

„ 50. Seen for the first time on Sep. 25, 1878.

„ 82. Double? 1880, Feb. 8.

„ 88. „ „

List of the principal Memoirs, &c., on Nova Cygni.

American Journal of Science and Arts—
Vol. 113, pp. 395–7, general note.
„ 115, „ 76–7, spectrum.

Astronomical Register—
Vol. 15, p. 124, J. E. Gore, magnitude.
„ 269, T. W. Backhouse, spectrum.
„ 17, pp. 42–3, Oswald Lohse's paper summarized.

Astronomische Nachrichten—
Vol. 89, pp. 9–14, J. F. Julius Schmidt, first accounts.
„ 13–14, E. Weiss, magnitude.
„ 13–14, E. Becker, meridian observation.
„ 37–40, H. Vogel, spectrum

Astronomische Nachrichten—
Vol. 89, pp. 41–44, J. F. Julius Schmidt, magnitude light-curve.
„ 61–62, J. Holetschek, meridian observation.
„ 61–64, Ralph Copeland, spectrum.
„ 63–64, P. A. Secchi, spectrum.
„ 79–80, Ralph Copeland, spectrum.
„ 159–60, J. F. Julius Schmidt, magnitude.
„ 90, „ 351–2, Ralph Copeland, spectrum.
„ 91, „ 61–2, J. F. Julius Schmidt, mag. and place.
p. 170, „ „ magnitude.
pp. 351–2, C. Bruhns and B. Peter, mag. and place.
„ 92, „ 9–10, J. F. Julius Schmidt, magnitude.
„ 96, p. 267, „ „ magnitude.

Bulletin International de l'Obs. de Paris *—
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„ 12, Littrow, merid. obs.
„ 15–16, Cornu, spectrum.
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Vol. 83, pp. 1097–8, J. F. Julius Schmidt, first account.
„ 1098, U.-J. Le Verrier, note.
„ 1172–4, M. A. Cornu, spectrum.
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„ 84, „ 107–8, P. A. Secchi, spectrum.
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p. 7, E. C. Pickering, spectrum.

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1877. pp. 241–59, H. C. Vogel, spectrum.
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„ 38, „ 210–1, „
„ 39, „ 34–7, T.W.Backhouse, spectrum.

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„ 158–59, M. A. Cornu, spectrum.
„ 166, 186, 206, general notes.
„ 244, brightness.
„ 295–96, T. W. Backhouse, spectrum.
„ 303, translation from Ast. Nach.
„ 315–16, translations and reprints from A. N.
„ 361, reprint from Ast. Nach.
„ 16, „ 400–403, abstract from Monatsb. d. Akad. z. Berlin.
„ 413–14, J. Norman Lockyer, Star or nebula?
„ 17, „ 46, magnitude and place.

Observatory—
Vol. 1, p. 185, W. H. M. Christie, general note.
„ 2, pp. 198–9, general note.
„ 322–3, „

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Vol. 10, pp. 1– 7, collected papers.
„ 194–200, „
„ 250– 2, „
„ 11, „ 97–109, „
„ 204– 6, „

Wochenschrift für Astronomie—
Vol. 31, pp. 51–2, collected papers.
„ 107–9, H. C. Vogel, spectrum.

RALPH COPELAND.
J. GERH. LOHSE.

NOTE ON THE COMPUTATION OF SPECIAL PERTURBATIONS.

BY ORMOND STONE.

The methods usually employed in the computation of special perturbations may be divided into two classes; that in which the elements are varied and those in which the co-ordinates are varied. The object of this note is to put the solution in such a form as shall, in some measure, combine the advantages of both.

Differentiating twice the equation—

$$p = r\,(1 + e_0 \cos v) \qquad (1)$$

considering, as Hansen does, that e_0 is constant, and putting—

$$c = r^2 \frac{dv}{dt} \qquad (2)$$

we have—

$$\frac{d^2p}{dt^2} = \frac{p}{r} \cdot \frac{d^2r}{dt^2} - \frac{e_0 \sin v}{r} \cdot \frac{dc}{dt} - \frac{e_0 \cos v}{r^3} \cdot c^2 \qquad (3)$$

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* We are indebted to Admiral Mouchez for these numbers of the Bull. Int.

Let R be the component of the disturbing force resolved in the direction of the radius-vector, and S the component in the plane of the orbit, but perpendicular to the radius-vector; then, as is well known—

$$\frac{d^2r}{dt^2} = \frac{c^2}{r^3} - \frac{\mu_0}{r^2} + \mathrm{R} \tag{4}$$

$$\frac{dc}{dt} = \mathrm{S}r$$

Substituting these values in (3), we have—

$$\frac{d^2p}{dt^2} = \frac{p}{r}\,\mathrm{R} - e_0 \sin v\,\mathrm{S} + \frac{c^2 - \mu_0 p}{r^3} \tag{5}$$

which since $\mu_0\, p_0 = c_0^2$, may be written—

$$\frac{d^2\delta p}{dt^2} + \frac{\mu_0}{r^3}\cdot\delta p = \frac{p}{r}\,\mathrm{R} - e_0 \sin v\,\mathrm{S} + \frac{(c_0 + c)\,(\delta c + \delta c_1) - \mu_0\,\delta p_1}{r^3} = \mathrm{A}_1 \tag{6}$$

where p_1 is the value of p at the time t_1, $\delta p = p - p_1$, $\delta p_1 = p_1 - p_0$, $\delta c = c - c_1$ and $\delta c_1 = c_1 - c_0$.

If we put $\mu_1 = \frac{c_1^2}{p_1}$ and $\delta\mu_1 = \mu_1 - \mu_0$, equation (6) becomes—

$$\frac{d^2\delta p}{dt^2} + \frac{\mu_1}{r_1^3}\,\delta p = \mathrm{A}_1 + \frac{\delta\mu_1\delta p}{r^3} + \mu_1\left(\frac{1}{r_1^3} - \frac{1}{r^3}\right) = \mathrm{A} \tag{7}$$

We may also write—

$$\frac{d^2x_1}{dt^2} + \frac{\mu_1}{r_1^3}\cdot x_1 = 0 \tag{8}$$

$$\frac{d^2y_1}{dt^2} + \frac{\mu_1}{r_1^3}\cdot y_1 = 0$$

where x_1 and y_1 are the rectangular co-ordinates in the orbit which osculates with the true at the time t_1.

Combining (7) successively with each of the equations (8) and integrating,* we have—

$$c_1\,\delta p = y_1\int \mathrm{A}x_1\,dt - x_1\int \mathrm{A}y_1\,dt \tag{9}$$

If we put—

$$x_1 = r_1 \cos v_1 \tag{10}$$

$$y_1 = r_1 \sin v_1$$

$$\mathrm{P} = r_1\,\mathrm{A}$$

equation (9) becomes—

$$\delta p = \frac{r_1}{c_1}\sin v_1\int \mathrm{P}\cos v_1\,dt - \frac{r_1}{c_1}\cos v_1\int \mathrm{P}\sin v_1\,dt \tag{11}$$

whence—

$$p = p_1 + p_s\frac{r_1}{c_1}\sin v_1 - p_c\,\frac{r_1}{c_1}\cos v_1 = a\cos^2\phi_0 \tag{12}$$

where the meaning of a, p_s and p_c is self evident.

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* Oppolzer, Lehrbuch zur Bahnbestimmung, Vol. II., p. 257.

Introducing the mean and eccentric anomalies we have—

$$\begin{aligned} \mathrm{M} &= \mathrm{E} - e_0 \sin \mathrm{E} \\ x &= a \cos \mathrm{E} - a e_0 \\ y &= a \cos \phi_0 \sin \mathrm{E}. \end{aligned} \tag{13}$$

If we differentiate each of these and substitute in the equation—

$$c = x \frac{dy}{dt} - y \frac{dx}{dt}$$

we have—

$$c = n\, a^2 \cos \phi_0 = \frac{np^2}{\cos^3 \phi_0} \tag{14}$$

where $n = \frac{dm}{dt}$ is the mean motion counted from a fixed perihelion; whence—

$$\frac{d\delta n}{dt} = \frac{n}{c} \cdot \frac{d\delta c}{dt} - \frac{2n}{p} \cdot \frac{d\delta p}{dt} \tag{15}$$

In applying these formulæ the elements employed may be considered constant for short intervals of time, and only changed at the end of each interval. If this be done we may put A = A₁, and write—

$$\begin{aligned} \mathrm{P} &= p_1 \mathrm{R} - e_0 r_1 \sin v_1 \mathrm{S} + \frac{(c_0 + c_1)(\delta c + \delta c_1) - \mu_0 \delta p_1}{r_1{}^2} \\ \frac{d\delta m}{dt} &= \delta n = \frac{n_1}{c_1} \delta c - \frac{2n_1}{p_1} \delta p \\ &= \frac{n_1}{c_1} \left(\delta c + \frac{2(p_c \cos v_1 - p_s \sin v_1)}{1 + e_0 \cos v_1} \right) \end{aligned} \tag{18}$$

Recapitulating and adding the equations depending upon Z, the component of the disturbing force perpendicular to the plane of the orbit, we have—

$$\begin{aligned} \frac{d\delta c}{dt} &= \mathrm{S} r_1 \\ \frac{dp_c}{dt} &= \mathrm{P} \sin v_1 \\ \frac{dp_s}{dt} &= \mathrm{P} \cos v_1 \\ \frac{d\delta m}{dt} &= \frac{n_1}{c_1} \left(\delta c + \frac{2(p_c \cos v_1 - p_s \sin v_1)}{1 + e_0 \cos v_1} \right) \\ \frac{d\delta \Omega}{dt} &= \mathrm{Z} \frac{r_1}{c_1} \cdot \frac{\sin(v_1 + \pi_1 - \Omega_1)}{\sin i_1} \\ \frac{d\delta i}{dt} &= \mathrm{Z} \frac{r_1}{c_1} \cos (v_1 + \pi_1 - \Omega_1) \end{aligned} \tag{19}$$

For the determination of the remaining elements, we have at the end of each interval—

$$\begin{aligned} n &= n_1 + \frac{d\delta m}{dt} \\ a &= \sqrt{\frac{c_1}{n_1 \cos \phi_0}} \\ \pi &= \pi_1 + 2 \sin^2 \tfrac{1}{2} i_1 \cdot \delta \Omega \end{aligned} \tag{20}$$

UEBER EIN ALTES MANUSCRIPT DER PULKOWAER STERNWARTE.

Von. A. WAGNER.

Der Freundlichkeit eines Herrn Goschkewitsch, ehemals russischen Consuls in China verdankt die Pulkowaer Sternwarte schon seit geraumer Zeit den Besitz eines arabischen Manuscriptes, welches mir für die Geschichte der Astronomie des Mittelalters nicht ohne Bedeutung zu sein scheint. Ich bin leider nur im Stande ziemlich dürftige Nachrichten von demselben zu geben, hoffe aber dass auch diese vielleicht im Stande sind das Interesse eines Freundes der Geschichte der Astronomie, der zugleich Kenner des Arabischen ist, zu wecken, und ihn zu weiterer Nachforschung in dieser Richtung zu veranlassen.

Der Verfasser des Manuscriptes ist bisher nicht zu ermitteln gewesen. Wir wissen nur, dass Herr Goschkewitsch dasselbe gleichzeitig mit einem chinesischen Manuscripte, das darauf Bezug zu haben scheint und das auch im Besitz der Sternwarte ist, in China, wenn ich nicht irre in Tientsing, erworben hat. Es ist auf ziemlich dickem Papier, welches durch Zusammenkleben mehrerer Lagen dünnen Papiers hergestellt ist, wie es scheint mit Tusche geschrieben, und zeigt Spuren seiner chinesischen Herkunft auch in chinesischen Schriftzeichen, die sich auf einer Seite des Manuscriptes vorfinden.

Ich habe das Manuscript ein arabisches genannt; es kann aber auch ein persisches sein, denn est besteht nur aus Zahlentabellen mit den dazu gehörigen Ueberschriften. Da die Perser sich der arabischen Schrift bedienen und für Astronomie und Mathematik arabische Termina technica benutzen, so lässt sich in soweit keine Entscheidung über die Nationalität des Verfassers treffen. Ein Titelblatt fehlt, und ebensowenig ist eine Einleitung oder Erklärung des Gebrauchs der Tafeln vorhanden. Es scheint mir aber, wie ich bald zeigen will, Grund zu der Annahme vorhanden zu sein, dass die Tafeln persischen Ursprungs sind.

Das Manuscript besteht gegenwärtig aus 20 Blättern, die erst neuerdings mit einer Paginirung versehen worden sind. Es trägt die Spuren vielfacher Benutzung an sich, indem die untern Ecken verschiedener Blätter stark abgegriffen sind. Einer unserer Orientalisten meinte nach dem Character der Schriftzüge das Manuscript dem XII^ten^ Jahrhundert zuschreiben zu können; aus Gründen, die dem Inhalte des Manuscriptes zu entnehmen sind, dürfte seine Abfassung auf die zweite Hälfte des XIII^ten^ Jahrhunderts zu verlegen sein.

Das Manuscript enthält vollständige Tafeln für die Bewegung der Sonne, des Mondes und der fünf den Alten bekannten Planeten. Ich will jetzt so viel von dem Inhalt der Tafeln angeben, als ich, ohne Orientalist zu sein und ohne zu viel Zeit darauf zu verlieren, habe ermitteln können.

Die erste, äussere Seite des Manuscriptes ist unbeschrieben, ich habe sie daher mit 0 bezeichnet.

P. 1 und 2 enthalten jede acht Columnen, von denen die zweite auf jeder Seite offenbar das Argument zu den tabulirten Grössen bildet. In allen Columnen mit Ausnahme der zweiten sind die Zahlen durch die arabischen Buchstaben ausgedrückt. Columne 2 enthält die Jahreszahlen der muhamedanischen Zeitrechnung durch Ziffern ausgedrückt, aber so, dass die Einer, Zehner, Hunderte und das Tausend besondere Specialcolumnen erhalten haben, mit besondern Ueberschriften welche dieses in arabischer Sprache bezeichnen. Die erste Columne giebt die Wochentage des Jahresanfangs der Epoche an. Die erste Zeile der tabulirten Werthe hat keine Jahresbezeichnung; sie gilt aber für den Anfang der muhamedanischen Aera, und der Wochentag ist gleich 6, was wohl ein Freitag bedeuten soll. Das nächstfolgende Argument ist 600 und von da an wachsen dieselben stets um 30 bis zum Jahre 1440 Mohameds. Die Tafeln sind demnach nicht auf häufigere Benutzung für die zeit vor 600 eingerichtet, woraus schon folgt dass die Tafeln nach dem Jahre 600 = 1204 n. Chr. abgefasst sind.

Die dem Argumente entsprechenden Grössen sind der Reihe nach die folgenden—

Auf Seite 1, Col. 3 Mittlere Länge der Sonne,
4 Mittlere Länge des Mondes,
5 Mittlere Anomalie des Mondes,
6 Doppelte mittlere Elongation des Mondes,
7 Mondsknoten,
8 Epicykel des Saturn,

und auf der gegenüberstehenden Seite 2—

Col. 3 Epicykel des Jupiter,
4 Epicykel des Mars,
5 Epicykel der Venus,
6 Epicykel des Merkur,
7 8 Zwei Quantitäten welche für das Jahr 660 = 0 gesetzt sind und deren jährliche Bewegungen für das muhamedanische Jahr respective 47″·878 und 58″·225 betragen. Die eine dieser Quantitäten ist offenbar die Praecession, die andere wahrscheinlich Bewegung des Apogaeums der Sonne.

Pag. 3 und 4 enthalten die Bewegungen derselben 12 Grössen, die auf den vorhergehenden Seiten vorkommen, für die einzelnen Jahre des 30jährigen Cyclus der mohamedanischen Zeitrechnung. Als Schaltjahre sind in diesem Cyclus angesetzt die Jahre 2, 5, 7, 10, 13, 16, 18, 21, 24, 26, 29. In den Argumentencolumnen befindet sich wiederum die Jahreszahl in Zifferschrift mit den entsprechenden Ueberschriften für Zehner und Einer und daneben der Wochentag des letzten Tages des betreffenden Jahres, den ersten Tag des Cyclus = 1 gesetzt, sowie die Bezeichnung der Schaltjahre.

Die nächsten beiden Seiten 5 und 6 enthalten die Bewegungen für die einzelnen Monate des Jahres in doppelter Weise, nämlich einmal für die bekannten arabischen Monate, welche abwechselnd 30 und 29 Tage haben und wo im Schaltjahre der letzte Monat 30 Tage statt 29 erhält, und das zweite mal für Monate die mit den persischen Namen* bezeichnet sind, welche die folgenden sonst nicht bekannten Dauern erhalten—

0	Ferwerdîn,	31 Tage.	6	Mir,	30 Tage.
1	Ardebehescht,	31 „	7	Abân,	30 „
2	Chordâd,	31 „	8	Ader,	29 „
3	Tîr,	32 „	9	Dei,	29 „
4	Mordâd,	31 „	10	Bahmen,	30 „
5	Scharîr,	31 „	11	Asfendârmed,	30 „

Im Schaltjahr erhält der letzte Monat 31 Tage.

Auf den folgenden Blättern des Manuscriptes kommen arabische Ziffern nicht weiter vor.

Die Seiten 7 und 8 enthalten die Bewegungen für die einzelnen Tage des Monats, und p. 9 und 10 die für die Sexagesimaltheile, wobei das Ganze gedrängter gehalten und die beiden letzten Columnen der vorhergehenden Doppelseiten, als überflüssig, weggelassen sind.

Von den folgenden Tafeln scheinen mir die auf p. 11 und 12 die Mittelpunktsgleichung der Sonne nebst den Differenzen zu enthalten; der Maximalwerth tritt bei den Argumenten $3^s\ 2°$ und $8^s\ 28°$ mit 2° 0′ 47″ ein. Die Tafel p. 25 bezieht sich auf den Mond und giebt offenbar die Breite des Mondes; die zu Grunde gelegte Neigung der Mondbahn ist 5° 2′ 20″.

Andere Tafeln sind den Tafeln auf p. 11 und 12 ähnlich construirt; ich bin augenblicklich nicht im Stande ihre Bedeutung im Einzelnen anzugeben. Da die Schrift aber sehr deutlich ist, so lassen sich alle Zahlen leicht lesen, und es würde demnach durch Vergleichung mit andern alten Tafeln nicht schwer fallen die Bedeutung der einzelnen Theile festzustellen. Es genüge hier zu erwähnen, dass sie sich gleichfalls theils auf die fünf alten Planeten beziehen, unter anderem auch das zur Berechnung der Stillstandspunkte Nöthige enthalten, theils die Hilfsmittel zur Berechnung der Conjunctionen und Finsternisse von Sonne und Mond zu enthalten scheinen, mit einem Worte Tafeln, ganz ähnlich denen, wie sie z.B. in den verschiedenen Ausgaben der Alphonsinischen Tafeln erhalten sind.

Da bei den Tafeln kein Verfasser genannt ist, so hoffte ich zunächst aus der denselben zu Grunde gelegten Sonnenbewegung einigen Aufschluss über ihren Ursprung zu erhalten.

Auf p. 1 findet sich die mittlere Länge der Sonne—

für das Jahr der Flucht:		3^s	26°	5′	8″
„ „	600	5	14	25	19
„ „	1200	7	2	45	29
„ „	1350	1	14	50	32

Das mohamedanische Jahr enthält bekanntlich 354 $\frac{11}{30}$ Tage. 1200 Jahre also 425240 Tage, während welcher Zeit die Sonne 1164 Umläufe + 3^s 6° 40′ 21″ gemacht hat. Dem entspricht ein tropiches Jahr von 365^d 5^h 48^m 46^s. Diese Jahreslänge, die zufällig mit der in der neueren Zeit scharf ermittelten genau übereinstimmt, kommt so viel ich habe ausfindig machen können, bei keinem der alten Astronomen vor. Ich gebe hier zur Vergleichung die von Herrn Houzeau im " Répertoire des constantes de l'Astronomie " angeführten Längen des tropischen Jahres von Ptolemäus an—

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* Ich habe die Lesung der Namen aus Ideler's Chronologie Bd. II p. 515 entlehnt, mit dem die persische Schreibweise im Allgemeinen übereinstimmt.

— 140	Ptolemäus,	365^d	5^h	55^m	14^s
+ 880	Al Battani,			46	30
1091?	Sûrya-Siddhanta,			52	14
1210	Thebit ben Corrah,			48	49
1230	Aboul Hassan,			54	1
1250	Alphonsus,			49	16
1271	Nassir Eddin,			49	2
1346	Chrysococca,	365^d	5^h	48^m	57^s
1437	Ulug Beg,			49	45
1543	Copernicus,			49	6
1551	Reinhold (Tab. Prut.),			49	16
1600	Tycho Brahe,			48	45·3
1627	Kepler (Tab. Rud.),			48	57·6

Es ist hierbei zu bemerken, dass nach Weidler Thebit ben Corrah das tropische Jahr für veränderlich gehalten hat und von ihm nur die mittlere Dauer des siderischen Jahres = 365^d 6^h 9^m 12^s angegeben ist, dass die von Herrn Houzeau für diesen Astronomen aufgeführte tropische Jahreslänge also nicht von Thebit selbst abgeleitet sein kann.

Man sieht also jedenfalls, dass der Verfasser der Tafeln aus einer besondern Quelle geschöpft hat, und es dürfte von Interesse sein dieser Quelle nachzuspüren. Könnten es vielleicht die Ilkahanischen Tafeln des Nassir Eddin sein? Die im obigen Verzeichniss aufgeführte Jahreslänge des Nassir Eddin stimmt damit nicht; sie ist aber nicht direct den Ilkahanischen Tafeln entnommen, sondern kann nur aus einer Angabe des Shah Cholg, von Greaves (London 1652 p. 42) publicirt, abgeleitet sein, da Bailly, der als Quelle aufgeführt wird, nur diesen hat benutzen können. Die Ilkahanischen Tafeln scheinen bis jetzt nicht publicirt zu sein, obgleich Jourdain in seinem " Mémoire sur l'Observatoire de Méragah " dieselben bereits 1810 als auf der Pariser Nationalbibliothek vorhanden erwähnt.* Es ist aus dieser Abhandlung und wohl auch schon früher bekannt, dass Nassir Eddin in hohem Ansehen bei Holagu Chan, dem mongolisch-tartarischen Eroberer Persiens stand, während Holagu's Bruder Kublai Chan, nach Gaubil in China Houpitié genannt, die Herrschaft der Mongolendynastie in China ausbreitete und befestigte. Gaubil in seiner " Histoire abrégée de l'Astronomie Chinoise "† erwähnt dass Kublai an seinem Hofe ein mathematisches Tribunal aus dem Occident gehabt habe, und stellt die Vermuthung auf, dass die zu jener Zeit verbesserten Methoden der Chinesischen Astronomie unter dem Einflusse dieser persischen Astronomen entstanden wären. Er erwähnt ferner, dass der abendländische (persische) Astronom

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* Persische Manuscripte Nos. 169 u. 173. Siehe Monatliche Correspondenz Bd. 23. Nach einer freundlichen Mittheilung des Baron V. Rosen befinden sich noch Manuscripte der Ilkahanischen Tafeln in Leiden, in Florenz (Bibl. Medicea Laurentiana et Palatina), in Gotha (unvollständig) und wahrscheinlich auch in der Bodleyan Library in Oxford. [See note on p. 129.]

† Souciet, Observations Math., Astron. &c. T. II. p. 129.

Tsa-ma-tou-lin (nach Gaubil Gema-le-din) eine Astronomie zum Gebrauch der Chinesen abfasste, und in Peking Instrumente verfertigte, die von denen der Chinesen verschieden waren. Könnte dieser Astronom nicht auch der Urheber unserer Tafeln sein? Und was ist weiter von ihm bekannt? Gaubil scheint von ihm als von einer bekannten Persönlichkeit zu sprechen, und auch Sédillot erwähnt seiner.[*]

Wir finden übrigens auch in den Tafeln selbst noch Hindeutungen auf jene Zeit und jene Beziehungen. Ich habe schon oben erwähnt, dass die in den Columnen 5 und 6 der Seiten 2, 4, 6, 8, aufgeführten Quantitäten, von denen die eine ohne Zweifel die Präcession bedeutet, für das Jahr 660 = 0 gesetzt sind. Der Grund dafür kann doch wohl nur der gewesen sein, den Gebrauch der Tafeln zu der Zeit, in welcher sie verfasst wurden, möglichst bequem zu machen. Das Jahr 660 Mohameds entspricht aber dem Jahre 1261 unserer Zeitrechnung. Kublai Chan starb wie im ersten Bande des Souciet p. 201 erwähnt im Jahre 1294, 80 Jahre alt; die Anfertigung der Tafeln fällt also wahrscheinlich gerade in seine Regierungszeit.

Ein zweiter Umstand, der unsere Aufmerksamkeit verdient, ist die von mir schon erwähnte Form der persischen Monate. Es findet sich die Erklärung dieser Monatsdauern in einer Notiz von Ideler,[†] wonach unter Omar Alcheijam auf Vorschlag von acht Astronomen die Monate des persischen Sonnenjahres wahre Sonnenmonate sein, d. h. dem Verweilen der Sonne in den Zeichen des Thierkreises entsprechen sollten. Auch Chrisococces berichtet ähnliches vom Sultan Melixa.[‡] Diese Form der Monate, die in Persien selbst nicht in Gebrauch gekommen oder wenigstens nicht im Gebrauch geblieben ist, hat sich in unseren Tafeln erhalten. Ferner erwähnt Gaubil auf p. 133 des bereits citirten Werkes, dass die Mohamedaner Chinas unbewegliche Sonnenmonate haben, deren Namen, wie man sich dort überzeugen kann, mit Rücksicht auf die Chinesische Aussprache vollkommen mit den in unseren Tafeln vorkommenden übereinstimmen. Die Monatslängen für Monat (3) Tîr und (7) Abân weichen freilich von den in unseren Tafeln aufgeführten ab, indem Gaubil für beide 31 Tage angiebt, während in unseren Tafeln Tîr 32 und Abân 30 Tage hat. Der unregelmässige Verlauf der Zahlen bei Gaubil, für den man keinen Grund angeben kann, lässt aber eher bei Gaubil einen Schreib- oder Lesefehler vermuthen.

Noch ist zu bemerken, dass als Wochentag der Aera, d. h. der Flucht Mohameds 6 oder der üblichen Bezeichnungsweise entsprechend ein Freitag angesetzt ist, also der 16te Juli, während bekanntlich einige orientalische Astronomen ihre Aera auch mit einem Donnerstag, dem 15ten Juli beginnen.

Das anfangs erwähnte chinesische Manuscript hat neben dem chinesischen Titel die etwas unleserliche offenbar schon im 17ten Jahrhundert von einem der Jesuiten gemachte Aufschrift: "Praecepta antiqua, annis superioribus ex Maurorum libris excerpta, quibus usque in hodiernum diem adhuc utuntur . . . nihilominus anno 1668 Maurus . . . hodie metuebat uti hisce praeceptis, nisi . . . . ." Nach der Erklärung eines hiesigen Sinologen, des Herrn Professor Wassiljeff bedeutet der chinesische Titel des Werkes: Anleitung zur westlichen Astronomie, kann also gleichfalls auf die Perser gedeutet werden. Man dürfte demnach in diesem Werke einige Aufklärung über unsere Tafeln zu finden hoffen, aber leider sind Personen, welche die Kenntniss der chinesischen Sprache mit dem Interesse für Astronomie verbinden, gar zu selten.* Auf p. 45–46 dieses Manuscriptes befindet sich eine in chinesischen Ziffern sauber geschriebene Tafel, zu welcher der Commentator folgende Bemerkung gemacht hat: "Vide tabulas Maurorum qui habentur inter tabulas latitudinis planetarum quae est exactior quam haec." Ich schliesse daraus das schon der Jesuit unser Exemplar der persischen Tafeln besessen und verglichen hat. Die eben erwähnte Tafel ist eine Tafel der Mondbreiten, wobei die Chinesen die Neigung 5° 2′ angenommen aber nur die Minuten angesetzt haben, während in der persischen Tafel auch die Secunden gegeben sind. Auch ist zu bemerken, dass die Chinesen in ihrer Tafel die sexagesimale und nicht die sonst nach Gaubil bei ihnen übliche centesimale Gradtheilung angewandt haben.

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[*] Prolégomènes des tables astron. d'Ouloug-Beg, Paris 1853, p. XVIII.

[†] Handbuch der Chronologie Bd. 2, p. 526 (Berlin 1826).

[‡] Bullialdus, Tabulae Philolaicae p. 214 (Anhang zur Astronomia Philolaica, Parisiis 1645).

Zum Schluss will ich noch, um die Vergleichung unserer Tafeln mit anderen älteren Tafeln zu ermöglichen die Tafelwerthe für's Jahr 660 Moh. und die Bewegung in 30 muham. Jahren mittheilen. Es können diese Grössen auch dazu benutzt werden die Länge des Ortes, für dessen Meridian die Tafeln berechnet sind, wenn auch nur genähert zu bestimmen. In unserer Bibliothek sind leider keine andern arabischen Tafeln vorhanden, mit denen ich unsere Tafeln direct vergleichen könnte.

	660			In 30 Jahren.		
Mittl. Länge der Sonne,	241°	15′	20″	38°	25′	1″
„ „ des Mondes,	265	12		38	15	
Mittlere Anomalie des Mondes,	115	41		293	47	
2 (☾ — ☉)	47	54		359	40	
Mondsknoten,	18	31		202	58	
Epicykel des Saturn,	209	16		42	16	
„ des Jupiter,	267	46		234	39	
„ des Mars,	218	36		227	1	
„ der Venus,	239	9		74	15	
„ des Merkur,	215	37		267	44	
Sonnenapogaeum, ?				0	29	7
Praecession,				0	23	56

Pulkowa, $\frac{13}{25}$ Mai 1882.

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* Nach einer Mittheilung des Baron V. Rosen befindet sich in Paris ein Manuscript arabischer astronomischer Tafeln, deren Einleitung oder Erklärung chinesisch geschrieben ist. Die Vergleichung dieses Manuscriptes mit unseren Tafeln wäre gleichfalls sehr wünschenswerth.

Notes on the preceding Paper.

The length of the tropical year, $365^{d}\ 5^{h}\ 48^{m}\ 46^{s}$, deduced by M. Wagner from the annual increase of the mean longitude of the Sun as given in the MS., is nearly identical with the value found by Ebn-Junis, while Ko Show-King, the astronomer of Koblai Khan, adopted $365^{d}\ 5^{h}\ 49^{m}\ 12^{s}$. I have not Souciet's work at hand, but find the following remarkable passage in Sédillot's "Matériaux pour servir à l'histoire comparée des sciences mathématiques chez les Grecs et les Orientaux" (T. II p. 644, Paris 1849): "Cochéou-king savait que la sienne était un peu plus longue que celle qui lui était communiquée [*i.e.* by Gemal-eddin], mais elle lui donnait une intercalation commode de 97 jours en 400 ans." M. Wagner's conjecture, that the tables in question date from the reign of Koblai Khan, is therefore probably correct.

As to the length of the tropical year given in M. Houzeau's "Répertoire" under Nassir Eddin's name, M. Houzeau quotes Bailly's Astr. mod. I p. 591 as his authority; but Bailly states expressly that Thebit only determined the length of the *sidereal* year. It will be remembered that Thebit and after him most astronomers nearly down to Tycho Brahe believed the equinoxes to be subject to an oscillating motion (trepidatis).

Among the Oriental Codices in the Bodleyan Library Heilbronner (Historia Matheseos universæ p. 616) mentions "Nasiroddini Thusii Tabulæ Astronomicæ."*

If M. Wagner is right in supposing the figure 0° 23′ 56″ to be the Precession in 30 Mahomedan years, the Author of the tables has had a better knowledge of the correct value of this constant than most of the Arabian Astronomers possessed. For one Julian year his precession would be 49″·35, or 1° in 72·95 years, while Ebn Junis found 51″·24 or 1° in 70¼ years (Caussin p. 170), Nassir Eddin 1° in 70 years, while most others adopted 1° in 66 years.

The inclination of the Moon's orbit, which by Hipparchus, Ptolemy (and even by Al Battani) was put = 5° 0′, was first found to vary by the sons of Musa ben Schaker (about 850) and by Abul Hassan Ali ben Amadjour (early in the tenth century). Ebn Junis who mentions this, adds that he had measured it himself several times and found 5° 3′ or 5° 8′ (Sédillot, Matériaux p. 286).

J. L. E. D.

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* It is probably this manuscript which I had recently an opportunity of seeing at Oxford. It forms No. 249 of the Huntingdon division of the Bodleyan Library, and is thus described in the Catalogue—"Codex bombycinus, fine truncus, foliis tamen 140 constans. In eo habetur opus, *Methodus facilis* inscriptum, complectens Tabulas motuum septem Planetarum; quae apud Orientales magno in pretio sunt. Praecepta non adduntur, nisi pro Luna. Initium a Marte ducitur. Auctoris nomen *Nasireddin Althusi*." It is written in Arabic—black, with the arguments in red. Although it is somewhat blotted and has a few wormholes, it is still perfectly legible.

R. C.

BEMERKUNG ZU DR. H. KOBOLD'S AUFSATZ ÜBER DAS PROBLEM DER DREI KÖRPER.

VON PROFESSOR AUG. WEILER.

Die Zeitschrift Copernicus No. 16 und 17 enthält den Aufsatz, "Eine neue Behandlung des Problems der drei Körper," in welchem der Verfasser die Absicht hat, die wahre Anomalie des störenden Körpers als unabhängige Veränderliche in die Differentialgleichungen der Bewegung einzuführen.

In diesem Aufsatze ist die Analysis bei dem ersten Schritte, welch ender Verfasser unternimmt, verfehlt. Die zu transformirende Differentialgleichung S. 70 ist:—

$$\frac{d^2x}{dt^2} + k^2 (1 + m) \frac{x}{r^3} = k^2 (1 + m) \frac{d\Omega}{dx}$$

soll die Veränderliche v' an die Stelle von t gebracht werden, so ist zum Behuf der Transformation die Gleichung—

$$\frac{dv'}{dt} = \frac{k\sqrt{1 + m'}\sqrt{p'}}{r'^2}$$

gegeben, welche S. 71 zum Ausgangspunkte genommen ist.

Der Verfasser setzt nun—

$$\frac{d^2x}{dt^2} = \frac{d^2x}{dv'^2}\frac{dv'^2}{dt^2},$$

wo es doch heissen sollte:—

$$\frac{d^2x}{dt^2} = \frac{d^2x}{dv'^2}\frac{dv'^2}{dt^2} + \frac{dx}{dv'}\frac{d^2v'}{dt^2}.$$

In dem Letzteren ist noch zu schreiben:—

$$\frac{d^2v'}{dt^2} = -\frac{2k\sqrt{1 + m'}\sqrt{p'}}{r'^3}\frac{dr'}{dt} = \frac{2k}{r'}\sqrt{\frac{1 + m'}{p'}}\, e' \cdot \sin v' \frac{dv'}{dt}$$

oder—

$$\frac{d^2v'}{dt^2} = \frac{2k^2(1 + m')}{r'^3}\, e' \cdot \sin v'.$$

Der Verfasser hat also das mit $2e'$ multiplizirte Glied offenbar vergessen. Wenn hier $e' = 0$ gesetzt werden dürfte, so würde man mit demselben Rechte $v' = p' = a'$ schreiben oder die Bahn des störenden Körpers als kreisförmig betrachten. Dies kann aber die Absicht des Verfassers nicht gewesen sein. Denn wollte man die Bahn des störenden Körpers als kreisförmig ansehen, so wäre das Verhältniss $dv' : dt$ ein konstantes, und die beabsichtigte Transformation könnte nicht mehr zum Gegenstand einer Abhandlung gemacht werden.

BEOBACHTUNGEN VON SONNENFLECKEN AM ASTROPHYSICALISCHEN OBSERVATORIUM ZU O'GYALLA.

Gruppe.	M. Z. O'Gyalla (bürg.)	p	ρ	l	L	b	
				Rotationsperiode 284.			
16	Februar 2·5328	194°·77	210″·0	154°·75	354°·00	− 17°·13	Kleiner dunkler Fleck; Febr. 5 doppelt.
	„ 3·4850	225·90	371·7	169·54	355·18	− 16·85	
	„ 4·4850	236·41	558·3	184·64	356·04	− 16·22	
	„ 5·5272	239·92	730·1	200·20	356·74	− 15·91	
	Februar 4·0076				355·49	− 16·53	$\xi = 15°·1725 \quad \xi_0 = 14°·106$
		Mittl. Abw. einer Beob.			± 0·163	± 0·009	$\Delta b = + 0°·429$
17	Februar 2·5328	93·90	417·1	123·85	323·10	− 12·57	Einzelner Fleck.
14	Januar 31·5059	53·83	867·5	87·67	315·83	+ 18·23	Ein sich auflösender Fleck. Anfangs einfach und mit einem Hofe umgeben; von Febr. 2 an in zwei kleine Fleckchen getrennt.
	Februar 1·5085	47·94	762·2	101·71	315·57	+ 18·53	
	„ 2·5328	38·71	628·7	116·36	315·61	+ 18·51	
	„ 3·4850	24·91	509·8	129·59	315·23	+ 18·54	
	„ 4·4850	1·42	416·8	143·79	315·19	+ 18·36	
	„ 5·5275	330·14	414·2	158·32	314·84	+ 18·01	
	Februar 3·0074				315·38 ± 0·093	+ 18·36 ± 0·211	$\xi = 14°·0795 \quad \xi_0 = 14°·051$
18	Februar 2·5328	87·88	824·6	90·38	289·63	− 12·38	Einzelner Fleck.
	„ 3·4850	88·24	688·3	104·54	290·18	− 12·47	Kleinere Flecke.
		87·89	740·2	99·93	285·57	− 12·47	
	„ 4·4850	88·95	587·2	113·62	285·02	− 12·32	Grosser Fleck am Ende einer langen Gruppe.
	„ 5·5275	99·27	267·0	136·31	292·83	− 12·17	Grosser Fleck im W. einer Gruppe; die sich bis L = 234° b = − 12°·5 erstreckte.
	„ 8·4935	239·30	432·4	180·57	294·77	− 12·43	
	„ 9·5072	241·20	626·4	195·66	295·41	− 12·91	
	„ 10·4839	242·03	780·7	210·38	296·19	− 13·37	
	„ 11·5123	241·93	890·3	224·68	295·83	− 13·24	
	„ 12·4905	241·34	951·0	237·82	295·00	− 12·97	
	Februar 9·6692				294·97 ± 0·560	− 13·02 ± 0·501	$\xi = 14°·6500 \quad \xi_0 = 14°·197$
19	Februar 5·5275	54·77	725·7	106·63	263·15	+ 11·16	Hauptfleck einer kleineren Gruppe, der sich allmählig auflöste.
	„ 8·4935	354·22	291·0	151·76	265·96	+ 10·83	
	„ 9·5072	312·10	345·3	166·75	266·50	+ 11·18	
	„ 10·4839	290·49	489·9	180·99	266·80	+ 11·57	
	„ 11·5123	279·27	648·8	195·47	266·62	+ 11·68	
	„ 12·4905	273·30	778·8	208·73	266·07	+ 11·82	
	Februar 9·6692				265·85 ± 0·819	+ 11·37 ± 0·377	$\xi = 14°·7165 \quad \xi_0 = 14°·232$

Gruppe.	M. Z. O'Gyalla (bürg.)	p	ρ	l	L	b	
	Februar 4·4850	59°·74	894″·9	86°·11	257°·51	+ 12°·67	Flecke in der Mitte der Gruppe.
	„ 5·5275	55·37	787·5	100·86	257·38	+ 12·52	
		56·90	870·4	91·47	247·99	+ 13·79	
		64·16	897·8	85·92	242·44	+ 8·21	
	Februar 8·4935	46·97	463·3	129·59	243·79	+ 7·06	Grosser behofter Fleck. Febr. 8–11 besass er einen doppelten Kern, während er Februar 5 und nach Febr. 11 sich als einfacher runder Fleck darstellte.
	„ 9·5072	25·00	297·9	144·11	243·86	+ 7·15	
	„ 10·4839	337·96	238·8	158·14	243·95	+ 7·69	
	„ 11·5123	295·77	348·6	173·20	244·35	+ 7·66	
	„ 12·4905	279·59	511·1	186·83	244·01	+ 8·08	
	„ 13·4805	271·81	673·4	201·03	244·09	+ 8·05	
	„ 14·4804	266·58	811·4	215·46	244·26	+ 7·87	
	„ 15·4848	263·07	910·1	229·79	244·26	+ 7·68	
	Februar 11·8816				244·06	+ 7·64	$\xi = 14^\circ{\cdot}3311$ $\xi_0 = 14^\circ{\cdot}295$
					± 0·129	± 0·377	
	Februar 8·4935	50·53	566·5	122·25	236·45	+ 8·44	Behofter Fleck. Febr. 10 und 11 war der Kern doppelt.
	„ 9·5072	37·03	398·0	136·52	236·27	+ 8·31	
	„ 10·4839	7·64	273·0	150·13	235·94	+ 8·50	
	„ 11·5123	317·55	273·5	164·85	236·00	+ 8·21	
	„ 12·4905	289·81	414·1	178·81	235·99	+ 8·55	
	Februar 10·4975				236·13	+ 8·40	$\xi = 14^\circ{\cdot}1477$ $\xi_0 = 14^\circ{\cdot}384$
					± 0·117	± 0·140	
20	Februar 8·4935	126·47	297·9	143·04	257·24	— 20·04	Vorangehender eines Doppelflecks.
21	Februar 9·5072	42·63	680·3	117·40	217·15	+ 16·55	Fleck mit kleinem Begleiter.
28	Februar 14·4804	268·68	490·0	189·62	218·42	+ 2·26	Sehr unbeständige Gruppe kleiner Flecken.
	„ 15·4848	263·68	680·6	205·03	219·50	+ 3·16	
	„ 18·4907	254·78	949·8	242·49	214·07	+ 2·03	
23	Februar 11·5123	108·79	485·5	142·13	213·28	— 16·65	Behofter Fleck; Febr. 12 war der Kern getheilt.
	„ 12·4905	155·77	177·9	157·28	214·46	— 16·91	
	„ 13·4805	211·30	266·2	172·00	215·06	— 16·84	
	„ 14·4804	228·07	442·1	186·25	215·05	— 16·80	
	Februar 12·9909				214·46	— 16·80	
	Februar 15·4848	232·70	521·1	193·35	207·80	— 15·97	Behofter Fleck enstanden aus vier kleinen.
24	Februar 13·4805	337·97	199·6	159·68	202·74	+ 5·28	Kleiner Fleck.
26	Februar 13·4805	95·60	306·5	142·37	185·43	— 13·22	Verwaschener Fleck.
22	Februar 9·5072	61·71	941·4	81·87	181·62	+ 10·48	Behofter Fleck. Febr. 10–12 mit doppeltem Kerne.
	„ 10·4838	58·32	879·5	94·16	179·97	+ 11·21	
	„ 11·5123	53·63	762·8	109·08	180·23	+ 11·29	
	„ 12·4905	47·79	617·9	123·36	180·54	+ 11·15	
	„ 13·4805	35·88	458·0	137·29	180·35	+ 10·40	
	„ 14·4804	10·56	318·6	151·75	180·55	+ 10·29	
	„ 15·4848	328·41	292·7	165·92	180·39	+ 10·35	
	„ 18·4907	272·50	717·8	209·21	180·79	+ 10·73	
	Februar 13·7747				180·62	+ 10·77	$\xi = 14^\circ{\cdot}3484$ $\xi_0 = 14^\circ{\cdot}244$
					± 0·142	± 0·438	

Gruppe.	M. Z. O'Gyalla (bürg.)	p	ρ	l	L	b	
27	Februar 13·4805	266°·58	811″·4	118°·95	162°·01	+ 24°·11	Kleiner, undeutlicher Fleck.
	„ 15·4848	5·82	502·4	148·91	163·38	+ 21·75	
25	Februar 12·4905	89·65	926·4	85·28	142·46	— 17·54	Grosser, behofter Fleck.
	„ 13·4805	89·48	840·5	99·44	142·50	— 17·57	
	„ 14·4804	90·93	711·6	113·60	142·40	— 17·86	
	„ 15·4848	94·56	547·6	128·61	143·08	— 17·85	
	„ 18·4907	189·66	210·7	170·98	142·56	— 17·66	
	Februar 14·8854				142·60 ± 0·265	— 17·70 ± 0·152	$\xi = 14^\circ{\cdot}2995$ $\xi_0 = 14^\circ{\cdot}072$
29	Februar 15·4848	254·78	903·2	92·49	106·96	— 17·40	Einzelner Fleck.
	„ 18·4907	91·60	492·3	135·24	106·82	— 15·98	Gruppe zahlloser Fleckchen
30	Februar 26·5054	238·67	471·1	201·96	59·21	— 11·03	Gruppe von drei kleinen Flecken.
31	Februar 26·5054	111·83	173·0	165·19	22·44	— 13·80	Einzelner kleiner Fleck.
		84·84	343·2	152·60	9·85	— 12·03	Behofter Fleck mit doppeltem Kerne. März 1 verschwunden.

Rotationsperiode 285.

Gruppe.	M. Z. O'Gyalla (bürg.)	p	ρ	l	L	b	
34	März 6·4925	63·6	828	122·2	225·5	— 0·7	Regelmässiger, behofter Fleck.
	„ 7·4850	62·1	707	135·2	224·4	— 1·3	
	„ 11·5066	289·6	157	192·8	224·5	— 0·5	
	„ 12·5307	263·3	354	207·5	224·6	— 0·2	
	„ 13·4903	255·9	540	221·4	224·9	— 0·1	
	„ 15·4785	249·5	829	249·0	224·1	0·0	
	„ 16·4757	247·7	912	262·1	223·0	+ 0·1	
	März 12·2085				224·43 ± 0·557	— 0·35 ± 0·574	$\xi = 14^\circ{\cdot}119$ $\xi_0 = 14^\circ{\cdot}346$ $\Delta b = +0{\cdot}159$.
36	März 11·4987	230·0	511	217·6	249·5	— 14·3	Behofter Fleck im Westen der Gruppe.
	„ 12·5359	233·0	688	232·7	249·8	— 14·0	
	„ 13·4903	233·5	813	246·1	249·6	— 14·0	
	März 12·5083				249·6	— 14·1	
39	März 11·4987	82·9	940	106·5	138·3	— 17·8	Hauptfleck einer sich auflösenden Gruppe.
	„ 12·5307	82·8	870	121·7	138·8	— 18·2	
	„ 13·4903	83·6	767	135·2	138·7	— 18·3	
	„ 15·4785	90·2	468	162·5	137·6	— 17·8	
	„ 16·4757	103·0	302	176·0	136·8	— 17·6	
	„ 17·4858	145·7	182	190·2	136·6	— 17·5	
	„ 18·4956	200·7	243	203·8	136·0	— 16·8	
	März 15·0650				137·54 ± 0·387	— 17·71 ± 0·498	$\xi = 13^\circ{\cdot}870$ $\xi_0 = 14^\circ{\cdot}071$
43	März 26·5120	78·0	236	187·0	4·7	— 9·7	Behofter Fleck mit länglichem Kerne.
	„ 29·4858	235·7	408	229·1	4·4	— 9·3	
	„ 31·4636	237·3	740	256·7	3·8	— 9·0	
	April 1·4709	236·9	854	270·3	3·0	— 9·1	
	März 29·9831				4·0	— 9·3	

Rotationsperiode 286.

Gruppe.	M. Z. O'Gyalla (bürg.)	p	ρ	l	L	b	
49	April 1·4709	61·9	923	132·5	225·2	+ 0·2	Regelmässiger, behofter Fleck.
	„ 2·4835	59·8	838	147·4	225·6	+ 0·4	
	„ 5·4825	45·0	360	190·1	225·5	+ 1·3	
	„ 6·5294	18·7	171	204·7	225·2	+ 1·4	
	April 3·9916				225·4	+ 0·8	

Gruppe.	M. Z. O'Gyalla (bürg.)		*p*	*ρ*	*l*	L	*b*	
57	April	20·4793	228°·49	712″·7	273°·62	95°·12	— 15°·15	Grosser Fleck im W der Gruppe.
	„	22·4888	228·49	870·5	293·82	86·64	— 16·70	Grosser Fleck im O der Gruppe.
	„	23·4819	228·39	925·5	305·62	84·26	— 17·04	
58	April	20·4793	192·45	453·7	244·67	68·17	— 26·54	Einzelner Kern des westlichen Fleckes.
	„	22·4888	213·53	681·2	269·98	62·80	— 25·18	Hauptfleck der Gruppe, dessen Kern sich am 22. in zwei Stücke spaltete.
	„	23·4819	217·37 216·34	794·3 813·5	283·33 285·50	61·97 64·14	— 25·39 — 26·58	
	„	24·4798	219·59 218·34	879·6 893·4	296·61 299·14	61·03 63·56	— 25·11 — 26·49	
	„	25·4850	220·20	931·9	309·54	59·62	— 25·22	
	April	23·9839				61·36 ± 0·173	— 25·23 ± 0·119	$\xi = 13^\circ{\cdot}2185$ $\xi_0 = 13^\circ{\cdot}789$.
	April	23·4819	211·64	783·5	280·75	59·39	— 29·59	Fleck im O. der Gruppe.
	„	24·4798	214·35	868·4	294·03	58·27	— 29·66	
	April	23·9804				58·83	— 29·63	
59	April	20·4793	218·66	283·0	241·43	62·91	— 12·11	Kleiner sich auflösender Fleck.
	„	22·4888	235·35	643·5	269·79	62·61	— 11·80	
	„	23·4819	233·52	782·5	283·83	62·47	— 11·89	
	April	22·4833				62·66	— 11·93	
60	April	20·4793	31·78	468·0	200·81	22·31	+ 10·88	Behofter Fleck.
	„	22·4888	325·95	266·6	230·04	22·86	+ 11·28	
	„	23·4819	290·66	369·7	244·51	23·15	+ 11·67	
	„	24·4788	274·64	529·8	259·00	23·24	+ 11·99	
	„	25·4850	267·06	685·4	273·46	23·54	+ 12·37	
	„	26·4857	263·01	810·3	287·51	23·31	+ 12·65	
	April	23·8166				23·07 ± 0·164	+ 11·81 ± 0·074	$\xi = 14^\circ{\cdot}4488$ $\xi_0 = 14^\circ{\cdot}223$. $\Delta b = + 0^\circ{\cdot}306$.
61	April	20·4793	74·90	621·3	184·98	6·48	— 10·48	
	„	22·4888	84·95	244·8	213·44	6·26	— 9·66	
	„	23·4819	142·24	82·6	227·90	6·08	— 9·59	
	„	24·4788	221·36	210·4	241·30	5·54	— 9·49	
	„	25·4850	232·58	405·9	255·35	5·43	— 9·37	
	„	26·4857	235·78	585·3	269·25	5·05	— 9·23	
	April	23·8166				5·81 ± 0·139	— 9·64 ± 0·154	$\xi = 14^\circ{\cdot}0197$ $\xi_0 = 14^\circ{\cdot}264$. $\Delta b = + 0^\circ{\cdot}191$.

Durch eine längere Abwesenheit meinerseits wurde leider eine von Anfang März bis Mitte April reichende Unterbrechung der regelmässigen Beobachtungen herbeigeführt. Von den während dieser Zeit verzeichneten Flecken habe ich nur diejenigen häufiger beobachteten mitgetheilt, die auch bei einer früheren oder späteren Erscheinung beobachtet sind. Bezüglich der Bezeichnungen gilt das auf Seite 90 angegebene.

Dr. H. Kobold, Observator.

O'Gyalla, 1882, Juni 11.

A NEW DETERMINATION OF THE CONSTANT OF PRECESSION.

By J. L. E. DREYER, Ph.D.

When we consider how many theoretical and practical investigations the development of Astronomy has called forth during the present century, it cannot but seem remarkable, how few researches we find the object of which is an exact determination of the constant of Precession. From the time of Copernicus until the beginning of this century the knowledge of the value of this important constant advanced very little until Piazzi produced a fairly exact determination of it. Not counting theoretical researches on the subject such as those of Laplace, Leverrier, Lehmann, and Stockwell, which possess great interest in themselves but cannot as regards accuracy compare with those deduced from observations, we have in reality only three determinations of the constant of Precession, those of Bessel, Struve and Nyrén. Of these only the two first have come into practical use, while the last shows a remarkable deviation from them. As it is no doubt universally acknowledged that we are yet far from knowing the value of the constant with sufficient accuracy, the reason why no recent researches in this direction have been made, is probably to be sought in the expectation that we shall at no distant future possess better materials for the purpose. I shall however in the following endeavour to show that we possess at the present moment materials enough for deducing a value of the constant, which at least may act as a confirmation of one or other of the older constants.

Before proceeding to describe the way in which I have found a new value it will not be useless to recall to the reader's mind the methods and materials used by Bessel, Struve and Nyrén.

Bessel's investigation appeared first in his "Fundamenta Astronomiæ," p. 285, and is founded on a comparison in R.A. of 2278 stars and in Declination of 2429 stars occurring both in Piazzi's Catalogue for 1800 and in the Catalogue for 1755 from Bradley's observations. This comparison gave for the epoch 1777·5 values of the well-known constants m and n. The latter constant is independent of constant errors in the two catalogues, while m depends on the absolute Right Ascensions. For this reason Bessel some years later returned to the question (Astr. Nachr. No. 92), as both the Königsberg Catalogues for 1815 and 1825 and Lindenau's new determination of the constant of Nutation showed the need of a positive correction to Piazzi's Right Ascensions. It appears that Piazzi founded his determination of the Equinox on observations made in the years 1803 to 1805 when the Nutation was very near its positive maximum, and accordingly his Right Ascensions became too small, the constant of Nutation used by him being too large. In the same manner Bradley's Right Ascensions turned out too large. The corrections to Bradley and Piazzi found by Bessel were respectively $-0''\cdot5$ and $+1''\cdot585$. The rectified values of m and n for 1777·5 compared with Laplace's Theory gave two values of the correction to the theoretical Lunisolar Precession, which were brought to equality by increasing *m* and decreasing *n* by equal amounts. Bessel's final result for the constant of General Precession, reduced to 1800 is—

$$50''{\cdot}2235.$$

This result is founded on an investigation entirely free from arbitrary assumptions, and not until after the publication of Dr. Auwers' new reduction of Bradley's observations (and possibly not even then) will it be practicable to deduce a sensibly different result from a renewed discussion of the same materials if we except the modification which a new determination of the equinoxes of 1755 and 1800 will introduce, to which matter we shall return later on. We have certainly since 1826 (when Bessel's paper appeared) got a new and better determination of the constant of Nutation by C. A. F. Peters, but this would not influence the Right Ascensions much, those of Bradley only requiring a correction of $-0''{\cdot}3$ instead of $-0''{\cdot}5$, while Bessel's correction to Piazzi would be diminished by about $0''{\cdot}33$. It may also be mentioned here that Bessel's constant has met with a confirmation by Mädler, who finds as a mean of 2193 Proper motions of Stars between ± 30° Declination $-0''{\cdot}0003$, while the mean should have been Zero if Bessel's *m* had been absolutely correct.* As Mädler's investigation is quite independent of his or any other hypothesis about the construction of the Sidereal System, this confirmation of Bessel's result would no doubt be of great importance; but as remarked by Chauvenet† it cannot be considered as certain that the sum of all the Proper Motions ought to be Zero, as the unequal distance of the stars from us will affect the apparent *amount* of Proper Motion differently in different parts of the heavens.

Otto Struve's "Bestimmung der Constante der Praecession mit Berücksichtigung der eigenen Bewegung des Sonnensystems" appeared in 1842 in the Mémoires de l'Académie Impériale de St. Pétersbourg. For this determination were used the positions of exactly 400 stars observed in Dorpat between 1822 and 1838 and occurring in Bradley's Catalogue. The Right Ascensions of the latter were corrected by $-0''{\cdot}31$ owing to Peters' constant of Nutation being adopted. Except 32 Standard Stars, 32 Stars observed in Dorpat to determine the Mean Refraction and 39 other stars, all the rest were Double Stars. We need not here describe in detail Struve's discussion of the accuracy of the Dorpat Observations, but the way he utilized the material for determining the constant of Precession deserves special consideration, as Struve alone has taken into account the motion of the Solar System through space. It is clear that this is not necessary if a sufficient number of stars is employed to justify us in supposing them uniformly distributed over the 24 hours of Right Ascension, as the influence on the R.A. in a certain part of the heavens would be counteracted by the influence exercised in the region diametrically opposite. As regards the Declinations a uniform distribution in zones of north and south Declination would be necessary. Such a distribution could of course not be

* Beobachtungen d. Kais. Univ. Sternwarte zu Dorpat, XIV., p. 11.

† Spherical and Practical Astronomy, 4th Ed., Vol. 1., p. 707.

expected in the case of a small number of stars, and Struve was therefore obliged to pay special attention to the motion of the Solar System.

Supposing the direction of this motion known and that all the stars employed are at an equal distance from us, and if we denote by q the amount of this motion in 70 years as seen from a star 90° from the Apex of the Solar System (A, D), it is easy to express the apparent motion of a star by—

$$d\alpha = -q \cos \mathrm{D} \sin (\mathrm{A} - \alpha) \sec \delta \qquad = \mathrm{B}q.$$

$$d\delta = -q \sin \mathrm{D} \left(\cos \delta - \frac{\sin \delta \cos (\mathrm{A} - \alpha)}{\tan \mathrm{D}} \right) = \mathrm{C}q.$$

If we call the difference found by comparing the two catalogues of Stars in R.A. g and in Decl. g', each star gives two equations of the form—

$$\mathrm{K}\Delta\pi + \mathrm{L}q = g$$

$$\mathrm{K}'\Delta\pi + \mathrm{L}'q = g'$$

where $\Delta\pi$ is 70 times the sought correction to the general precession for 1790.

The quantities K and K′ are proportional to P and P′, the Precession of each star in R.A. and Decl. during the interval between the epochs of the two catalogues. Struve introduces the quantities m and n into the equations and multiplies the equations for R.A. with cos δ in order not to give undue weight to stars of high declination. The equations are then—

$$\frac{\mathrm{P} \cos \delta}{70m} \Delta m + \mathrm{L} \cos \delta q = g \cos \delta.$$

$$\frac{\mathrm{P}'}{70n} \Delta n + \mathrm{L}'q = g'.$$

The co-efficients L and L′ have now to be determined, as they cannot be put equal to B and C, which amounts to assuming all the stars to be at the same distance from us.

So far there is in Struve's proceeding nothing arbitrary, but this can hardly be said of the rest of the investigation. First the mean proper motions in a great circle are determined for each class of magnitude, and these motions are supposed to be inversely proportional to the mean distances of the stars of each class. These mean distances were determined by W. Struve from an examination of the distribution of the stars in Harding's Atlas, and by means of them the Proper Motion of one class is reduced to that of another. In doing this Struve adopts the motion of stars of the 6th magnitude as origin as being founded on the greatest number of stars. The agreement between observation and calculation is not very satisfactory, as the following table shows—

Magn.	Mean Dist.	Observed PM.	Computed PM.
1	1·00	36″·1	43″·2
2	1·71	10·9	25·3
3	2·57	11·0	16·8
4	3·76	8·4	11·5
5	5·44	6·7	8·0
6	7·86	5·5	5·5
7	11·34	4·5	3·8

Calling the mean distance of a star of a certain class E and multiplying the co-efficient by 7 to avoid small fractions, Struve's equations become—

$$\frac{\mathrm{P}\cos\delta}{70m}\,\Delta m + \frac{7\,\mathrm{B}\cos\delta}{\mathrm{E}}\,q' = g\cos\delta,$$

$$\frac{\mathrm{P}'}{70n}\,\Delta n + \frac{7\,\mathrm{C}}{\mathrm{E}}\,q' = g'.$$

Struve gives the equations for the single stars different weights (the brighter less, the fainter greater) according to two different systems depending on the adopted value of the accidental error of the absolute Proper Motions, but the results according to the two systems of weights do not differ much. Finally the Right Ascensions gave $\Delta\pi = +\,1''{\cdot}16 \pm 0''{\cdot}67$ and the Declinations $+\,0''{\cdot}66 \pm 0''{\cdot}86$, adopted mean $= +\,0''{\cdot}97 \pm 0''{\cdot}53$. The general Precession for 1790 is—

$$50''{\cdot}23492 \pm 0''{\cdot}00757$$

while Bessel's value for the same epoch is 50″·22106. In his paper on the constant of Nutation Peters combines Struve's values of m and n with his own determination of the obliquity of the Ecliptic and of the planetary precession and finds for $1800 + t$—

$$\text{Prec. Gen.} = 50''{\cdot}2411 + 0''{\cdot}0002268\,t,$$

which value, as already mentioned, has come into very general use.

Although it cannot be denied that Struve's investigation is carried out with great ability, it appears that the result can scarcely claim more confidence than Bessel's. First it should be remembered that for an exact determination of the constant of Precession two sets of data are required, viz., two well-founded Equinoxes and a number of stars observed near both their epochs. The greater the number of stars used, the more probable it is that accidental errors of observation and unknown or imperfectly determined Proper Motions will become eliminated from the result. Even if the Solar System had no motion whatever it seems undesirable to use so small a number of stars as 400, and if one were anxious to use a small number, the brightest stars ought at any rate to be excluded, as the condition of uniform distribution cannot possibly be fulfilled as far as they are concerned. Another objection is, that about three-fourths of Struve's stars were Double Stars among which are many binaries. Until some observer of Double Stars has commenced from time to time to observe the relative positions of the principal components of binaries and neighbouring stars, it seems in no way proved that the motion of those components around the centre of gravity of the system does not sensibly affect their absolute places in the heavens. Considering in how many cases of binaries the two components are only slightly different in brightness, it seems very improbable that the change of place of the principal star should be insensible when that of the companion is so easily seen. Such motions will hardly ever be detected by meridian observations as they are not progressive and are therefore more easily mixed up with errors of observation, systematic deviations between catalogues &c., but there seems little likelihood that they should not in the course of seventy years have some influence (though possibly but a slight one) on the amount of Precession derived from a small number of stars, chiefly double.

As regards the influence of the motion of the Solar System through space, Struve's result depends to a considerable extent on the adopted relative distances of the stars of the first seven classes, and the hypotheses on which these are founded, appear to be very uncertain. These are, the supposition of the brightness of the stars as generally being a function of their distance, and secondly the hypothesis of uniform distribution of the stars within the system of the Milky Way, so that a conclusion can be drawn from the number of stars of equal brightness as to their relative mean distance. With the first hypothesis the few tolerably well-determined annual parallaxes seem to disagree, as hardly one of them belongs to a very bright star (with the one doubtful exception of α Centauri), although of course it is likely enough that the majority of bright stars are nearer to us than the majority of faint ones. As to the second hypothesis, it is only necessary to remember the very irregular construction of the Milky Way, the undoubted want of resolvability of some parts of it (*e.g.* in Cygnus) and the large number of Globular Clusters in or close to the Milky Way, especially in the hours 17 and 18 of R.A. These and other circumstances which it will not be necessary to enter into here, make Struve's hypotheses rather doubtful.

The method adopted by Struve therefore does not appear to be the best for the determination of so very important a constant as that of Precession, and further researches on the subject seem to be most desirable. We shall now shortly consider an investigation by Dr. Magnus Nyrén published at Upsala in 1869 and entitled "Försök til bestämning af Precessionskonstanten medelst ljussvaga stjärnor."*

As the title indicates, this investigation differs from the two already considered by employing much fainter stars, chiefly of the 7th, 8th and 9th magnitudes. Stars of these classes are so numerous, that errors of observation, physical proper motions and the influence of the motion of the Solar System may be supposed to be eliminated with even greater probability than in the case of Bradley's and Piazzi's 2400 stars. Provided the observations are good, it will not be of any consequence that the majority of the stars employed have generally only been observed once by each observer. The method first followed by Nyrén is therefore perfectly legitimate. The catalogues used by him are Weisse's and Schjellerup's Catalogues of Stars between ± 15°. Declination. Bessel's Observations from which the mean positions for 1825·0 in the first-mentioned catalogue were deduced, were made between August 21, 1821 and February 12, 1825, while Schjellerup's observations were made between September 7, 1861 and December 6, 1863, so that the interval between the two epochs was very nearly forty years. The two catalogues have about 5300 stars in common, and Weisse's positions for these were reduced from 1825 to 1865 by the mean of the precessions given in the two catalogues. Great care was taken to free Bessel's observations from the systematic errors, whose existence in many zones were revealed through the comparison with Schjellerup. Although the investigation comprised about 250 zones and the Author might perhaps have been justified in supposing that such constant errors would have been eliminated, Nyrén endeavoured to clear Bessel's observations of them by comparisons with Struve's "Positiones Mediæ" and the Armagh Catalogue. The mean epoch of the Dorpat observations is about 1830·7 or about seven years later than Bessel's epoch, but this was taken into account afterwards in fixing the interval between Bessel's and Schjellerup's epochs as 36·0 years. As Robinson's epoch is considerably later, Nyrén only made use of stars from the Armagh Catalogue which also occur in Bradley, and for which the sum of Precession and Proper Motion is well known. The differences Weisse – Struve and Weisse – Robinson generally agreed very well and thus guaranteed the reliability of the corrections deduced. For each zone the mean was taken of all the correction-differences belonging to it, and this was applied as a correction to the zone. Where a zone did not contain a sufficient number of correction-stars it was compared with other zones which had already been corrected, and in all cases the difference between the Fundamental Systems, on which the various catalogues were founded, was taken into account by applying Argelander's reductions to Wolfers. Nyrén's paper contains a complete list of all the corrections to Bessel's zones thus determined, and after applying them the agreement with Schjellerup becomes much closer. No account was taken of the Proper Motions, and only in a few cases, where the difference between Bessel and Schjellerup amounted to one or more seconds of time, was an examination made by means of other authorities to see if either of the two observers had committed an error of one or more seconds. It can hardly be considered quite correct that no attention was paid to the Proper Motions, for though the apparent motions arising from the motion of the Solar System become eliminated by the regular distribution of the stars employed, this can of course only more or less approximately be the case with the real, physical Proper Motions, the less so as many of these have a common direction (star-drift) for great numbers of neighbouring stars. But as Nyrén has used more than 5000 stars, the effect of this omission can scarcely be of much importance.

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* Also in French (Détermination du coefficient constant de la précession au moyen d'étoiles de faible éclat) in Mélanges mathématiques et astronomiques, Vol. IV., p. 590 et seq.

By giving all the means obtained for each hour equal weight Nyrén finds :—

$$\text{Schj.} - \text{Weisse} = -\,0^{s}{\cdot}1348 \pm 0^{s}{\cdot}0056.$$

As regards the Equinoxes, Nyrén refers Weisse's positions to Wolfers by adding the difference B. – W. = + $0^{s}{\cdot}014$ given by Argelander, while he has himself determined an Equinox for Schjellerup, whose epoch is too far removed from Wolfers. The Copenhagen Observations were founded on the positions of

Fundamental Stars between ± 15° occurring in the Nautical Almanac for the years 1861–63. These Nyrén compares with the Greenwich Seven-Year-Catalogue, with the several annual volumes of Greenwich Observations and with the Paris Observations of 1856–59. He finds the correction to Schjellerup equal to $+ 0^{s}{\cdot}0259$ and consequently the final result—

$$\text{Schj.} - \text{Weisse} = - 0^{s}{\cdot}0949.$$

Considering this as arising from error in the value of m adopted (the mean of Struve's and Bessel's values) and putting the interval between Schjellerup and Bessel equal to 36 years only (on account of the Dorpat Observations having been used as a control), Δm becomes $= - 0''{\cdot}0394$. By means of the data given by C. A. F. Peters for the planetary precession Nyrén finds the General Precession for 1800—

$$50''{\cdot}1882.$$

This result is very different from Bessel's and Struve's, and the difference is evidently far too great to be ascribed to want of accuracy in the observations employed. Both on account of the interest attached to the matter and also because it is of importance for the new determination which I shall communicate below, we shall now consider whether constant errors in either of the two catalogues are likely to be the cause of the unexpected result.

Nyrén has compared Schjellerup with about 100 stars observed in Washington in the years 1862–63. After allowing for the different standards he finds—

$$\text{Wash.} - \text{Schj.} = + 0^{s}{\cdot}033.$$

He has also by means of Bessel's constant and the Proper Motions found by comparing Bradley and Robinson derived from 184 Stars—

$$\text{Armagh} - \text{Schj.} = - 0^{s}{\cdot}011,$$

while the reductions of the two Fundamental Catalogues to Wolfers are about equal, so that the difference should have been zero. In order to try whether a different estimation of the transits of fainter and brighter stars can have anything to do with the matter, Nyrén lastly separates the corrected differences Schj. − W. within eight hours symmetrically situated into two classes, one containing stars not fainter than the eighth magnitude, the other comprising the remainder. In this way however no indication of any difference depending on magnitude was found.

Since the publication of Nyrén's paper, several circumstances bearing on this question have come to light.* First it should be mentioned that the good agreement between the Right Ascensions of Schjellerup and those of Copeland and Börgen (in the Göttingen Catalogue of Stars in the Zones − 0° and − 1°) makes it extremely improbable that there should be any constant errors in Schjellerup's results. The Göttingen zones were also founded on the Nautical Almanac, and from about 600 stars the difference was—

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* In a paper "On personal Errors" (Proc. R. Irish Acad. 1876, p. 518), I have described a fruitless effort made by me in 1874 to detect the existence of a personal error depending on magnitude in Bessel's and Schjellerup's original observations.

C. and B. — Schj. = — $0^s{\cdot}005$.

That Bessel's personal error was not sensibly different for bright and faint stars can also be proved. In Vol. VII. of the "Vierteljahrsschrift der astr. Gesellschaft" Argelander in a review of Santini's fifth catalogue (0° to − 3°) has communicated some interesting facts. This catalogue is deduced from observations made by Trettenero exactly at the same time at which Schjellerup observed his zones, and they are also founded on the Nautical Almanac. The following difference was found—

$$\text{Schj.} - \text{Si.}_5 = + 0^s{\cdot}096$$

and after correcting for difference of equinox—

$$\text{Weisse} - \text{Si.}_5 = + 0^s{\cdot}135.$$

Argelander was hereby induced to examine Bessel's positions in Weisse's first catalogue and Struve's "Positiones Mediæ" after excluding those double stars of which different components had been observed. 627 stars gave—

$$\text{Struve} - \text{Bessel} = - 0^s{\cdot}0431.$$

As Struve's "Correctiones Ultimae" are included in this and his reduction to Wolfers in consequence is Zero, while Bessel's reduction is − $0^s{\cdot}0124$, Argelander finds—

$$\text{Bessel's Fund. Cat.} - \text{Bessel's Zones} = - 0^s{\cdot}0307 \pm 0^s{\cdot}0064.$$

In order to meet the possible objection that observations of double stars are so different from those of ordinary stars, that a foreign element may be introduced thereby, Argelander has next left them out and found from 244 single stars or stars observed as single—

$$- 0^s{\cdot}0295 \pm 0^s{\cdot}0103.$$

From this it seems that it may safely be concluded that although Bessel's personal error in other respects and chiefly for its large amount, was very remarkable, it was not a function of the brightness of the stars observed.

This question of the different influence of personality on transits of faint and bright stars has of late attracted a good deal of attention. Argelander has examined a number of his observations of variable stars and found that he (with eye and ear) observed stars down to 9·1 mag. in the same way, but stars of 9·2 and still more those of 9·3 mag. slightly earlier. For stars below the 9·3 mag. the accidental errors became too prominent to allow a personal error to be distinguished.* A difference with the same sign appeared through the above-mentioned comparison of Trettenero with Schjellerup and Bessel.† As the change of personal error however in this case takes place *per saltum* between 7 and 7·5 mag., some other anomalous influence must here have been at work. The stars of 6 and 7 mag. in the Padua Catalogue are besides so few that their deviation from the very numerous fainter ones is possibly accidental. The thorough discussion by Dr. Gill of the meridian observations of the Mars comparison stars is of so recent a date that we need only mention the result here.* Within the probable limits of errors of observation faint and bright stars appear to be observed by eye and ear in the same way, while for *all* the chronograph observations the Right Ascensions of faint stars are too great. Although this result is derived from observations made at thirteen observatories and therefore entitled to great weight, it is as yet impossible to say whether it represents a general law, particularly as later observations made at Leyden by three observers† have not shown any difference between the two methods of observing, although the results otherwise as to quantity and sign agreed with Dr. Gill's.

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* Astr. Nachr. LXXIV., p. 268.

† Vierteljahrsschrift d. astr. Gesellschaft VII., p. 19.

Although the possibility of a sensibly different estimation of transits of faint and bright stars has been proved, such a different estimation appears at any rate in eye-and-ear observations to occur very rarely. It seems therefore, when we remember the above-mentioned researches by Nyrén and Argelander, and the comparison with the two Göttingen observers, that we shall be perfectly justified in concluding, that neither Bessel's nor Schjellerup's Right Ascensions are affected by errors depending on magnitude, and that the deviation of Nyrén's constant of Precession from Bessel's and Struve's must have some other cause.

This other cause must be sought in some error in the equinoxes, as it seems most improbable that the stars of the 8th and 9th magnitudes should really have a different precession from the brighter ones, or in other words, that the latter should have a common rotatory motion in the same direction in which the planets of our Solar System move. But it is not difficult to see, that Nyrén's Equinox of 1862, as derived through an improvement of the Nautical Almanac System by means of Greenwich Observations, can scarcely be free from constant error, as the N. A. positions depend almost entirely on observations made at Greenwich, so that Nyrén in reality has only corrected Greenwich by means of Greenwich. It has been shown by Newcomb‡ that in such a way systematic errors can only be sensibly diminished in the course of a great many years. But a later investigation by Nyrén, "Das Aequinoctium für 1865,"§ makes the case perfectly clear. It is there shown that the Greenwich observations of the Sun require a very considerable positive correction to make them agree with observations made at Pulkova, Edinburgh, Cambridge, Paris and Washington, and it must be due to the chapter of accidents, that the Paris results for the years 1856–59 agreed fairly well with those at Greenwich. One sees at once that this positive correction will reduce the great difference Schj. — Weisse to a small quantity and thus bring Nyrén's constant near to Bessel's and Struve's. Though Nyrén seems to have seen this himself, he has not examined the case further.

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* Memoirs R.A.S. XLVI., pp. 78–79.

† Vierteljahrsschift XIV., p. 413.

‡ Positions of Fundamental Stars (Washington Observations 1867, App. III.).

§ Mémoires de l'Académie de St. Pétersbourg, 1876.

The series of 29 absolute Right Ascensions to be found in Newcomb's paper, "On the Right Ascensions of the Equatoreal Fundamental Stars and the Corrections necessary to reduce the Right Ascensions of different Catalogues to a mean homogeneous System" must be considered free from systematic errors, as it rests on a very thorough discussion of all the Fundamental catalogues from Bradley down to our time. The corrections to various Star-Catalogues deduced by Newcomb contain both the corrections for equinox and the terms depending on R.A. and Declination. It seems therefore likely that star-positions to which these corrections have been applied, really become referred to "a mean homogeneous system" or at least to a system closely bordering on such.

The application of this system to the subject which occupies us here is made by Newcomb in a note, "Reduction of the Constants of Precession found by Bessel, Struve and Nyrén, to a common Equinox"* and is very simple.

We have already seen that Bessel corrected the "Fundamenta" by $-0''\cdot50$ and Piazzi by $+1''\cdot585$. As the reductions of these two catalogues to Newcomb are respectively $-0''\cdot15$ and $+1''\cdot50$, Bessel's Precession in 45 years has to be corrected by $-0''\cdot08-0''\cdot35=-0''\cdot43$ and in 100 years by $-0''\cdot96$.

The Dorpat R.A.'s are corrected by Newcomb by $-0''\cdot63$. As Struve has added $-0''\cdot31$ and $-0''\cdot42$ to the Fundamenta and Dorpat, his result for the 70 years' Precession gets the correction—

$$(+0''\cdot42-0''\cdot63)-(0''\cdot31-0''\cdot15)=-0''\cdot37$$

and the 100 years' Precession $-0''\cdot53$.

As regards Nyrén the correction to Bessel's first equinox is $-0^{s}\cdot037$, while Nyrén corrected Weisse by $-0^{s}\cdot014$, consequently the correction to Nyrén's earlier epoch is $-0^{s}\cdot023$. Schjellerup (as we have seen) was corrected by $+0^{s}\cdot026$, while Newcomb found (by 20 Nautical Almanac Stars between ± 15°) $+0^{s}\cdot073$, the second epoch therefore gets the correction $+0^{s}\cdot047$. The difference Schj. — Weisse $=-0^{s}\cdot095$ is thus reduced to $-0^{s}\cdot025$, and Nyrén's Precession in R.A. in 36 years has to be increased by $0^{s}\cdot070=1''\cdot05$. This annual general Precession therefore becomes increased by $\frac{1}{36}\cdot1''\cdot05\sec 23°\,28'=0''\cdot0318$.

Strictly speaking Newcomb's corrections to Bessel's and Struve's results should be applied to their values of the constant m, while the constants of Precession deduced by these two astronomers depend both on R.A. and Declination. However, as remarked by Newcomb, it will be very near the truth to consider the corrections given as applicable to their general Precessions. We have thus for 1800—

Bessel	50″·214
Struve	50·232
Struve-Peters . .	50·236
Nyrén	50·220

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* Vierteljahrsschrift XIII., pp. 107–110.

From the above it will be evident, that we are still far from possessing a perfectly satisfactory knowledge of the true value of the Constant of Precession. It needs therefore no further explanation why I shall endeavour in the following to effect a new determination of this quantity, but it will be necessary to set forth my reasons for founding this determination on Lalande's and Schjellerup's observations.

As already mentioned we require for such a determination two collections of star-positions sufficiently accurate and which can be referred to well established Equinoxes. In order to make errors in the latter as harmless as possible, it is necessary to select two epochs as far apart as possible. Just at present one would scarcely be justified in using Bessel's Bradley, as we may expect at any moment to see Auwers' new reduction published, in which the Equinox of 1755 is probably better determined. After Bradley there are however no suitable materials until we reach the last ten years of the 18th century, for (as remarked by Bessel), although the work was being continued in Greenwich, the spirit of Bradley was gone. I did not consider it desirable to use star catalogues from the earlier years of the present century, particularly as Bessel's Zones have already been used and I should only have had to choose between a few Fundamental Catalogues, which I wished especially to avoid using on account of their small number of stars not too different in brightness. There was however a very rich collection of observations of fixed stars left, which as to accuracy can almost compare with the very best modern ones and which in every respect are at least fully as suited to the purpose as Bessel's Zones. I allude to the observations made at the Observatory of the École Militaire and published in Lalande's "Histoire Céleste."*

To nearly all the 50,000 positions contained in this work corresponding ones can be found in modern catalogues. Of the latter none appeared to me in fulness, accuracy and uniform distribution of the stars over the heavens to compare with Schjellerup's Catalogue of Stars. This has about 3,300 stars in common with Lalande, among which are several hundred which by one or both of the observers were observed twice or oftener. Being fairly equally distributed with respect to the equator and in R.A. and deduced from observations as accurate as any we possess, this great number of star-positions might be expected to give a value of the Constant of Precession which as to weight might compare not unfavourably with those already published. The interval of time between the two epochs of observation is also favourable, being 66 years. Those of Lalande's observations which refer to the equatoreal belt, were made between November 7, 1793 and January 8, 1801, the greater part however in the years 1793–98, while Schjellerup's observations, as already stated, comprise the time from September 7, 1861, to December 6, 1863. It seemed advisable only to use the Right Ascensions, as the foundation of these appeared more secure than that of the declinations.

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* For historical accounts of this series of observations see the introductions to Baily's Catalogue and Gould's Reduction of d'Agelet's observations.

That Baily's Catalogue could not be used, I knew of course beforehand, as it is computed from the tables of Hansen and Nissen (published in 1825 by Schumacher) which depend too much on Lalande's own statements as to the auxiliary quantities necessary for the reduction. The excellence of the new tables of reduction computed by the late von Asten became very apparent through their extensive use, which the reduction of about 3600 Right Ascensions by their aid entailed. The constant comparison with Baily showed also how absolutely necessary the new reduction was, as deviations of $0^s\cdot4$ from the new R.A.'s were by no means scarce, while the new single results for a star observed several times almost always agreed much better *inter se* than was the case with Baily's. It is very evident that Baily's catalogue can now only be used as a mere Index to the stars observed by Lalande.

It is scarcely necessary to mention that I have carefully taken into account the extensive list of corrections to the Histoire Céleste, communicated by Argelander in Vol. VII. of the Bonn Observations. Wherever a star had been observed more than once, I nearly always took the mean of the single results, and only in a few cases where one of these differed considerably from the others and as resting on a single wire-transit might be supposed to have less weight, I have left it out. Otherwise I have not attempted to give the single stars different weight according to whether they were observed once or twice, over one or more wires &c. By far the greater part of the stars were only observed once, and, as remarked by Argelander, the circumstance that many transits are only given in round seconds without fractions, does not prove anything with regard to the accuracy, as the nought-decimal has been added or left out perfectly at random. Nor did it seem of any use to give a R.A. depending on two or three wires greater weight than one depending on a single wire, especially as the large number of stars made it more than probable that the final result would not be sensibly affected by such weighting.

Having computed the Mean Right Ascensions for 1800·0 (during which part of the work I was ably assisted by Mr. H. J. Carpenter) I next proceeded to reduce them all to Schjellerup's epoch 1865·0. As von Asten in preparing his tables had made use of Bessel's Constant of Precession, while Schjellerup had used Struve's, I thought it most correct to adopt the mean of the two constants, as both would enter into the final result whether I employed one or the other. By computing a large number of Precessions for 1800 I found that Baily had used Bessel's constant and that the Precessions in his catalogue were sufficiently accurate. I therefore made use of these except in cases where Argelander's corrections to the Histoire Céleste altered the positions of the stars sufficiently to affect the third decimal of Precession. In all such cases I recomputed the Precession for 1800 with Bessel's value. The means of these and Schjellerup's Precessions were next taken (whereby the secular variation was taken into account in a simple manner), and an eye was kept on the difference between these Precessions, so that errors in the third decimal can scarcely have escaped me in more than a very few cases. A constant control was also kept by means of the Precessions in the Catalogues belonging to the Berliner Akademische Sternkarten and in Weisse's Catalogue, and both the Precession for 1800 and that for 1865 were recomputed wherever any doubt appeared.

The Right Ascensions for 1865 thus produced were then compared with Schjellerup and the differences written out. The latter should strictly speaking have been multiplied by $\cos\delta$, but as all the stars are within ± 15° and $\cos 15° = 0{\cdot}966$, a reduction to the Equator could only be of slight importance and *e.g.* only diminish a difference of $0^{s}{\cdot}70$ (which occurs very rarely) to $0^{s}{\cdot}675$. As I have done away with all larger differences by adding or subtracting $1^{s}{\cdot}0$, wherever a comparison with other authorities proved the existence of such an error, or by correcting for Proper Motion, I considered the omission of this reduction justifiable. Besides, stars of higher declination than 10° only form a small portion of the entire number, and the slightly greater weight they get, is of no importance.

As already mentioned, I have in every case where I could detect Proper Motions by comparing with all the catalogues at my disposal,[*] applied a corresponding correction to the difference Schj. — Lal. This might perhaps have been omitted, trusting that the effect of the Proper Motions would be eliminated in the vast number of stars, but for the reasons alluded to above when speaking of Nyrén's paper, I preferred not to rely on this as far as the larger Proper Motions were concerned.[‡] It was impossible to check Lalande's zones in the way Nyrén controlled Bessel's, as there are no contemporary observations to use. Of more modern observations W. Struve's "Positiones Mediæ" are the nearest, but their mean epoch is about 30 years later than Lalande's. It is besides known from investigations by Auwers and von Asten[†] that Lalande's quadrant at the time when his nephew observed with it, was free from the deviations from the plane which disfigured it at the time of d'Agelet (and probably up to the rebuilding of the Observatory in 1788). From this circumstance together with von Asten's great care in constructing the tables, we may doubtless conclude that the newly reduced Right Ascensions for 1800 are not to any great extent affected by constant errors.

I give below a list of *all* the adopted corrections for Proper Motion or errors of whole seconds. That there are a number of errors of $\pm 1^{s}{\cdot}0$ in Lalande's observations can cause no surprise, as they are by no means very scarce in modern observations. As to the Proper Motions I must make the reservation that I have by no means examined every single star for this purpose, which would have entailed immense labour, but all differences (Schj. — Lal.) greater than $0^{s}{\cdot}7$ have been examined. In determining the amount of Proper Motion I have not reduced the positions taken from other catalogues to a common standard, and the quantities given in the list below must only be considered as provisional. I have just commenced observing these stars in order to determine definite values of the Proper Motions, but the influence of a small number of slight improvements in the adopted values could not possibly have any material effect on the result of the present investigation.

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[*] Unfortunately I could not consult the Annales de l'Observatoire de Paris.

[†] Vierteljahrsschrift II., p. 7, and von Asten's preface, p. III.

[‡] I have since found that the sum of all the Proper Motions employed is very nearly Zero, so that the final result would have been practically the same, even if I had not corrected for P.M.

In the following list the corrections to the Histoire Céleste given in Vol. VII. of the Bonn Observations are not included. I have also left out a few erroneous identifications in Schjellerup's Catalogue, as they are easily detected (with one or two exceptions). I also pass over the improvements in the Precessions.

The abbreviations are: L. Lalande, S. Schjellerup, B. Bessel (*i.e.* Weisse), Si Santini, Lam. Lamont, Arg. Argelander's list of 250 stars with PM, Σ Struve's Catalogus Novus and Gött. the Göttingen Catalogue. All positions are for 1865. The numbers are Baily's.

427 PM = $+ 0^{s}{\cdot}023$ evident.
455 B and S agree perfectly. L has to be corrected by $+ 1^{s}$ (he has only one wire).
755 Also observed by Rümker. PM = $+ 0^{s}{\cdot}03$ adopted.
865 PM = $+ 0^{s}{\cdot}031$.
970 B and S agree so well that L certainly requires $- 1^{s}$ (only one wire).
1005 L $+ 1^{s}$ (one wire). B and S agree.
1020 PM adopted = $+ 0^{s}{\cdot}014$.
1251 = Arg 10; PM = $+ 0^{s}{\cdot}0036$.
1299 PM = $+ 0^{s}{\cdot}0545$ (Mädler, A.N. 668).
1911 S — L = $+ 1^{s}{\cdot}18$. I have not found this star elsewhere, and as L has only one wire (no fraction) I have added $+ 1^{s}{\cdot}0$.
1992 PM = $+ 0^{s}{\cdot}012$. L has two observations.
2160 PM = $- 0^{s}{\cdot}012$. L has three observations.
2308 PM = $+ 0^{s}{\cdot}016$. (Mädler.)
2335 Corr. = $+ 1^{s}$ to L; one wire and no fraction.
2587 Left out. No plausible correction to L.
2635 = Arg 14, PM = $+ 0^{s}{\cdot}0227$.
2957 B and S agree well; L $+ 1^{s}$ (one wire).
2999 S — L = $+ 1^{s}{\cdot}31$. As B, Si, S and Rümker (Neue Folge) agree well, there cannot possibly be a PM greater than $+ 0^{s}{\cdot}01$ at most. I have had to add 1^{s} to L, though he has three wires.
3546 L to be corrected by $- 10^{s}$.
3620 PM = $- 0^{s}{\cdot}029$.
4050 A strange discordance here: L $33^{s}{\cdot}14$, B $35^{s}{\cdot}49$, S $34^{s}{\cdot}38$. I have given L $+ 1^{s}$ (one wire).
4269 = Arg 26, PM = $+ 0^{s}{\cdot}0233$.
4848 PM = $+ 0^{s}{\cdot}017$ adopted.
5074 L $+ 1^{s}$ (one wire).
5075 L $+ 1^{s}$, B agrees well with S.
5290 PM = $+ 0^{s}{\cdot}015$ adopted. No PM in Decl.
6117 PM = $+ 0^{s}{\cdot}023$ according to B and Gött.
6136 PM = $+ 0^{s}{\cdot}018$ (Mädler).
6167 S has to be corrected by $- 1^{s}$, as B and L agree.
6293 L $+ 1^{s}$, B and S agree exactly (L one wire and no fraction).
6320 PM = $- 0^{s}{\cdot}019$ adopted.
6778 = Arg 42, PM = $+ 0^{s}{\cdot}0037$ used, but as L has but one wire and B agrees fairly well with S, the PM seems doubtful.
7623 PM = $+ 0^{s}{\cdot}020$ adopted.

8215 From L, Rümker and S the PM is = $+ 0^s{\cdot}018$. I have not used B, as he has interchanged the two neighbouring stars. L agrees with DM in putting them *sp*, *nf*. S has only observed one.

9051 S — L = $+ 1^s{\cdot}44$. As B is only $0^s{\cdot}3$ smaller than S, I have given L + 1^s. Probably PM in Decl.

9090 PM = $+ 0^s{\cdot}015$ adopted.

9253, 9784 } L must have + 1^s (one wire), B and S agree.

9960 PM = $+ 0^s{\cdot}048$ adopted.

11080 PM = $+ 0^s{\cdot}040$ adopted.

11424 PM = + 0·015 agrees well with B.

12018 PM = $- 0^s{\cdot}021$.

12039 L has to be corrected by + 1^s (otherwise S — L = $+ 1^s{\cdot}13$). B agrees well with S.

12283 PM = $+ 0^s{\cdot}020$.

12393 Not in B. L has two accordant wires, and as the Declinations also show trace of PM, I have adopted PM in RA = $+ 0^s{\cdot}016$.

13872 PM = $+ 0^s{\cdot}015$.

14938 L + 1^s (one wire and no fraction); B and S agree.

16363 S — L = $+ 1^s{\cdot}30$. B agrees closely with S. L + 1^s adopted (one wire).

16953 S — L = $+ 1^s{\cdot}03$. As B is $0^s{\cdot}27$ larger than S and L has only one wire and no fraction, I have given L + 1^s.

18060 According to B and S, L must certainly be corrected by + 1^s (one wire).

18547 S — L = $- 1^s{\cdot}20$. S — B = $+ 0^s{\cdot}03$; L must therefore be corrected by — 1^s (one wire).

18713 L to be corrected by + 1^s (one wire) according to B and S.

19749 A distant companion to Regulus. In the "Positiones Mediæ," p. CCXIV., Struve proves, by comparing his own observations with one by C. Mayer in 1778, that the two stars are relatively immovable. In March, 1882, I myself found $\Delta\alpha = 9^s{\cdot}55$ (L has $9^s{\cdot}54$ in 1796). PM of α Leonis = $- 0^s{\cdot}018$ adopted. This star was also observed by d'Agelet in 1783.

19896 L, B, Lam and S all agree well by adopting PM = $+ 0^s{\cdot}014$.

21000 L has two good wires. Not in B but in Lam. PM = $+ 0^s{\cdot}014$.

21358 L, B, Lam and S give PM = $- 0^s{\cdot}017$.

21373 L has two accordant wires. Not in B but in Lam. PM = $+ 0^s{\cdot}015$. The Declinations also indicate PM.

21455 PM = $- 0^s{\cdot}021$ evident from B and Lam.

21468 L has two wires. By adding 1^s to the first one, they agree much better *inter se* as also with the RA of d'Agelet, B and S.

21584 = Σ 1529. L has observed both components, Σ and Gött the *f* one, S the *p* one. From this, combined with the Mens. Microm., I find PM = $- 0^s{\cdot}019$.

21640 L — 1^s according to Armagh and S.

21899 = Σ 1547. PM = $- 0^s{\cdot}020$ (Pos. Med., p. 28).

21935 S to be increased by 1^s, as L and B agree.

22080 PM adopted = $- 0^s{\cdot}027$.

22312 L has two good observations. Occurs also in B and in Bond's Zone (p. 192). PM = $- 0^s{\cdot}016$.

22341 β Virginis. PM = $+ 0^s{\cdot}030$ (Mädler).

22585 = Arg 113. PM = $+ 0^s{\cdot}007$.

22662 = Arg 116. PM = $- 0^s{\cdot}012$.

22701 = Arg 118. PM = $- 0^s{\cdot}034$.

22954 = Arg 120. PM = $+ 0^s{\cdot}007$.

23297 Though L has two accordant wires B and S agree in demanding a correction of + 1^s.

23369 S + 1^s agrees well with L and B.

23552 = Σ 1661, PM = $- 0^s{\cdot}024$.

23866 PM = $- 0^s{\cdot}033$ adopted.

23890 PM = + 0^{s}·020.
23967 L must be increased by 1^{s}, as B agrees perfectly with S and the second Armagh Catalogue.
24005 Both L and B must have — 1^{s} according to Lam, Si, and S.
24168 = Arg. 127. PM = — 0^{s}·057.
24504 PM = — 0^{s}·036. L has only one wire, but B confirms the PM.
24700 PM = — 0^{s}·015 agrees with B.
25146 S — L = — 0^{s}·83. I have not supposed any PM, although Rümker, Lam, S and Gött do not exclude the possibility of a very slight PM.
25885 PM = — 0^{s}·025.
26069 Left out. Error of about 15^{s} in L.
27093 S + 1^{s} agrees with B and S.
28634 In his notes S makes PM = — 0^{s}·025. As however L, B (2 obs.) and Gött agree closely, S must be corrected by + 1^{s} and there is no PM.
28785 S — L = — 0^{s}·89. S confirmed by Gött, but Lam is nearer to L. Not in B. No correction applied.
29070 S — L = — 1^{s}·11, but B is even 0^{s}·19 greater than L, while Si and the second Armagh Cat. agree with S. From these and Rümker and Lam, PM = — 0^{s}·020 ±, which was adopted.
29492 L to be corrected by — 6^{s}·0.
29581 = Σ 2021. S and Σ have not observed the same component, L must have + 1^{s}.
29617 L + 1^{s}, as B and S agree.
30111 = *h* Herculis. PM = — 0^{s}·015.
30191 PM = — 0^{s}·031.
30694 = Arg 181. PM = — 0^{s}·047.
31055 PM = — 0^{s}·060.
31177 Two observations by B and two by S demand L increased by 1^{s}.
31741 The *third* wire (not the second one) to be altered by 1^{s}. Thus the RA for 1800 becomes 17^{h} 16^{m} 40^{s}·29. Compare what Argelander says about Michel Lalande's corrections (Bonner Beob. VII., p. 169).
31804 S — 2^{s} (as mentioned in the notes).
32378 = Σ 2202. No PM; Michel Lalande's correction not to be used.
$\left.\begin{array}{l}32422\\32717\end{array}\right\}$ L + 1^{s} (one wire, no fraction), as B agrees closely with S.
32715 *Not* observed by S, whose observation belongs to DM + 0°, 3804 (0^{s}·7 *f* and 2′ *n*).
33169 = Σ 2272. PM = + 0^{s}·0144 (Cat. Åboensis).
33701 L + 1^{s} (one wire) agrees closely with B and S.
34118 PM = — 0^{s}·018.
34947 L + 1^{s}, as B and S agree (otherwise S — L = + 1^{s}·48).
35290 Left out, no plausible correction found.
36908 Si agrees with S, these and B give PM = — 0^{s}·015.
37044 = μ Aquilæ, PM = + 0^{s}·0155 (Mädler).
37113 PM = + 0^{s}·015 (Pos. Med., p. 315).
37296 S to be corrected by + 1^{s}, as L, B and Lam agree.
38004 L must be corrected by + 1^{s} to agree fairly with B and S.
38100 PM = — 0^{s}·024 adopted.
38139 PM = — 0^{s}·024.
38318 No PM (as supposed by S), as L, Si, Lam, and Yarnall agree closely. S to be corrected by — 1^{s}.
38534 L + 1^{s}, as B and S agree.
38651 S — 1^{s} agrees well with B and L.
$\left.\begin{array}{l}38610\\38625\end{array}\right\}$ The RA's of these two neighbouring stars are both 1^{s} too small. B and S agree.
39773 L to be corrected by + 1^{s} (one wire) to agree with B, Lam and S.

39832 S + 1^s agrees closely with L, B and Lam.

39955 PM = − $0^s\cdot011$ (B, Lam, Si).

40080 B and S agree well, therefore L + 1^s (one wire).

40209 = Arg 214, PM = + $0^s\cdot0094$.

40217 S − L = + $1^s\cdot35$, not found elsewhere. As L has two good wires and the declinations differ 5″, I have assumed PM = + $0^s\cdot020$.

40307 Argelander corrects the transit 24^s to $20^s\cdot4$. I prefer to assume a misprint, $21^s\cdot0$ for 24^s, as only thus is it possible to make L agree with B and S.

40394 L + 1^s (one wire) agrees well with B, Gött and S.

40405 L has two accordant wires ; S − 1^s agrees better with L and Gött.

40636 L − 1^s (one wire) agrees with B and S.

40796 Not in B but in Lam. PM = − $0^s\cdot017$ (L has two good wires).

40911 L − 1^s agrees well with B and S.

41126 L (2 obs.), B and Lam indicate PM + $0^s\cdot016$.

41348 = Arg 220. PM = + $0^s\cdot030$.

41655 S corrected by − 1^s, as L agrees well with Gött.

41928 Left out. The Globular Cluster M. 2.

41936 L + 1^s (one wire, no fraction) agrees closely with Lam and S.

42443 = Σ 2825, PM = − $0^s\cdot015$. The duplicity hardly remarked by either observer.

42602 L + $2^s\cdot0$ (Argelander says : "+ 2^s oder rohe Beobachtung").

42846-7 differ by $1^s\cdot07$. The second one is evidently the correct one (two wires).

43115 S − L = + $1^s\cdot51$ (after applying Argelander's correction). L must further be corrected by + 1^s, though B − S = − $0^s\cdot49$ does not exclude the possibility of PM.

43382 L − 1^s (one wire, no fraction) agrees closely with B, S and Gött. No PM possible.

43730 Lam, Gött and S agree, L must be corrected by + 1^s (one wire, no fraction).

43906 L + 1^s (one wire) agrees closely with B, S and Gött.

44276 = Σ 2928 = Arg 231 : PM = + $0^s\cdot019$.

44304 = Arg 232. PM = + $0^s\cdot012$.

44626 L to be corrected by + 1^s, as 44628 agrees with B and S.

44710 PM = + $0^s\cdot025$.

45059 L − 1^s (Bonner Beob. VII., p. 139).

45113 L + 1^s (one wire), as B precludes the possibility of PM.

45191 PM = − $0^s\cdot015$ (B and Gött).

45630 = Σ 2998. PM = − $0^s\cdot022$ (Mädler).

45638 = Arg 243. PM = + $0^s\cdot0365$.

45656 PM = + $0^s\cdot020$ agrees well with B.

45680 PM = + $0^s\cdot020$ (B and Yarnall).

45758 not in B. Lam and Gött agree with S in demanding a correction of L by + 1^s (one wire, no fraction).

46615 L − 2^s agrees well with B and S (L one wire).

46650 = Arg 247. PM = + $0^s\cdot064$.

46916 As Rümker, Si, S and B agree well, L (one wire, no fraction) must be corrected by 1^s.

46952 S − L = + $1^s\cdot31$. B, Si and S do not preclude a very slight PM, but it seems far more likely that L is 1^s too small.

47226 PM = − $0^s\cdot015$ agrees well with B.

When the above corrections had been applied there of course still remained a number of differences S. − L. which were larger than might have been expected from the general good agreement between the two observers and the accuracy of the observations. No doubt some of these large differences arise from unnoticed

Proper Motions, which can scarcely have any sensible influence on the final result, but others must arise from simple errors of observation which we may be justified in expecting will be eliminated among the large number of stars employed.

In order to give a clear idea of the differences between Schjellerup and Lalande I have within every hour of R.A. collected them in groups of ten and taken the mean. These means are given in the following tables, and at the end of each column the mean for the whole hour is given. It should be remembered, that the last group of each hour generally does *not* contain ten differences :—

0^{h}.	1^{h}.	2^{h}.	3^{h}.	4^{h}.	5^{h}.	6^{h}.	7^{h}.
133 Stars.	72 Stars.	80 Stars.	142 Stars.	121 Stars.	108 Stars.	128 Stars.	112 Stars.
+ 0^{s}·024	+ 0^{s}·097	+ 0^{s}·261	+ 0^{s}·270	+ 0^{s}·058	+ 0^{s}·098	+ 0^{s}·034	+ 0^{s}·177
+ 0·125	+ 0·124	+ 0·141	+ 0·203	+ 0·157	− 0·034	+ 0·112	+ 0·058
− 0·074	+ 0·058	− 0·086	+ 0·088	+ 0·158	+ 0·156	+ 0·038	+ 0·175
+ 0·081	+ 0·114	+ 0·108	+ 0·077	+ 0·147	+ 0·069	+ 0·178	− 0·131
+ 0·103	+ 0·109	+ 0·102	+ 0·195	− 0·100	− 0·022	− 0·005	+ 0·090
+ 0·088	− 0·046	− 0·092	+ 0·268	+ 0·193	+ 0·035	+ 0·128	+ 0·037
+ 0·074	+ 0·060	+ 0·079	+ 0·184	+ 0·220	− 0·047	+ 0·102	− 0·073
+ 0·048		+ 0·211	+ 0·198	+ 0·048	− 0·030	+ 0·199	+ 0·036
+ 0·174			− 0·011	+ 0·116	+ 0·252	− 0·056	+ 0·137
+ 0·108			+ 0·154	+ 0·119	− 0·098	+ 0·070	+ 0·220
+ 0·218			+ 0·130	+ 0·074	+ 0·200	+ 0·094	+ 0·131
− 0·024			+ 0·131	+ 0·083		− 0·095	
+ 0·291			+ 0·011			− 0·059	
			+ 0·111				
+ 0^{s}·099	+ 0^{s}·073	+ 0^{s}·092	+ 0^{s}·143	+ 0^{s}·106	+ 0^{s}·051	+ 0^{s}·059	+ 0^{s}·077

8^{h}.	9^{h}.	10^{h}.	11^{h}.	12^{h}.	13^{h}.	14^{h}.	15^{h}.
125 Stars.	135 Stars.	117 Stars.	132 Stars.	156 Stars.	115 Stars.	107 Stars.	137 Stars.
+ 0^{s}·028	− 0^{s}·037	+ 0^{s}·150	− 0^{s}·042	− 0^{s}·113	− 0^{s}·125	− 0^{s}·033	+ 0^{s}·038
− 0·100	− 0·014	+ 0·093	− 0·030	− 0·190	− 0·159	− 0·029	+ 0·061
+ 0·074	+ 0·096	+ 0·101	− 0·093	− 0·110	+ 0·006	− 0·071	+ 0·066
+ 0·084	+ 0·113	+ 0·106	− 0·150	+ 0·038	− 0·065	+ 0·119	− 0·093
− 0·016	− 0·005	− 0·152	+ 0·007	− 0·111	− 0·034	+ 0·187	− 0·090
+ 0·050	− 0·105	+ 0·101	+ 0·021	− 0·001	− 0·089	− 0·062	+ 0·111
+ 0·001	− 0·047	+ 0·157	− 0·074	− 0·180	− 0·074	− 0·064	− 0·083
+ 0·075	+ 0·019	+ 0·031	− 0·018	− 0·043	− 0·041	+ 0·049	− 0·158
+ 0·075	+ 0·084	+ 0·024	+ 0·010	− 0·074	+ 0·004	+ 0·063	+ 0·045
+ 0·095	+ 0·028	− 0·029	− 0·031	− 0·024	− 0·002	+ 0·161	− 0·092
+ 0·122	+ 0·107	− 0·217	+ 0·068	+ 0·033	+ 0·243	+ 0·027	− 0·151
− 0·050	− 0·050	+ 0·049	+ 0·046	− 0·106	+ 0·150		+ 0·056
+ 0·182	− 0·010		+ 0·081	− 0·076			+ 0·084
	+ 0·302			+ 0·027			+ 0·121
				+ 0·059			
				+ 0·023			
+ 0^{s}·042	+ 0^{s}·024	+ 0^{s}·034	− 0^{s}·014	− 0^{s}·055	− 0^{s}·023	+ 0^{s}·032	− 0^{s}·009

16^{h}.	17^{h}.	18^{h}.	19^{h}.	20^{h}.	21^{h}.	22^{h}.	23^{h}.
149 Stars.	145 Stars.	185 Stars.	176 Stars.	207 Stars.	174 Stars.	140 Stars.	169 Stars.
+ 0^{s}·152	+ 0^{s}·256	+ 0^{s}·132	+ 0^{s}·010	+ 0^{s}·153	+ 0^{s}·274	+ 0^{s}·153	+ 0^{s}·051
− 0·156	− 0·072	− 0·060	− 0·068	− 0·035	+ 0·132	− 0·180	+ 0·130
+ 0·092	+ 0·057	− 0·029	+ 0·032	+ 0·031	+ 0·266	− 0·084	+ 0·014
+ 0·002	+ 0·092	+ 0·097	+ 0·124	+ 0·117	− 0·079	− 0·034	+ 0·098
− 0·002	− 0·065	+ 0·061	+ 0·005	+ 0·087	+ 0·309	+ 0·248	+ 0·057
− 0·010	+ 0·100	+ 0·005	+ 0·087	+ 0·035	+ 0·064	+ 0·227	− 0·095
+ 0·103	+ 0·071	+ 0·075	+ 0·038	− 0·100	− 0·105	+ 0·090	+ 0·078
− 0·108	+ 0·108	+ 0·139	+ 0·030	− 0·037	− 0·013	+ 0·050	+ 0·158
− 0·007	+ 0·029	+ 0·032	+ 0·115	+ 0·063	− 0·064	+ 0·250	− 0·273
+ 0·108	+ 0·182	− 0·019	− 0·037	+ 0·028	− 0·038	+ 0·071	− 0·071
+ 0·054	− 0·001	+ 0·093	− 0·055	− 0·063	+ 0·185	+ 0·024	− 0·218
+ 0·102	+ 0·135	+ 0·096	− 0·006	+ 0·265	− 0·014	+ 0·072	− 0·187
+ 0·118	+ 0·053	+ 0·115	+ 0·109	+ 0·217	− 0·012	+ 0·100	+ 0·178
+ 0·267	+ 0·099	+ 0·095	+ 0·021	− 0·085	+ 0·070	− 0·123	+ 0·056
+ 0·124	+ 0·124	+ 0·035	+ 0·191	+ 0·006	− 0·036		+ 0·030
		− 0·090	− 0·064	− 0·006	− 0·035		+ 0·053
		− 0·062	+ 0·130	+ 0·106	− 0·021		+ 0·036
		+ 0·050	− 0·052	− 0·036			
		+ 0·010		+ 0·039			
				+ 0·176			
				− 0·091			
+ 0^{s}·055	+ 0^{s}·076	+ 0^{s}·042	+ 0^{s}·036	+ 0^{s}·043	+ 0^{s}·050	+ 0^{s}·062	+ 0^{s}·006

As will be seen by a glance at the above tables, the stars are not quite as uniformly distributed over the heavens as might have been wished, there being a minimum at 1^{h}–2^{h} and a maximum at 19^{h}–20^{h}. As however these hours must be considered as exceptions, and as the differences within the hours 1^{h} and 2^{h} agree well *inter se* and with the majority of the rest, there is no reason to give them different weight. Besides, the stars within each hour are fairly uniformly distributed, and by giving all the 24 single results equal weight we fix 24 equidistant points around the equator.

It is not without interest to notice the prevailing minus-sign in the hours 11^{h}–13^{h}, while the plus-sign reaches a maximum at 3^{h}, nearly opposite. These signs agree with what we should expect from the motion of the solar system, but as the latter is directed towards R.A. 261° = 17^{h} 24^{m}, the maxima ought to be at 11^{h}·4 and 23^{h}·4, and the deviation of the plus maximum must be ascribed to special, independent Proper Motions.

The mean result of all the twenty-four hours is—

$$\text{Schjellerup} - \text{Lalande} = +\,0^{s}\cdot0459 \pm 0^{s}\cdot0061.$$

From this result we have now to deduct the errors in the equinoxes on which the Right Ascensions are founded.

Von Asten's tables, used by me for deducing Lalande's Right Ascensions, rest on positions of Standard Stars taken from Piazzi's Catalogue, and the correction applicable to the latter should therefore be added to Lalande's R.A. Schjellerup's

R.A.'s could no doubt without difficulty have been referred to a good equinox, as there is plenty of good material from the years 1855–70, but another question is whether this could have been combined with one from Piazzi's time, as Wolfers' Equinox of 1830 is too far distant. Besides, I have not been able to find out anything about Piazzi's own determination of equinox, and we should in any case be obliged to compare Piazzi's Right Ascensions with those of some other catalogue.

I therefore thought it best to found my determination of the Constant of Precession on Newcomb's "Equatoreal Fundamental Stars," which besides will make it comparable with the above-mentioned reduction of the older values "to a mean homogeneous system."

Newcomb's correction to Piazzi is $+ 0^{s}{\cdot}10$ and his correction to the Nautical Almanac stars used by Schjellerup is $+ 0^{s}{\cdot}073$,* consequently to the difference S. – L. we have to add $+ 0^{s}{\cdot}073 - 0^{s}{\cdot}100 = - 0^{s}{\cdot}027$ and thus find—

$$\text{Schjellerup} - \text{Lalande} = + 0^{s}{\cdot}019 = + 0''{\cdot}285.$$

From this difference we now deduce a correction for the value of m adopted by dividing it by the number of years between the two epochs of observations. By writing out the dates of those of Lalande's Zones which have been used in the present investigation, the mean epoch will be found to be very nearly 1796·7, while Schjellerup's is 1862·7. The interval is thus 66·0 years and—

$$\Delta m = + 0''{\cdot}0043.$$

The value of m used was—

$$\tfrac{1}{2}(46''{\cdot}0808 + 46''{\cdot}0437) = 46''{\cdot}06225.$$

Consequently the new value for 1829·7 is—

$$m = 46''{\cdot}0666.$$

In order to determine the Constant of Precession for the epoch 1800 from this, I borrow the following data for 1800 from C. A. F. Peters' paper "Numerus Constans Nutationis"—

$$\frac{d\lambda}{dt} = 0''{\cdot}15119 - 0''{\cdot}00048372\,t$$

$$\epsilon_0 = 23^\circ\ 27'\ 54''{\cdot}2$$

$$\frac{d\psi}{dt} - \frac{d\psi'}{dt} = 0''{\cdot}1387 - 0''{\cdot}0004436\,t$$

We first get for 1829·7—

$$m + \frac{d\lambda}{dt} = 46''{\cdot}2034$$

$$\frac{d\psi}{dt} = 50{\cdot}3688$$

$$n = 20{\cdot}0563$$

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* I have myself computed this correction and found $+ 0^{s}{\cdot}0734$ from 20 N.A. stars between ± 15° in good accordance *inter se.*

and accordingly for 1800 + *t*—

$$\begin{aligned}\text{Lunisolar Precession} &= 50''\cdot3752 - 0\cdot0002168\,t\\ \text{General Precession} &= 50\cdot2365 + 0\cdot0002268\,t\\ m &= 46\cdot0581 + 0\cdot0002849\,t\\ n &= 20\cdot0589 - 0\cdot0000862\,t\end{aligned}$$

The new value of the Constant of Precession—

$$50''\cdot2365$$

for 1800 is only 0″·0046 smaller than the value by Struve and Peters, which is now used in all the large Ephemerides,* and considering the somewhat uncertain foundation of the latter, the confirmation given it by the present investigation can scarcely be without some value. The new constant becomes almost identical with Struve's if to the latter we apply Newcomb's correction of − 0″·053, and as it is founded on more than three thousand stars, each class of magnitude being fairly uniformly distributed over the equatoreal belt of the heavens, and observed most carefully with an interval of 66 years, the new constant must be entitled to some weight. As Struve-Peters' value is the largest of all, the new one is more than likely to be somewhat nearer to the truth, but whether the third decimal ought to be diminished by a few more units, bringing the constant still nearer to Bessel's, must be left for the future to decide.

That Zone-observations will become of great importance for the further development of Sidereal Astronomy, is certain, and the value of the Histoire Céleste in particular will increase as each year distances us more from its epoch. Whenever the Zone work of the Astronomische Gesellschaft shall have been finished, it will be possible to use *all* Lalande's observations, and such a mass of stars must give very reliable results. But it would be a very useful undertaking for some astronomer having a Transit Circle of, say 8 inches aperture, to observe very faint stars (of 10th and 11th magnitude), and especially their Right Ascensions, as an examination and future repetition of such observations might furnish absolutely uniformly distributed materials for a determination of the important constant. Also in other respects would it be worth while to observe such very faint stars, as systematic movements within the System of the Milky Way would more easily be revealed in this way, if only change of personal error could be guarded against. That the stars down to the ninth magnitude inclusive are not distinguished by any motion which could show itself through a Constant of Precession different from that of the brighter stars, has been shown by the present investigation.

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* In the American Ephemeris (I believe) from the beginning, in the Nautical Almanac since 1857, in the Connaissance des Temps since 1877, and in the Berliner Jahrbuch from 1883.

RECENT LITERATURE.

"*Observations of the Transit of Venus, December* 8–9, 1874, *made and reduced under the direction of the Commission created by Congress. Edited by Simon Newcomb. Part I., General Discussion of Results.*" Washington, 1880, 157 pp. 4to and two plates. This volume appears to have been issued about a year ago but has only reached us quite lately. We may refer the reader to Vol. I., p. 167, of this journal for a list of its contents. The discussion of the photographic operations occupies 93 pages, it enters very fully on the method of investigating the relation between the apparent positions of Venus and the Sun on the sphere and the positions of their images on the photographic plate, and all the necessary details as to the constants of the photoheliographs employed are also given. The reduction of the photographs follows next in tabular form, and the comparisons of observed and tabular positions of Venus on the face of the Sun are presented in a form which admits of their ready translation into equations of condition. The fourth chapter contains first the results of observations on the model transit and secondly a summary of the times of contact and distances of centres derived from all the observations at the several stations. The latter are to follow in Part II., but as the reductions are still unfinished it was thought desirable to publish the contact observations without further delay in an appendix to Part I. We find here a complete copy of the original notes of the observers, no discussion being attempted for the present.

"*The Multiple Star* Σ 748. *By E. S. Holden.*" Washington, 1881 (22 pp. 4to, Wash. Obs., 1877, App. II.) This paper contains a discussion of the measures of the six stars in the trapezium of Orion made by Prof. A Hall in 1877–78, compared with those of previous observers. For the four principal stars a table is given of all the differences ΔP and Δd of each observer's results compared with W. Struve's, and although the residuals are difficult to reconcile with any particular scheme of geometrical change, they seem too large and agree too well with each other to attribute them exclusively to accidental errors. The observations of the fifth and sixth star also seem to indicate a physical connexion with the whole system.

"*Publicationen der K. Universitäts-Sternwarte zu Leipzig, Heft I.*," Leipzig, 1882 (127 pp. 4to and 2 plates). This part contains first Dr. H. C. Vogel's micrometric observations of Nebulæ between + 9° 30′ and + 15° 30′ (issued separately in 1876) and secondly observations of double stars by Engelmann, Vogel, Börgen and Koch.

"*Astronomische Mittheilungen von Dr. Rudolf Wolf, No.* 55." In this number are given the usual computations of "relative numbers" of Sun-spot frequency for 1881, a continuation of the summary of Sun-spot literature, and a very fine photolithographic plate reproducing twelve drawings of Jupiter selected from a series of about 250 drawings made by A. Weiler since 1877.

DUN ECHT CIRCULARS, Nos. 52 AND 53.

The spectrum of the nucleus of Comet Wells deserves the closest attention, as it shows a sharp bright line coincident with D, as well as strong traces of other bright lines, resembling in appearance those seen in the spectra of γ Cassiopeiæ and allied stars.

1882, May 29.

A Science Observer telegram announces that Mr. S. C. Chandler, Jr., finds the period of the variable star D. M. + 1° 3408, which forms the subject of Dun Echt Circular, No. 41, to be only 0·83 days. A minimum occurs on July 1, $12^h\ 57^m{\cdot}6$ G. M. T.

1882, June 29.

NOTES.

In Thomson and Tait's Elements of Natural Philosophy I., pp. 14–15, it is shown that the hodograph for the motion of a planet or comet is always a circle, whatever be the form and dimensions of the orbit. In other words, whether an observer be placed on a planet or a comet, the apparent path described by a fixed star in consequence of Aberration, will always be a circle, and only the position of the latter with respect to the mean place of the star (the hodographic origin) will depend on the excentricity of the orbit. It does not appear to be generally known that Bradley has had a remarkably clear idea of this phenomenon, as may be seen from his "Miscellaneous Works," edited by the late Prof. Rigaud, p. 287 and seq. Bradley here demonstrates the above theorem in a very simple manner, showing that the star will always be in a line through the mean place parallel to the tangent to the Earth's orbit, and as the line from the middle place to the apparent place is always perpendicular and proportional to the perpendicular on the tangent from the second focus of the Earth's orbit, the ends of these two lines must describe similar figures, *i.e.*, circles.

The annual visitation of the Dunsink Observatory by the Board of Trinity College, Dublin, took place on June 27. We extract the following from Dr. Ball's report. The MS. of Part V. of the Dunsink Observations is ready for the printer, it contains (1) Reconnoitring observations of 368 stars with a view of finding whether they have a large parallax, (2) on the parallax of Groombridge 1618 (*Copernicus* I., p. 16), (3) further researches on the parallax of 61 Cygni, including a discussion of Dr. Brünnow's observations, (4) on the parallax of P. III., 242, and (5) on the parallax of 6 Cygni B = Σ 2486. All these papers are by Dr. Ball. The results for 6 Cygni (which will be published in the next number of *Copernicus*) are of great interest, the parallax resulting from observations of the distance of a star near being $+ 0''{\cdot}504 \pm 0''{\cdot}060$ and from the position-angles $+ 0''{\cdot}383 \pm 0''{\cdot}130$. The parallax of P. III., 242, is insensible. For Gr. 1618 the position-angles have now been discussed and give $\pi = + 0''{\cdot}314 \pm 0''{\cdot}031$ (the result from the distances was $+ 0''{\cdot}334 \pm 0''{\cdot}036$). The equatoreal is now being employed to observe *Victoria* and *Sappho* for the solar parallax in accordance with Mr. Gill's proposal. The Meridian circle has been used by Dr. Dreyer to observe a number of stars between − 2° and − 23° Decl., nearly the whole working list having been communicated by Professor Schönfeld. Dr. Dreyer will leave Dunsink about Sept. 1, having been appointed to succeed the late Dr. Robinson at Armagh.

Professor Ormond Stone has recently been appointed to the Leander McCormick Observatory, of Charlotteville, Virginia. The 26-inch object-glass, which has been complete in the hands of the Clarks for the last eight years, will now undoubtedly be mounted and at work within a year or two.

The Secretary of the U.S. Navy has appointed Vice-Admiral Stephen C. Rowan to be Superintendent of the U.S. Naval Observatory, in place of Rear Admiral John Rodgers, deceased on May 5.

Professor Johann Carl Friedrich Zöllner died at Leipzig on April 25. He was born at Berlin on the 8th November, 1834, and held from 1872 the chair of Physical Astronomy at the Leipzig University. Besides a great number of papers (in the Leipzig *Berichte*) on subjects connected with Spectrum Analysis and the constitution of the Sun, he was the Author of several larger works, of which the most important was published in 1865, under the name of "Photometrische Untersuchungen." The photometer constructed by Zöllner compares the light of a star to that of an artificial star, produced by a paraffin lamp and varied as to intensity by turning two Nicols, while the colour is changed by turning a chrystal plate.

We are sorry to have to announce the death of Mr. Charles Edward Burton, which took place very suddenly in Castleknock Church, county Dublin, on July 9. Mr. Burton was born at Barnton, Cheshire, on Sept. 16, 1846, and occupied himself, from an early age, with the study of Astronomy, although his very delicate health put many difficulties in the way of such pursuits. From 1868 to 1869 he worked at Lord Rosse's Observatory, and from 1876 to 1878 he was Assistant at Dunsink Observatory, but he was obliged to resign both these posts owing to ill health. He took part in the Venus expedition to Rodriguez and after his return he remained in Greenwich for nearly a year to measure the British photographs of the Transit. Mr. Burton was a successful photographer himself and had much practice in the manufacturing of silvered glass specula, with which he observed Jupiter and Mars as often as possible, at his father's parsonage at Loughlinstown, county Dublin. In a few weeks Mr. Burton was to have started for South Africa to observe the coming transit.

The Observatory founded by Mr. Warner, at Rochester, N.Y., and of which Mr. Lewis Swift is the Director, was ready for work in May last. The principal instrument is a 16-inch refractor by Clark. Mr Swift has hitherto worked under the greatest difficulties, his observing place having been on the top of a very large cider mill half a mile from his residence, in the open air, without measuring apparatus, maps, clocks, or any similar implements.

We learn from Professor C. H. F. Peters that ten of his large charts of telescopic stars are now in the hands of a photolithographer at Washington.

The two Belgian Venus expeditions have started, one under M. Houzeau for Texas, the other under M. Niesten for Chili. These expeditions are furnished with a new kind of heliometer devised by M. Houzeau, in which the two halves of the O. G. have very different focal lengths, one being fourteen feet, the other only about six inches. In this way the observer will see a large and a small image of Venus and the Sun, and the measures are taken by placing the large image of Venus concentric with the small image of the Sun. In this way the difficulties attending the placing in contact of the limbs will be avoided. The mountings of these heliometers were made by Mr. Grubb.

Professor H. A. Newton has been appointed Director of the new Observatory of Yale College. This institution possesses a heliometer by Repsold and an eight-inch refractor by Grubb. The annual report lately issued gives full details of the activity of the "Horological and Thermometric Bureaus" in charge of Dr. Waldo.

Nos. 37–38 of the "Science Observer" contain further particulars of Mr. Chandler's determination of the period of the variable star D.M. + 1° 3408. From his own observations of four well indicated minima and from a large number of observations by Mr. Sawyer (including two complete minima), Mr. Chandler finds the period to be $20^h\ 7^m\ 41^s{\cdot}6$ with a probable error of $1^s{\cdot}3$ and the epoch, 1881, July 17, $10^h\ 49^m$ Cambr. M. T. All the sensible variations are comprised in the short space of about four hours, the star during the other sixteen hours maintaining its maximum brightness.

DETERMINATION OF THE ANNUAL PARALLAX OF 6 CYGNI B = Σ 2486.

BY ROBERT S. BALL, LL.D., F.R.S., ASTRONOMER ROYAL FOR IRELAND.

In the course of certain reconnoitring observations in search of stars with an appreciable annual parallax the star 6 Cygni B came under review. It is No. 2789 of Groombridge and is also a well known double star, Σ 2486 (6, 6·5), the place for 1855·0 being $19^h\ 8^m\ 20^s{\cdot}4$, + 49° 35′·3. On Nov. 30, 1879, the distance of a small star *n f* of the 10·5 mag. was measured with the following result, where π is the annual parallax—

$$D = 171''{\cdot}4 - {\cdot}9 \cdot \pi$$

On May 4, 1880, the measures were repeated with the following result—

$$D = 169''{\cdot}9 + {\cdot}9 \cdot \pi$$

These observations having been made at the time when the object was in the apse of its parallactic ellipse, it was natural to enquire how far they could be explained by annual parallax. From the distances a parallax of considerable amount is suggested. It will of course be understood that these observations are not intended to give any determination of the actual value of the parallax; they would indeed point to a parallax of 0″·7 or 0″·8 which is larger than the parallax since determined, but they were sufficient to suggest that further enquiry was desirable. A systematic series of observations was therefore commenced on Oct. 3, 1880, and was continued to Dec. 22, 1881. It is the object of the present paper to discuss this series of observations.

6 Cygni is No. 196 of Argelander's list of 250 stars with a large proper motion (Bonn Observations, Vol. VII), the proper motion being 0″·636 in the position angle 346° 27′. Argelander's positions refer to the preceding star, but it is the following star which has been employed in the present investigation. The distance and position angle have been measured from the following star in the dark field. Argelander states that the two stars form a system and I have therefore adopted the proper motion which he gives for the preceding star as a first approximation to the motion of the following star. The mode of observing this object differed from that I had adopted in previous investigations inasmuch as each set was made in only one position of the micrometer in the present case, while in the others the micrometer was turned 180° when half the observations were completed; there were also two reversals of the wires in the latter case while there was only one in the former. The reason why I departed in this case from the practice which was found to answer well in the others may be easily stated. The measures of 6 Cygni were made in a dark field with bright wires. Now with our instrument the measures

cannot be made so accurately with the dark field as they can with the bright field. I therefore thought that I might legitimately adopt a simpler method of observing in the dark field, believing that the errors thereby introduced would be inappreciable in comparison with the errors incidental to these measures themselves. In some respects I now wish that the more accurate method had been followed, as the probable error of the single observations would perhaps have been lessened. Another remark should be made with respect to the observations of position angle. It was my practice to insert this object in the working list along with many other stars on the reconnoitring list, and I not unfrequently made only a single determination of the parallel (by four readings usually) so long as the telescope was in the same position. Indeed, throughout the work I regarded the observations of position angle as of subsidiary importance, my care being chiefly directed to the measures of the distance. It was therefore to be anticipated that the measures of the distance would afford a more satisfactory result than the measures of the position angles, and this anticipation has been confirmed by the results.

The following are the observations obtained, corrected for refraction and for reduction to epoch—

Date.	Distance.	Position.	Date.	Distance.	Position.
1880, Oct. 3,	170″·899	78° 6′·31	1881, Oct. 14,	171″·108	78° 30′·65
„ „ 19,	171·113	78 9·22	„ „ 14,	171·073	78 30·41
„ „ 20,	170·309	77 54·98	„ „ 15,	171·138	78 28·83
„ Nov. 3,	171·210	78 10·32	„ „ 15,	169·988	78 21·59
„ „ 21,	171·563	78 0·48	„ „ 15,	171·231	78 23·60
1881, Apr. 1,	169·700	78 8·20	„ „ 16,	171·258	78 15·86
„ „ 1,	169·753	78 29·01	„ „ 17,	170·950	78 20·86
„ „ 5,	170·293	78 16·62	„ „ 17,	171·484	78 22·12
„ „ 5,	170·361	78 6·96	„ Nov. 1,	171·623	78 9·53
„ „ 16,	169·864	78 22·49	„ „ 5,	171·224	78 24·33
„ May 2,	170·080	78 12·91	„ „ 25,	170·638	78 14·68
„ „ 4,	169·821	78 20·35	„ „ 25,	170·881	78 2·36
„ „ 6,	169·398	78 32·15	„ Dec. 3,	171·559	78 26·25
„ „ 7,	170·213	78 25·89	„ „ 3,	171·839	78 13·23
„ „ 9,	170·252	78 28·87	„ „ 15,	170·463	78 24·60
„ „ 9,	169·607	78 19·37	„ „ 22,	171·429	78 32·78
„ „ 22,	170·604	78 31·82	„ „ 22,	171·131	78 31·86
„ June 8,	170·016	78 29·52			
„ Oct. 2,	171·353	78 19·77			
„ „ 2,	170·176	78 19·78			

The mean date of the observations was 1881·5207. This was taken as the epoch. The assumed mean distance is 170″·692 and position angle 78° 18′·610.

The annual proper motion has next to be considered. If we assume that the comparison star is at rest and if we attribute to 6 Cygni B the proper motion of Argelander, then the annual increase of the distance is + 0″·02, and of the position angle + 12′·796. It thus appears that nearly the entire proper motion is devoted to change of the position angle. A glance at the observations will show that the distance is subject to no regular increase, while the position angle is obviously larger towards the end of the Series than it was a year previously. This may be taken as *prima facie* evidence, that the large proper motion is not participated in by the comparison star.

The correction to be applied to the distance for parallax is—

$$-\,[9{\cdot}99938]\cos(\odot - 206^\circ\,59'{\cdot}16)\,\mathrm{R}\pi,$$

and the equations of condition are of the form—

$$x + bx' - [9{\cdot}99938]\,\mathrm{R}\pi\cos(\odot - 206^\circ\,59'{\cdot}16)$$
$$-\,[9{\cdot}99938]\,\kappa\sin(\odot - 206^\circ\,59'{\cdot}16) + \mathrm{D}'' - 170''{\cdot}692 = 0,$$

where $-\,x$ is the correction to the adopted distance, b is the fraction of year elapsed since the epoch, x' is the correction to the assumed proper motion, π the parallax, κ the aberration and D'' the observed distance corrected for refraction and reduction. The observations yield the following equations of condition—

EQUATIONS OF CONDITION FROM DISTANCES.

					Residuals.
1	x − ·7650 x′	− ·9581 π	+ ·2808 κ	+ ·223 = 0	+ ·042
2	− ·7212	− ·9938	+ ·0087	+ ·436	+ ·141
3	− ·7185	− ·9935	− ·0087	− ·368	− ·670
4	− ·6802	− ·9584	− ·2500	+ ·532	+ ·162
5	− ·6309	− ·8304	− ·5382	+ ·884	+ ·478
6	− ·2743	+ ·9642	− ·2599	− ·986	− ·423
7	− ·2743	+ ·9642	− ·2599	− ·933	− ·370
8	− ·2606	+ ·9809	− ·1931	− ·394	+ ·199
9	− ·2606	+ ·9809	− ·1931	− ·326	+ ·267
10	− ·2332	+ 1·0028	− ·0064	− ·823	− ·157
11	− ·1894	+ ·9718	+ ·2618	− ·608	+ ·131
12	− ·1839	+ ·9627	+ ·2945	− ·867	− ·122
13	− ·1785	+ ·9526	+ ·3265	− 1·290	− ·539
14	− ·1757	+ ·9471	+ ·3424	− ·475	+ ·277
15	− ·1703	+ ·9353	+ ·3738	− ·436	+ ·321
16	− ·1703	+ ·9353	+ ·3738	− 1·081	− ·324
17	− ·1347	+ ·8335	+ ·5656	− ·085	+ ·683
18	− ·0881	+ ·6410	+ ·7737	− ·674	+ ·065
19	+ ·2295	− ·9521	+ ·3014	+ ·656	+ ·404
20	+ ·2295	− ·9521	+ ·3014	− ·521	− ·773
21	+ ·2623	− ·9904	+ ·0992	+ ·411	+ ·069
22	+ ·2623	− ·9904	+ ·0992	+ ·376	+ ·034
23	+ ·2651	− ·9917	+ ·0818	+ ·440	+ ·090
24	+ ·2651	− ·9917	+ ·0818	− ·710	− 1·060
25	+ ·2651	− ·9917	+ ·0818	+ ·533	+ ·183
26	+ ·2678	− ·9926	+ ·0647	+ ·560	+ ·204
27	+ ·2706	− ·9933	+ ·0473	+ ·252	− ·111
28	+ ·2706	− ·9933	+ ·0473	+ ·786	+ ·423
29	+ ·3116	− ·9679	− ·2119	+ ·925	+ ·483
30	+ ·3226	− ·9498	− ·2797	+ ·525	+ ·068
31	+ ·3773	− ·7927	− ·5930	− ·062	− ·552
32	+ ·3773	− ·7927	− ·5930	+ ·181	− ·308
33	+ ·3992	− ·7014	− ·7003	+ ·859	+ ·379
34	+ ·3992	− ·7014	− ·7003	+ 1·139	+ ·658
35	+ ·4321	− ·5390	− ·8349	− ·238	− ·687
36	+ ·4513	− ·4326	− ·8965	+ ·728	+ ·310
37	+ ·4513	− ·4326	− ·8965	+ ·430	+ ·012

NORMAL EQUATIONS.

$$\begin{array}{rrrrr}
+\,37{\cdot}0000\,x & -\,{\cdot}0000\,x' & -\,8{\cdot}8113\,\pi & -\,2{\cdot}6079\,\kappa & {\cdot}0000 = 0 \\
{\cdot}0000\,x & +\,5{\cdot}1102\,x' & -\,4{\cdot}0579\,\pi & -\,1{\cdot}9890\,\kappa & +\,3{\cdot}1411 = 0 \\
-\,8{\cdot}8113\,x & -\,4{\cdot}0579\,x' & +\,30{\cdot}2489\,\pi & +\,4{\cdot}8179\,\kappa & -\,15{\cdot}9515 = 0 \\
-\,2{\cdot}6079\,x & -\,1{\cdot}9890\,x' & +\,4{\cdot}8179\,\pi & +\,6{\cdot}5593\,\kappa & -\,4{\cdot}4572 = 0
\end{array}$$

· Solving these we find—

$$\begin{aligned} x &= + 0''\cdot1441 \pm 0''\cdot050 \\ x' &= - \; 0\cdot0814 \pm \; 0\cdot143 \\ \pi &= + \; 0\cdot5039 \pm \; 0\cdot060 \\ \kappa &= + \; 0\cdot3420 \pm \; 0\cdot127 \end{aligned}$$

The sum of the squares $[n\,n]$ is 16·015, and the sum of the squares of the residuals is 6·202. The probable error of one observation is $0''\cdot2924$. The smallness of the probable error seems to suggest that this star really possesses a parallax of considerable amount; it is therefore a matter of great interest to see how far this conclusion can be corroborated by the observations of position angles.

The equations of condition are formed in the usual manner from the type—

$$x + bx' - m' \mathrm{R} \cos(\odot - \mathrm{M}')\pi - m' \sin(\odot - \mathrm{M}')\kappa + 60\,(p'' - p_0'') \sin \mathrm{D} = 0$$

and they are as follows—

EQUATIONS OF CONDITION FROM POSITION ANGLES.

1	$x - \cdot7650\,x'$	$- \cdot2497\,\pi$	$- \cdot9130\,\kappa$	$- \cdot1246 = 0$
2	− ·7212	− ·0088	− ·9465	− ·0080
3	− ·7185	+ ·0252	− ·9462	− ·7170
4	− ·6802	+ ·2513	− ·9120	+ ·0206
5	− ·6309	+ ·5178	− ·7880	− ·4993
6	− ·2743	+ ·2298	+ ·9182	− ·3426
7	− ·2743	+ ·2298	+ ·9182	+ ·6907
8	− ·2606	+ ·1665	+ ·9318	+ ·0668
9	− ·2606	+ ·1665	+ ·9318	− ·4129
10	− ·2332	− ·0111	+ ·9465	+ ·3408
11	− ·1894	− ·2669	+ ·9088	− ·1627
12	− ·1839	− ·2980	+ ·8993	+ ·2032
13	− ·1785	− ·3286	+ ·8888	+ ·7857
14	− ·1757	− ·3438	+ ·8832	+ ·4731
15	− ·1703	− ·3739	+ ·8712	+ ·6176
16	− ·1703	− ·3739	+ ·8712	+ ·1459
17	− ·1347	− ·5572	+ ·7703	+ ·7415
18	− ·0881	− ·7554	+ ·5851	+ ·5977
19	+ ·2295	− ·2694	− ·9074	− ·0882
20	+ ·2295	− ·2694	− ·9074	− ·0877
21	+ ·2623	− ·0768	− ·9434	+ ·4311
22	+ ·2623	− ·0768	− ·9434	+ ·4192
23	+ ·2651	− ·0603	− ·9446	+ ·3390
24	+ ·2651	− ·0603	− ·9446	− ·0205
25	+ ·2651	− ·0603	− ·9446	+ ·0793
26	+ ·2678	− ·0441	− ·9455	− ·3067
27	+ ·2706	− ·0277	− ·9462	− ·0602
28	+ ·2706	− ·0277	− ·9462	+ ·0023
29	+ ·3116	+ ·2157	− ·9212	− ·6488
30	+ ·3226	+ ·2789	− ·9037	+ ·0790
31	+ ·3773	+ ·5681	− ·7513	− ·4348
32	+ ·3773	+ ·5681	− ·7513	− 1·0465
33	+ ·3992	+ ·6660	− ·6627	+ ·1257
34	+ ·3992	+ ·6660	− ·6627	− ·5208
35	+ ·4321	+ ·7879	− ·5049	+ ·0228
36	+ ·4513	+ ·8431	− ·4015	+ ·4168
37	+ ·4513	+ ·8431	− ·4015	+ ·3711

Normal Equations.

$$\begin{array}{rrrrr}
+\ 37{\cdot}0000\,x & {\cdot}0000\,x' & +\ 2{\cdot}4837\,\pi & -\ 8{\cdot}5154\,\kappa & +\ 1{\cdot}4893 = 0 \\
{\cdot}0000\,x & +\ 5{\cdot}1098\,x' & +\ 1{\cdot}9455\,\pi & -\ 3{\cdot}8185\,\kappa & -\ {\cdot}0171 = 0 \\
+\ 2{\cdot}4837\,x & +\ 1{\cdot}9455\,x' & +\ 5{\cdot}9728\,\pi & -\ 4{\cdot}6946\,\kappa & -\ 2{\cdot}3771 = 0 \\
-\ 8{\cdot}5154\,x & -\ 3{\cdot}8185\,x' & -\ 4{\cdot}6946\,\pi & +\ 27{\cdot}0927\,\kappa & +\ 5{\cdot}1311 = 0
\end{array}$$

Solutions.

$$\begin{array}{l}
x = -\ 0''{\cdot}112 \pm 0''{\cdot}05 \\
x' = -\ 0{\cdot}291 \pm 0{\cdot}13 \\
\pi = +\ 0{\cdot}383 \pm 0{\cdot}13 \\
\kappa = -\ 0{\cdot}199 \pm 0{\cdot}06
\end{array}$$

The probable error of one observation is 0″·276 of arc or 5′·6 of position angle.

Combining the results of the two sets of equations we have for the parallax of 6 Cygni B—

$$\pi = +\ 0''{\cdot}482 \pm 0''{\cdot}054.$$

It is very desirable that this result should be tested by other investigations, and I hope to make another series of observations. The present determination is to be regarded as merely provisional, though I think it can hardly be doubted that a parallax of very considerable amount really exists.

It is impossible not to be reminded of 61 Cygni which is in the same constellation and the parallax of which is about the same amount. It will also be observed that Σ 2486 and 61 Cygni are doubles of the same general character and have each a large proper motion.

UEBER DEN ZWEITEN DIFFERENTIALQUOTIENTEN DER WAHREN ANOMALIE.

Für ein *unendlich kleines* Increment $-\ dt$ und für das entsprechende $-\ dv'$ gehe x von x_0 in x_{-1} über, für $+\ dt$ aber und $+\ dv'$ in x_{+1}, so hat man—

$$x_{-1} = x_0 - \frac{dx}{dt}\,dt + \frac{d^2x}{dt^2}\cdot\frac{dt^2}{2}\cdots$$

$$x_{+1} = x_0 + \frac{dx}{dt}\,dt + \frac{d^2x}{dt^2}\cdot\frac{dt^2}{2}\cdots$$

$$x_{-1} = x_0 + \frac{dx}{dv'}\cdot dv' + \frac{d^2x}{dv'^2}\cdot\frac{dv'^2}{2}\cdots$$

$$x_{+1} = x_0 + \frac{dx}{dv'}\cdot dv' + \frac{d^2x}{dv'^2}\cdot\frac{dv'^2}{2}\cdots$$

Setzt man die Summe der beiden ersteren Relationen gleich der Summe der letzteren, so folgt—

$$\frac{d^2x}{dt^2}\cdot dt^2 = \frac{d^2x}{dv'^2}\cdot dv'^2$$

oder—

$$\frac{d^2x}{dv'^2} = \frac{d^2x}{dt^2} \cdot \frac{dt^2}{dv'^2} \cdot$$

Wenn nun überhaupt die Taylor'sche Reihe richtig ist (was ich für mein Theil nicht in Abrede stellen möchte) so sind obige Ausdrücke für die Summe $x_{-1} + x_{+1}$ nur um eine Grösse der *dritten* Ordnung (wenn man will, oder noch höherer) von einander verschieden, welche selbst nach einer Division mit dv'^2 noch unendlich klein bleibt, und das *endliche* Verhältniss $\frac{d^2x}{dv'^2}$ nur um ein unendlich Kleines erster oder höherer Ordnung beeinflusst.

Auf welche andere Weise kann und darf aber dann die Veränderlichkeit von $\frac{dv'}{dt}$ noch mitspielen, als dadurch, dass man

$$\frac{dv'}{dt} = \frac{k\sqrt{1+m'} \cdot \sqrt{p'}}{r'^2}$$

setzt, wie Dr. Kobold dies gethan hat?

W. Klinkerfues.

Göttingen, Juli 30, 1882.

ON SOME HITHERTO UNKNOWN OBSERVATIONS OF A SUPPOSED SATELLITE OF VENUS.

By H. C. F. C. SCHJELLERUP.

Without intending to discuss the reliability of the observations of a body resembling a satellite of Venus, I consider it well to put on record some hitherto unknown observations of this kind.

In a book entitled "Der Venusmond und die Untersuchungen über die früheren Beobachtungen dieses Mondes" (Braunschweig 1875) Dr. Schorr has put together in tabular form all the known observations which may be interpreted as pointing to the existence of a satellite of Venus. On comparing this list with the old ledgers of observations at the Copenhagen Observatory I have found that seven observations made in 1761 by Roedkiær and one made in 1768 by C. Horrebow and two other observers have never been published. The observations of 1764 were made known the same year in Vol. IX. of the Transactions of the Roy. Danish Society of Sciences, but it is most remarkable that Professor Horrebow and his second assistant Roedkiær in their reports to the Society do not make the slightest allusion to the observations made only three years previously. Horrebow even says after having mentioned two observations by Cassini, one by Short and four by Montaigne, that up to 1764 there were no other proofs of the existence of the satellite. I can offer no suggestion as to why the observations of 1761 were not mentioned, but as they seem at any rate as reliable as those of 1764 I have thought it desirable to publish them. I give them exactly as they are in the ledgers and have added the original observations of 1764, which it also seems of interest to record.

To anyone wishing to explain these observations it will always cause the greatest difficulty that all the observations, with the exception of those of Fontana, Cassini and Short, were made within the space of ten years (1759–68), while nothing of the kind has ever been seen since. To the same period belongs another observation which seems to have been overlooked by modern writers,[*] although it is published in the well-known French "Encyclopédie," edited by Diderot and d'Alembert, under the article "Venus." We there read as follows—

> Parmi les apparitions . . . une des plus remarquable est sans contredit celle du P. la Grange, jésuite. Ce savant cultivoit à Marseille l'astronomie depuis nombre d'années ; muni d'excellens instrumens & entr'autres du télescope de 6 pieds de foyer du P. Pezenas, construit par M. Short en 1756, dont l'effet est de grossir 800 fois, & égale celui d'une lunette qui auroit 1600 pieds. Son expérience reconnue & son exactitude dans les observations, rendent précieuses celles que nous allons rapporter.
>
> Il n'y vit point de phase comme l'avoient apperçue tous les autres observateurs ; & ce qui n'est pas moins surprenant, c'est qu'il lui parut que ce petit astre suivoit une route perpendiculaire à l'écliptique. Cette direction qui par ce qui précede se concluoit des observations de Limoges,[†] parut si étrange au P. la Grange, qu'il ne fit point difficulté d'abandonner toutes les conséquences qu'il avoit déduites de ses observations. Elles furent faites des 10 au 12 février 1761, à trois jours différens.

I now give all the Copenhagen Observations. It may be mentioned here that there are no ledgers extant for the years 1762 and 1763 ; they were no doubt destroyed during the bombardment of Copenhagen in 1807, as we possess a description of the damage done to Professor Bugge's residence, in which it is said that two volumes of observations for 17 . . (years left blank) were lost.

1761 d. 28 Junii. In quadrante Venerem observans[‡] Roedkiær vidit albedinem quandam, quæ Venerem sequebatur. Distantiam inter hanc et limbum Veneris superiorem invenit 0·66,[§] et observavit 11″ transitum inter hanc et Venerem. Postea tubo 17′ eandem interum vidit, atque ex facie ejus, quæ non adeo falcata erat ac Veneris, sed fere dimidia facie splendens, suspicatur Observator, se satellitem Veneris vidisse. Coeteri nos eandem albedinem videre non potuimus, licet Venerem sæpe observaverimus quadrante, rota meridiana et tubo 17 pedum.

1761 d. 30 Junii, A.M. Venerem observans Roedkiær eandem videbat albedinem, quam antea diebus 28^{vo} & 29^{no} Junii viderat. Distantiam ejus a Venere determinabat ¼ circiter diametri Veneris. Erat isto die prope cornu superius Veneris in quadrante. Sed ista albedo nondum coeteris nobis apparuit.

1761 d. 2 Julii, A.M. Per totam hanc noctem, et etiam per plurimas antecedentes Boserup et Roedkiær Venerem expectarunt orientem spe Satellitem ejus detegendi, sed, etsi tubis 17 & 22 pedum usi fuerint, nihil tamen viderunt.

1761 d. 19 Julii. Statim post ortum Veneris Roedkiær observabat lumen aliquod debile figuræ confusæ et indistinctæ una cum Venere in tubo 17 pedum existens, etsi optime videretur, etiam Venere in tubo non existente. Distantia hujus luminis a Venere erat, ut æstimabat, 40 fere semidiam. Veneris. Venus supra illam in tubo ad sinistram sita erat. Lumen valde debile erat, sed tamen satis magnum, ut persvasus sit se stellam non observasse.

1761 d. 20 Julii. Roedkiær Venerem orientem quidem observabat, sed nubes Venerem quovis momento tegebant.

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[*] See, however, Peters' Zeitschrift für populäre Mittheilungen, II., p. 248.—Ed.

[†] By Montaigne. See the table at the end of this paper.

[‡] No time given, altitude = 51° 52′ 21″.

[§] As $1^p = 130''{\cdot}44$, the distance was 86″.

1761 d. 23 Julii. Roedkiær hac nocte Venerem plus integra hora observabat tubo 17 pedum et interdum tubo machinæ parallacticæ, sed ne minimum quidem signum Satellitis videbat.

1761 d. 25 Julii. Roedkiær Venerem per longum satis tempus observavit, sed Satellitem detegere non potuit.

1761 d. 5 Augusti. Statim post hanc observationem[*] stellam quandam machina parallactica observabat Roedkiær. Hanc stellam pro Satellite Veneris habebat—

Venus h.	22	40	41
Stella —	22	40	46

Distantiam stellæ a limbo Veneris superiore observabat circumv. 7·18.[†] Postea Boserup stellam invenit eandem, atque admonuit, Satellitem Veneris esse infra hanc stellam. Hinc Roedkiær machina parallactica sequentem fecit observationem—

Stella supra observata . . . h.	22	58	11
Satelles	22	58	30

Distantia Satellitis micrometro machinæ parallacticæ determinata 21·66, quod facit in minutis.[‡]

Postea a Roedkiær determinata distantia Satell. a Venere—

in rectasc. 24″ temp.
in declin. 28·84 [§]

1761 d. 8 Augusti. Roedkiær h. 2 matut. iterum vidit Satellitem Veneris, et machina parallactica observavit—

Satelles . . . . h.	23	23	31
Ven. limb. sequ.	23	23	40

Distantiam a Veneris limbo inferiore determinavit—

$$\begin{array}{r}12{\cdot}92\\13{\cdot}50\\\hline 26{\cdot}42\end{array}$$ [‖]

1761 d. 12 Augusti. Hora 1 mat. Roedkiær Venerem orientem observavit et Satellitem ejus iterum invenit infra Venerem ad dextram situm fere eodem in loco, ubi prima vice eum observabat. Situm ejus accuratius determinare nubes impediebant.

1761 d. 13 Augusti. Hora 1 matutina Roedkiær observavit Venerem, et invenit in parte inferiori tubi lumen quoddam debile ad dextram Veneris, sed nubes Venerem subinde occultantes observationes ulteriores impedivere.

1761 d. 29 Augusti, A.M. Per hanc noctem Venerem observavit Roedkiær, sed Satellitem detegere non potuit.

1761 d. 24 Novembris, A.M. Roedkiær Venerem observabat tubo 17′ et 7′, sed Satellitem non inveniebat.

1761 d. 1 Decembris. Venerem observabat, sed Satellitem non invenit [Roedkiær].

1764 d. 3 Martii. Horâ 6 pomeridiana Roedkiær Venerem observans ejus detegebat Satellitem. Vitro utebatur objectivo utrinque convexo $9\frac{1}{2}$ pedum, oculari 3 pollicum. Phasis ejus prorsus erat similis Ven. scilicet globosa, ita ut diameter maxima esset verticalis in ambobus. Distabat in tubo ad sinistram Veneris $\frac{3}{4}$ diametri Veneris. Centrum Satellitis in eodem cum centro Veneris erat parallelo, magnitudo fere $\frac{1}{4}$ diametri Veneris.

1764 d. 4 Martii. Iterum hac vespera horâ itidem 6tâ Roedkiær videbat Satellitem Veneris. Distantia ejus ad sinistram erat $\frac{1}{2}$ diametri Veneris. Centrum ejus cum centro Veneris angulum faciebat semirectum plus minus: altius in tubo apparebat quam Veneris centrum. Phasin ejus etiam optime distinguere potuit Veneri conformem: usus est partim eodem vitro objectivo quo heri, partim objectivo menisco 14 pedum cum oculari 3 pollicum. Configurationes d. 3 & 4 Martii tales erant:

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[*] Transit of "Pegasi Collum."

[†] From 18 measures of the diameter of the Sun during the transit of Venus, June 6, 1761, assuming the Solar Radius = 958″·42 (Leverrier, Annales, VI., p. 39) I find 1^{p} = 69″·23 and accordingly the observed distance = 497″·0.

[‡] Full stop in MS., and a new line. The distance is 25′ 0″.

[§] Equal to 33′ 17″.

[‖] Equal to 30′ 9″.

Configuratio d. 3 Martii
Satell. ○ ◯ Venus

Configuratio d. 4 Martii
Satell. ○
◯ Venus.

N.B. Ita Satelles et Venus in tubo apparuere. Satellitem fuisse præsertim exinde patet, tam Veneris quam Satellitis diametrum notabiliter (tubo usus 14 pedum præ tubo 9½ pedum) amplificari, quod de nulla fixa valet.

1764 d. 9 Martii. Horâ 6½ vespertinâ Satelles Veneris visus est a P. H.,* Boserup and Roedkiær. Ob defectum necessariorum apparatuum situm Satellitis atque distantiam ejus a Venere observationibus determinare non potui. In tubis videbatur supra Venerem ad dextram, ita ut ex meo judicio faceret cum verticali Veneris circa 30°. Distantia a Venere erat 1¼ vel 1½ diam. Veneris. Satelles Veneris erat perquam parvus ita ut vix ⅙ diam. Veneris excederet.†

1764 d. 10 Martii. Horâ 6 vespertinâ videbamus tubo 9½ ped. ad dextram Veneris lumen quoddam, an vero Satelles Veneris esset, nec ne? C. et P. Horrebow affirmare non audent. Lumen istud faciebat cum verticali Veneris 45°. Roedkiær autem credebat, se Satellitem Veneris vidisse eodem tubo 9½ ped., in quo applicuerat duo filamenta sese ad angulum rectum secantia, et observabat sequentem transitum Veneris et Satellitis—

h. 5	56	13	Satelles ad horizontale,
	56	14	Limb. superior Ven. ad idem,
	56	18	Limb. inferior Ven. ad idem,
	56	31	Limbus anteced. Ven. ad verticale,
	56	35	Limbus sequens Ven. ad idem,
	56	36	Satelles ad idem filam.

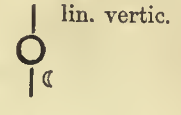
Horâ 7¼ vespertinâ lumen istud plane disparebat.

1764 d. 11 Martii. Hac vesperâ ab hora 5¾ ad horam 7 videbamus omnes tubo 9½ ped. lumen quoddam debile ad dextram Veneris circa 30 grad. supra horizontalem Veneris. Hoc lumen colore Veneri (non stellulis fixis quæ simul in tubo apparebant) simile erat, quamobrem omnes credidimus, quod Satelles Veneris esset. Multis modis tentatum est, an lumen fictitium esset, quod in tubo apparuit, sed contrarium deprehensum est.‡ Colore simile erat illi lumini, quod P. H. d. 9 Martii apparuit, sed multo minus erat. Distabat fere a Venere 1 vel 0¾ diam. Veneris. Horâ 7 disparebat.

1764 d. 12 Martii. Hac vesperâ Satelles Veneris non visus est, licet iisdem tubis usi simus ac antea. Aër tamen serenus erat; ita ut minimas stellas in tubo cum Venere videremus.

1768 d. 4 Januarii. Venus observabatur h. circiter 5¾ A.M. tubo Dolloniano 10 pedum, apparatu terrestri instructo, atque ejus facies talis erat:

scilicet infra Venerem paulo ad dextram â linea verticali conspiciebat parvum lumen, quod minime stella erat, (erant enim etiam stellæ in tubo, quæ longe aliam habebant apparentiam), quodque â Venere circiter magnitudine diametri Veneris distabat. Mox postea in tubo Islæano Astronomico 12 pedum §



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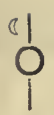
* Peter Horrebow, Jr., son of Prof. Peter Horrebow, Römer's disciple, and brother to Chr. Horrebow. He was for some years first assistant at the Observatory, until his brother's death in 1776. He died in 1812.

† This evening the "satellite" was observed through two telescopes, one of 9½ feet and one of 6 feet focal length. P. Horrebow also thought he saw it with the quadrant, which had a tube of 3 feet only, but he was not certain (Trans. R. Dan. Soc. IX. p. 400).

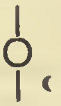
‡ In his report to the R. Society of Sciences Prof. Horrebow states that he looked at Jupiter and Saturn the same evening and saw them both well and without any false images. He also moved the telescope about while observing Venus, but the relative positions of Venus and the satellite remained unchanged. This was also the case while he let Venus pass through the entire field of view, and even when Venus was outside the field the satellite was visible (l.c. p. 402–3).

§ Under April 1, 1767, we read: "Hic tubus 12′ postea Islæanus dicendus est, quia D. de l'Isle eum ordinavit."

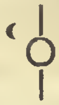
observabatur Venus, ibique ejus hæc erat facies: scilicet idem lumen, quod infra ad dextram in tubo terrestri conspiciebat, supra ad sinistram in tubo Astronomico apparebat, atque in distantia diametri Veneris. *



Præterlapsa horâ, vel horæ tribus quartis, magis ad dextram in Dolloniano, et magis ad sinistram



tubo Dollon.



tubo coelesti

in tubo astronomico apparebat hoc Veneri adhærens lumen. Hoc vero idem phænomenon in Venere observabant tres Observatores, C. H., O. B. et J.; † omnes certo videbant, hoc lumen stellam non esse, tentando omnibus certum videbat, hoc lumen non esse illusionem opticam, unde suspicabant, illud forsan esse Veneris Satellitem.

In conclusion it may not be devoid of interest to compare the above notes with the following summary of similar observations, which I take from Dr. Schorr's book with the exception of those of La Grange and Scheuten. The latter saw for several hours a "satellite" follow Venus across the Sun at the end of the transit in 1761. (See Bode's Jahrbuch, 1778, pp. 186–188).‡

Year	Date, Hour.	Relat. Diam.	Dist. from Venus.	Relative Position.	Figure, Brightness.	Observer.	Instrument, &c.
1645	Nov. 15, P.M.			Above	Dark	F. Fontana	
1672	Jan. 24, 19^h	$\frac{1}{4}$	1 diam.	West	Crescent	Cassini	34 feet telescope.
1686	Aug. 27, 16^h	$\frac{1}{4}$	$\frac{3}{5}$ diam.	East	Shapeless	Cassini	
1740	Nov. 3, 17^h	$< \frac{1}{3}$	10′ 20″	*p*	Same as Ven.	Short	Greg. Tel. 16 in., powers 50–240.
1759	May 20, 9^h		$1\frac{1}{2}$ diam.	Above	Fainter	A. Meier	Greg. Tel. 30 in. (Greifswald).
1761	Feb. 10, 11, and 12				Round	La Grange	6 feet tel. by Short, power 800.
1761	May 3, 9.30	$\frac{1}{4}$	20′	Below and tow. south	Same as Ven.	Montaigne	9 feet tel. (at Limoges).
1761	May 4, 9.30	$\frac{1}{4}$	21′	Below, 20° tow. north	Same	Montaigne	
1761	May 7, 9.30	$\frac{1}{4}$	25′	Above, 45° tow. north	Same	Montaigne	
1761	May 11, 9^h	$\frac{1}{4}$	25′	Above, 45° tow. south	Same	Montaigne	
1761	June 6, A.M.	$\frac{1}{4}$	27′	*nf*		Scheuten	
1764	Mar. 15, 7^h			East		Montbarron	Greg. Tel. 32 in., at Auxerre.
1764	Mar. 28, 7.30			West		Montbarron	
1764	Mar. 29, 7.30			West		Montbarron	

Copenhagen Observatory, July 1882.

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* It will be noticed that the crescent is not reversed in the figure. In the description there is nothing about the "satellite" being crescent-shaped.

† Chr. Horrebow, Ole Bützou and Ejolvor Johnsen. The last-named became second assistant in August 1767, two months after Roedkiær's death.

‡ See also Peters' Zeitschrift III., pp. 2 and 11, from which it would appear that something similar was seen in Huntingdonshire.—Ed.

JOTTINGS ON THE SCIENCE OF THE CHINESE.

By ALEXANDER WYLIE.

[The following paper was originally published in the *North China Herald* and then in the Shanghae Almanac for 1853. From special enquiries made at Shanghae it appears that both of these publications now exist in only one or two copies. It was published again in the *Chinese and Japanese Repository* (London, 1863–66), but that work has also become very scarce. It was also partly translated into German, with some alterations and additions, by Dr. K. L. Biernatzki, Crelle's Journal, Vol. 52, Art. 5, *Die Arithmetik der Chinesen,* but to the best of our knowledge this interesting paper, in its original form, seems to be very little known to recent writers on the History of Mathematical Science. As it contains a considerable amount of information of general interest, we have thought it desirable with the Author's permission to reprint the whole paper, the more so as we are able to avail ourselves of a manuscript copy made in Shanghae through the special courtesy of the Hon. Sorabjee S. Bengallee of Bombay, and the kind intervention of George Hamilton, Esq., of Skene, Aberdeenshire.—Editors.]

There are a few discoveries, such as the magnetic needle and the typographic art, which have been of indisputable service in the advancement of civilization, priority with respect to which, is by universal consent ascribed to the Chinese; a certain superiority in some of the arts, many will admit that these people possess; some questions in modern science have received a practical illustration in the ordinary habits of the "Flowery" race, and some few have been constrained to admit, that they have for time out of mind, possessed a perfect knowledge of facts, which have been but recently arrived at through the medium of theory in the West. Beyond this, few are disposed to accede to the pretensions of this people, and the suggestion of any degree of eminence in abstract science is by many thought to be unworthy of a moment's consideration. It is possible, however, that a little investigation might establish juster views. The object of the following desultory notes, made from time to time, in the course of some researches, entered upon with another purpose in view, is to draw attention to the state of the Arithmetical Science in China, a subject which has not been so fully explored as it might with advantage, and on which some erroneous statements have been current in modern publications.

The Roman Catholic missionaries who had from the commencement of the seventeenth century begun to acquire fame and influence at court, continued to retain and fortify their position, and on the change of dynasty taking place, they found themselves even more favourably circumstanced than before; by means of the works of Ricci, Schaal and others, much of the science of Europe had been laid open to the Chinese; their system of astronomy and methods of computation had at length overcome the scruples of the native mathematicians; the Astronomical Board was placed very much under their control, and the native methods, which had continued to be used till the end of the Ming dynasty, were now fairly supplanted by the western rules. Under these circumstances, the young Emperor Kang-he, who assumed the reins of Government at the age of fourteen, was brought in some measure under the tutorage of the foreign missionaries, and being naturally gifted with a high degree of intelligence, he readily entered into the spirit of their instructions, and became deeply imbued with a love for science in all its bearings.

During the brilliant reign of this emperor, men of genius were attracted from the remotest corners of the Empire, by the splendours of the imperial court and the encouragement offered to the devotees of Science; among others, appeared a man of no ordinary mental standing (1) Mei Wuh-gan,* a native of Hwuy-chow; not however in the character of an aspirant for office, or as a mere candidate for imperial favour, for being zealously attached to the interests of the Ming dynasty, he steadily refused, to the end of a long life, every offer to enter the service of their successors.

With this man, the emperor held frequent consultations on subjects congenial to the minds of both. Mei had become thoroughly conversant with the works of the foreigners, as is evident from the voluminous writings he has left behind him; but his shrewd mind did not allow him to stop here, for on ransacking the stores of antiquity, he came to the conviction that in overestimating the merits of the missionaries, his countrymen were unjustly neglecting the ancient native works; and that in fact there was very little in the recently introduced theories, which they had not themselves possessed for centuries before. This assertion Mei proceeded to prove and illustrate by extensive quotations from the remaining works of the ancients, and it was probably owing to the development of this fact in a great degree that we may ascribe the causes of the foreigners so soon after having declined in favour at the imperial court; for in a large mathematical work, the (2) *Leuh leih yuen yuen*, compiled by the personal efforts of Kang-he and his chief councillors, and published soon afterwards, the influence of such views is clearly to be traced. In one of the introductory chapters to that work, after acknowledging their obligations to Ricci, Schaal, Verbiest and other foreigners, for the good service they had rendered towards the perfection of the numerical sciences, the question is suggested,—Whence did these foreigners originally gain their knowledge? and the answer is complacently given,—that it has flown abroad from the Central land. The scanty notices of celestial phenomena recorded in the histories of the early monarchs are held to be confirmatory of this view; and if few written evidences of the flourishing state of science at that period, have come down to the present age, the cause is to be attributed to that catastrophe in the history of literature, the burning of the books by Che Hwang-te, the first emperor of the Tsin dynasty † above two centuries before the Christian era. Fortunately we are told, during the palmy days of the sages, when the transforming influence of the Central Kingdom had extended to the uttermost bounds of the habitable earth, their books on science had been translated into the languages of foreigners, and hence it happened that when all the native books were consigned to the flames, the foreigners became the sole depositories of these valuable treasures, the veritable productions of the celestial sages. In this way, the superiority of foreigners, where admitted, is ingeniously accounted for.

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* The figures in parenthesis refer to Plate V., on which the Chinese names occurring in this paper have kindly been copied by Prof. Schjellerup.—Ed.

† For a notice of this event, see Williams' "Middle Kingdom," Vol. 2, p. 212.

One may be inclined to smile at the statement above set forth, and if the assertion of some modern writers be correct, it will be perfectly legitimate to do so. If however instead of despatching the subject in that summary style, we take the pains to enquire how far Chinese authorities may be trusted in the matter, we may find that as a practical question, it is deserving of investigation, and facts may be brought to light which will show that the Chinese had some knowledge of mathematics at a very early date.

To ascertain the actual state of arithmetic among the Chinese, it might be sufficient to run over the contents of some of the principal modern publications on that science; but as they have confessedly gained something from the Roman Catholics during the past two centuries, and previous to that time, their intercourse with the Arabians during the Yuen dynasty, doubtless added to their stock of mathematical acquirements, it will be interesting to discover somewhat respecting the state of their knowledge anterior to these successive accretions.

Setting aside the vagaries of Fo-he, as little to the purpose, the earliest definite notice of the science of numbers is to be met with in the (3) *T'ung Keen Kan muh,* "General History of China," where it is stated, under the reign of (4) Hwang-te, that he caused his minister (5) Le-show to form the (6) *Kew chang,* "nine sections of Arithmetic." Many other improvements are attributed to this monarch, and among them the Sexagenary cycle. Although there is much vagueness about records of that early period which is scarcely admitted within the range of authentic history, yet it is a remarkable fact, that the present chronological era of cycles dates its commencement from the 61st year of that emperor (B.C. 2637.)

The Nine Sections attributed to Hwang-te have formed the nucleus of arithmetical science in China, and although it would be difficult to conjecture how far the ancient sage was acquainted with the substance of these sections or whether indeed it would be true that they ever came under his notice, the work has been handed down to us with abundant evidence from a very remote period and has been largely expounded and illustrated by an almost unbroken succession of authors down to the present day.

The history of (7) Yaou has been frequently quoted to show that the Chinese had made a considerable advancement in science by his time (B.C. 2300). From the (8) *Shoo-King* we learn that he had a Board of Astronomers for making the calculations and forming a calendar with a view of the motions of the heavenly bodies, that the solstices and equinoxes were marked, and that the length of the year was known within an hour of true time. All this indicates that some attention was paid to the science of numbers.

But the document which can be traced with certainty to the greatest age, in which we have any detail of mathematical principles, is a brief dialogue held between the famous (9) Chow Kung and a nobleman of the recently vanquished Shang dynasty named Kaou, generally known as (10) Shang Kaou, about B.C. 1100. It is entitled the (11) *Chow-pe,* Thigh-bone of Chow.* As it is an object of some interest on account of its great antiquity, the translation is here given of the first section, which forms an epitome of the whole.

"1, Formerly, Chaou-Kung, addressing Shang-Kaou, said,—I have heard it said, my lord, that you are famous at numbers, may I venture to ask you how the ancient Fo-he established the degrees of the celestial sphere? There are no steps, by which one may ascend the heavens, and it is impracticable to take a rule and measure the extent of the earth; I wish to ask then how he ascertained these numbers? 2, Shang-Kaou replied,—The art of numbering originates in the circle and quadrangle. 3, The circle is derived from the quadrangle. 4, The quadrangle originates in the right angle.† 5, The right angle originates in the multiplication of the nine digits. 6, Hence, separating a right angle in its component parts, if the base be equal to 3, and the altitude to 4, a line connecting the farther extremities will be 5. 7, Square the external dimensions, and half the amount will give the area of the triangle. 8, Add together all the sides and the result will equal the sum of 3, 4 and 5. 9, The square of the hypothenuse being 25 is equal to the squares of the two short sides of the triangle. 10, Thus the means by which (14) Yu restored order throughout the empire was by following out the principles of these numbers. 11, Chow-Kung exclaimed,—How truly great is the theory of numbers! May I ask what is the principle of the use of the rectangle? 12, Shang-Kaou replied,—The plane rectangle is formed by uninclined straight lines. 13, The direct rectangle is used for observing heights. 14, The reversed rectangle is used for fathoming depths. 15, The flat rectangle is used for ascertaining distances. 16, By the revolution of the rectangle, the circle is formed. 17, By the junction of rectangles, the square is formed. 18, The square pertains to earth, the circle belongs to heaven, heaven being round and the earth square. 19, The numbers of the square being the standard, the dimensions of the circle are deduced from the square. 20, The circular plate is employed to represent heaven; the celestial colours are blue and black, the terrestrial are yellow and red; the circular plate is formed according to the celestial numbers; it is blue and black outside, red and yellow inside, in order to represent the celestial and terrestrial stations. 21, Therefore he who understands the earth is a wise man, he who comprehends the heavens is a sage. This knowledge begins with the straight line, the straight line is a component part of the rectangle, and the numbers of the rectangle are applicable to the construction of all things. 22, Chow-Kung exclaimed, Excellent indeed!"

The sentences are numbered here for convenience in reference. Several things deserve attention. In 3 there is allusion to the quadrature of the circle; 4 and 5 refer to plane mensuration; 6 gives the ratio of the hypothenuse to the short sides of a right angle triangle; 7 states that a quadrangle bounding the three angles of a triangle contains double the area of said triangle; 8 states the whole to be equal to the sum of the several parts; 9 is the well-known 47th proposition of the 1st book of Euclid; 13, 14 and 15 show that the author understood the application of trigonometry to the measurement of distant objects; 16 gives the fundamental principle upon which the area of the circle is calculated; the circular plate mentioned in 20 seems to have been some ancient instrument for representing the appearance of the heavens and the earth, the form of which is not very well understood now.

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* This name which at first sight appears a little *outré*, receives its explanation from the two characters (12) *Kew-Koo* used to designate the base and altitude of a triangle, and which mean originally the Leg and Thigh. It may call to mind also a similar device in our English term, Napier's Bones.

† The term right angle is used here to designate (13) *Kew-Koo*, two short sides of a right angle triangle.

At the 11th page of the 2nd book of the (15) *Chow-le*, Chow Ritual, the authorship of which is attributed to Chow-Kung, in the list of the officers of instruction, the duties of the Royal Tutor are said to be, "to train up the eldest sons of the prince and the nobility, in the six arts, to wit, the 5 classes of rites, the 6 kinds of music, the 5 laws of archery, the 5 descriptions of carriage-driving, the 6 rules of writing and the 9 methods of numbering." The 9 methods of numbering are explained by the commentators to be the *Kew-chang* or Nine Sections.

With the exception of an occasional allusion in the ancient classics, we have little further respecting the science previous to the great conflagration, when Che Hwang-te ordered the destruction of all the native books. Subsequent to this period, the works are too numerous to admit of recording even their titles; a brief notice of some of the most prominent must suffice.

About a century before the Christian era (16) Chang Tsang published the (17) *Kew-chang swan shuh*, "Arithmetical rules of the nine sections," which professes to embody the principles of arithmetic employed by the Royal Tutor during the Chow dynasty. It is not given forth however as an original composition, but merely pretends to be a revised and corrected edition of a much older work, the author of which is now unknown. The work has been preserved by successive editions down to the present time ; but it is now extremely rare ; it is probably the oldest exemplar of the *Kew-chang* extant. Commentaries have been written on it by several authors.

During the third century (18) Sun-tsze, an author of considerable note published his (19) *Swan King*, "Arithmetical Classic." This work is referred to by almost every subsequent writer on the subject. Several commentaries have been written on it.

About the same time also appeared (20) *Shuh soo Ke e*, "Record of rules and calculations," by (21) Sew Kew, which has been explained and commented on by succeeding authors.

Before the sixth century (22) Hea How-yang published his (23) *Swan King* "Arithmetical Classic," in which he broached some innovations on ancient practice, not confining himself strictly within the limits of the *Kew-chang*, which had been invariably followed by all his predecessors.

About the fifth century (24) Lew Hwuy published (25) *Chang cha Keae tsih wang che shuh*, "The whole system of measuring by the observation of several beacons." This was republished in the eighth century under the title of (26) *Hae taou Swan King*, "The island arithmetical classic," the name being adopted from the first problem in the book which is on measuring an island from a distance.

(27) Woo Tsaou who was about contemporary with the two preceding wrote a (28) *Swan King*, "Arithmetical classic," which is much referred to.

At the beginning of the seventh century a treatise on trigonometry was published by (29) Chaou Tchwang, called the (30) *Chow pe Swan King*, "Arithmetical classic of the Chow trigonometry." This however is said to be but a subsequent edition of a very much older work, which is looked upon as the original native production on trigonometry.

In the seventh century (31) Chin Lwan wrote a treatise called the (32) *Woo King Swan shuh,* "Arithmetical rules of the Five classics," embodying the various problems alluded to in the *Yih-King, Shoo-King, She-King, Le-Ke, Chun-tsew, Heaou-King,* and *Lun-yu.* A commentary was written on this work by (33) Le Chun-fung.

About the same time as the preceding (34) Chang Kew-Keen wrote his (35) *Swan-King,* "Arithmetical classic," the conciseness of the text of which, it is said, renders it difficult to be understood. Several authors of celebrity have undertaken to write commentaries on and illustrate it.

Towards the close of the eighth century (36) Wang Heaou-tung, an imperial librarian during the Tang dynasty, issued his (37) *Tseih Koo Swan King,* "Arithmetical classic of ancient formulæ," with a commentary by himself; it consists of 20 problems on solid mensuration, in illustration of the 5th of the nine sections. The text is said by natives to be very abstruse. An edition of this work was published at Soo-chow in 1803 with all the problems worked out at full length and explained by (38) Chang Tun-jin.

During the Tang dynasty a Buddhist priest named (39) Yih-hing was raised to power in consequence of his mathematical attainments. His works on Astronomy, Arithmetic, Variation of the magnetic needle, and kindred subjects have gained him a lasting reputation. It was probably about or before this time that the Chinese received what little they seem to have got from India. Several of the highest terms in their series of numbers evidently bespeak an Indian origin as (40) *Hang-ho-shaou,* literally "Sands of the Ganges,"=10,000,000,000,000,000,000,000, 000,000,000,000,000,000,000,000,000,000, extending to 53 places of figures, (41) *A-seng-che,* reaching to 57 places, &c.

During the Sung dynasty, which lasted from 950 to 1280, several authors of note appeared, among others (42) Tsin Kew-Chaou about 1240 published the (43) *Soo shoo Kew chang,* "Nine sections of the art of numbers."

Ten years later (44) Yang Hwuy published a work entitled (45) *Tseang Keae Kew chang swan fa,* "Explanation of the arithmetic of the nine sections," besides, (46) *Tseang Keae jih yung swan fa,* "Explanation of arithmetic for daily use," and (47) *Shing choo tung pien pun muh* "Complete treatise on the mutations of multiplication and division." The works of the last mentioned two authors were republished in Shanghae three years since [in 1849].

At th ecommencement of the Yuen dynasty, about 1300 (48) Ko Show-king made considerable improvements on the methods of calculation hitherto adopted. The introduction of spherical trigonometry is ascribed to him, and although his production is now lost, there is still a work published in the Ming dynasty, called

(49) *Hoo she swan shuh,* "Arithmetical rules for arcs and versed-sines," embodying his principles with illustrations.*

During the Yuen dynasty (50) Ley Yay published (51) *Tsih yuen hae King,* "Mirror of the mensuration of circles," in which a species of algebra is applied to the solution of problems in trigonometry. While this dynasty was in power, it is probable that some improvements may have been introduced by the Arabs, as we know that they held much intercourse with China at that time.

It appears however that many of their books on science which they introduced were not translated into the native language, and consequently remained a dead letter after they were bequeathed by their original possessors. Some of these were found in the imperial library on the accession of Hung Woo, the founder of the Ming dynasty. This monarch employed two of his ablest scholars with the aid of Mahomeddan officers to translate them, but the subject was altogether so foreign to their attainments and the terms employed beyond their comprehension, that there is reason to think that the result of their labour would be but an unprofitable production. When they had completed their task, the book was called (52) *Keen yuen seen shing che shoo* "Book of the former sages on the Celestial origin." This book is now unattainable, but there is ground to believe that it was the translation of some works on Algebra.

In the earlier part of the Ming dynasty, arithmetical science seems to have been at a very low ebb, scarcely any work of importance having appeared, although several books were written on the manipulation of the abacus and its application to the various formulæ; so that on the arrival of the Jesuit missionaries in the seventeenth century, their recently perfected theories had the advantage of comparison with the native science at one of the most unfavourable periods of its history.

It is by no means an easy task to arrive at the exact meaning of most of the books here mentioned, for little assistance towards their interpretation can be gained from the ordinary run of teachers; few general principles are given, but they are left to be deduced from a variety of examples, and these for the chief part are stated with most perplexing brevity, besides being burdened with a number of obsolete terms and antiquated allusions; a separate rule is generally given for every problem, but a careful analysis of the various examples will enable the student to gain an insight into the principles they illustrate.

Those who are fond of tracing the origin of numeral characters will probably find in these records a strong corroboration of the digital theory, or that which assigns the number of fingers on the hand, as the foundation of decimal notation. In this respect the ancient Chinese characters approximate to the Roman numerals. It has been erroneously stated by some authors that the Chinese have used the (53) *Swan pan*, or abacus, from time immemorial, whereas its introduction appears to be of but comparatively recent date. In ancient times, calculations were carried on by means of (54) *Show* or tallies made of bamboo, and the written character is evidently a rude representation of these. From 1 to 5 the numbers are represented by the respective number of parallel strokes; from 6 to 9, inclusive, one stroke is drawn to represent 5, and the additional number is represented by so many strokes perpendicular to it, thus, ||| 3, ⊥ 6, |‾| 7.* Having thus obtained a simple but sufficiently effective system of figures, we find the Chinese in the actual use of a method of notation depending on the theory of local value several centuries before such theory was understood in Europe, and while yet the science of numbers had scarcely dawned among the Arabs.

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* It is asserted in Murray's China Vol. 3, page 259, that previous to his time the ratio of the circumference to the diameter of the circle had not been known nearer than 3 to 1. This is a mere figment.

In the Penny Cyclopædia, vol. 2, p. 338, it is said,—"The Chinese have no written method of expressing local value, though their *Swan pan* is a practical use of the principle." Sir John Davis in a paper read before the Asiatic Society in 1823 stated,—"The Chinese possess no real science of their own, and that they obtained none from the Hindus, is, I think, proved by the readiness with which they adopted that of the Europeans." This remark he goes on to support by several arguments, and he has been frequently quoted as an authority by writers on the subject. Again, the same author in his interesting work on the Chinese, vol. 2., p. 283, remarks,—"The Chinese numbers are *written* in words at length, that is, unlike the Arabic system of numeration, where the powers of the numbers increase or diminish decimally according to position," an example from any native work will be a sufficient reply to the above statements. The following is taken at random from the works of Tsin Kew-chaou of the Sung dynasty, and is a fair specimen of the the ancient notation :—

| ≣ 0 ≣ ||||| ≡ T

| ≣ |‾| 0 0 0 0
T × |||| ⊥ ×

in this subtraction sum it will be seen at a glance, that the author had the same view with regard to local value, including the use of the cypher, as that universally adopted by modern civilized nations; the above sum in European figures stands thus,—

$$\begin{array}{r}1,470,000\\ 64,464\\ \hline 1,405,536\end{array}$$

it is taken out of the middle of a complicated problem.

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[*] We find an incidental evidence of the antiquity of these symbols in the 33rd Vol. of the *Tso-chuen*, a historical work written by a disciple of Confuscius five centuries before the Christian era, where it is said—In the 30th year of Seang Kung, an old man being asked his age, replied—I am but a humble individual and unable to compute years; I only know respecting the year that I was born, that it was on the first day of the cycle in the first month, from which to the present time, 445 cycles lacking one third, have elapsed. On hearing this, the enquirer repaired to the palace for a solution; to which the Recorder Chaou replied (55). The character (56) *Hae* has 2 for its head, 6 for its body, the two lower limbs resembling the body, which is the number of days. Sze Wan-pih rejoined. Then his days are 2,666 decades. The antique form of the character will make this clearer If we analyze this figure we find it composed of ═ 2 for the head, ⊥ 6 for the body, while the two lower members are each T 6. Reversing the figure of the symbol does not alter its power.

In the old arithmetical works, no initiatory rules are given corresponding to our Addition and Subtraction; but these are supposed to be understood by the student, before commencing his methodical application. They have indeed the words (57) *Kea fa* addition and (58) *Keen fa* subtraction, but the reader must be careful not to confound these with the notions which the same terms convey to us. The chinese *Kea fa* is a substitute for our multiplication, performed by successive additions to the original sum, while *Keen fa* is a substitute for division; each of these are divided into five or six rules, the main feature of which consists in reducing the first figure of the working factor to unity, and treating the remaining figures as aliquot parts. The English idea of addition is conveyed by the words (59) *Ho*, or (60) *Ping*, "to combine." And here it may be remarked that in the modern use of many of the terms there is an obvious deflection from the ancient meaning of these same terms, the cause of which will be explained by subsequent remarks in this paper.

Their (61) *Shing fa*, Multiplication is the same in theory as ours, but instead of beginning at the right hand as we do, they begin at the left. Multiplication by a single figure is called (62) *Yin*.

Division by a single figure is termed (63) *Kwei*; where the divisor consists of more than one figure, or long division, it is called (64) *Choo*; these terms however do not cover the same extent of meaning as our word Division; they merely apply to division by concrete numbers, which as the word *Choo* implies, is in theory the same as repeated subtraction; division by abstract numbers or where a number is divided into any number of equal parts is expressed by (65) *Yo*. The working factor, whether multiplier or divisor is called (66) *Fa*; the worked factor whether multiplicand or dividend is (67) *Shih*; a single multiplication of the divisor in a division sum is termed (68) *Shang*.

The ancient *Kew-chang* consists of 246 problems in all, distributed under the following nine heads:—

I. (69) *Fang-teen*, Plane mensuration; which contains besides an exposition of multiplication and division, a series of examples with rules attached for measuring fields of various forms, quadrangular, triangular, circular, semi-circular, segment, annular, &c. The rule given for measuring the triangle is to multiply the base by half a perpendicular dropped from the opposite angle, or the converse of this. 6 rules are given for finding the area of the circle:—1. Multiply semi-circumference by radius; 2, Take $\frac{1}{3}$ of square of semi-circumference; 3, Take a twelfth of square of circumference; 4, Take $\frac{1}{4}$ of 3 times the square of diameter; 5, Take $\frac{1}{4}$ of product of diameter and circumference; 6, Take 3 times square of radius. Although the ratio of the circumference to the diameter of the circle is here set down at 3 to 1, native authors tell us that it was not that the compiler was ignorant of a much closer approximation to the true value, but the necessities of the case did not require it; in the (70) *Meih suh* of (71) Tsoo Chung-che, published about the end of the

6th century it is set down at 22 to 7, while Lew Hwuy still earlier, gives it 157 to 50. Two rules are given for segment fields;—1, Add the product of the sine and the versed-sine to the square of the versed-sine and halve the sum; 2, Multiply the sine and versed-sine and halve the amount. Two rules are given for annular fields:—1, Multiply half the sum of the outer and inner circumferences by the diameter; 2, Take a twelfth of the difference between the respective squares of the outer and inner circumferences.

II. (72) *Shuh-poo,* Proportion. This section is chiefly occupied with Rule of three questions, on the comparative value of grain of divers kinds and qualities. The various weights and measures are all regulated by the *Hwang tsung,* one of the musical tubes, which is divided into 90 parts in length, one division forming a *fun* (line), 10 *fun,* 1 *tsun* (inch), 10 *tsun,* 1 *shih* (foot), &c. The *Hwang tsung* contains 1200 grains of rice, and 10 of these tubes full are equal to a *ho*; 10 *ho* are equal to a *shing* (pint), &c. The 1200 grains contained in the *Hwang tsung* are equal in weight to 12 *choo* (penny-weights); 24 penny-weights make a *leang* (ounce); 16 ounces make a *kin* (pound), &c.

III. (73) *Shwae-fun,* Fellowship. This section teaches the division of property among several partners, in unequal shares, such as the relative proportions of 4 to 6, 2 to 8, 3 to 7, &c., in continued progression.

IV. (74) *Shaou-Kwang,* Evolution. The object of this section is to elucidate the extraction of the square and cube roots, which it does in a series of 24 problems, where the principles are applied, not merely to equilateral squares and cubes, but also to parallelograms and parallelopipeds of every possible dimension. The rules are very much the same as those in use at the present day in school-books in England, while the arrangement of the nomenclature certainly does not suffer by comparison with our system; for the various parts of the process taking their names from the geometrical figure, the operator is enabled to see at every step, the particular object of what he is performing. The next lower term to the given power is called (75) *fang* (square); in the cube, the next term is the (76) *leen* (angle or line); while the lowest in each is the (77) *yu* (corner or unit). There is no reference to any higher power than the cube.

V. (78) *Shang-Kung,* Solid mensuration. This section treats principally of the calculation of public works, such as building walls and fortifications, raising turrets and mounds, excavating pits and fosses, the rules being applied to ascertain the cubic measurement of all rectilineal bodies where the surfaces are not at right angles to each other, or where a mean is required between two dimensions, such as the prism, the pyramid, the cone, and their respective frustra, &c., &c.; but the application is not confined merely to substances, or even spaces, for several problems are given on the comparative speed of different kinds of travelling, on foot, on horseback, and by boat; also the comparative qualities of labour, &c.; all worked out on the same fundamental principle.

VI. (79) *Keun-shoo,* Alligation. The main object of this section seems to be the levying of an average impost, according to the amount of land, the number of the population, the distance and the portability of the tribute; it is also applied to the mixture of various articles differing in price, so as to attain a given standard, and to find the respective numbers of several kinds in a given assemblage. The following is one of the examples of the last-mentioned division:—Suppose there are a number of rabbits and pheasants confined in a cage, in all 35 heads and 94 feet: required, the number of each?—Ans., 23 pheasants, 12 rabbits.

VII. (80) *Yin-nuh,* Surplus and deficiency. This consists of a kind of proportion, where the required sum is obtained through the relative ratios of the remainder or deficiency of two series; the questions take the following form:—A number of men bought a number of articles, neither of which are known; it is only known that if each man paid 8 cash, there would be a surplus of 3 cash; and if each man paid 7 cash, there would be a deficiency of 4 cash; required, the respective numbers?—Ans., 7 men, 53 articles.

VII. (81) *Fang-ching,* Equations. This section is remarkable as containing an exposition of the use of the terms (82) *ching* (plus), and (83) *foo* (minus). In a series of 18 problems, it gives the method of ascertaining the value of unknown quantities, from certain conditions of combination, depending on the number of terms in the equation. The following is one of the simplest examples:—If 5 oxen and 2 sheep cost 10 taels of gold, and 2 oxen and 8 sheep cost 8 taels; what are the prices of the oxen and sheep respectively?—Ans., each ox, 1 tael and $\frac{13}{21}$; each sheep, $\frac{20}{21}$ of a tael.

IX. (84) *Kew koo,*[*] Trigonometry; this section is illustrated by 24 problems, some of which are curious in themselves, and a selection is here given, preceded by the corresponding rules, from which, the character of the whole may be judged. Nothing is attempted beyond the right-angled triangle, the capabilities of which however, are tolerably well exhausted. In the following examples, let a represent the base, b the altitude, and c the hypothenuse.[†]

1. Given, the difference of altitude and hypothenuse, and the base, to find the altitude.

$$\left(a^2 - \overline{c - b}^2\right) \div 2\overline{c - b} = b$$

Ex. There is a pool 10 feet square, with a reed growing in the centre, which rises a foot above the surface; when drawn towards the shore, it reaches exactly to the brink of the pool; what is the depth of water? Ans. 12 feet.

2. To find the hypothenuse.

$$\left(a^2 + \overline{c - b}^2\right) \div 2\overline{c - b} = c$$

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[*] It is not pretended that the names here given are a translation of the Chinese characters; they have been adopted from an analysis of the several rules, and although these do not exactly correspond with the rules of the same name in Western treatises, yet as there is generally a near approximation it has been thought more convenient to use them than to transfer the sounds of the characters, or to give a literal translation; as for example these would read:—1, Squaring fields. 2, Rice and money. 3, Declining divisions. 4, Paucity of breadth, &c.

[†] Algebraic symbols are used here for the sake of brevity; it is not meant to imply that these are in the original.

Ex. On opening a two-leaved door, when the inner edge of the leaves are a foot from the door-frame, there is an opening of 2 inches between the leaves; what is the width of the door? Ans. Each leaf, $50\frac{1}{2}$ inches.

3. Another method.

$$(a^2 \div \overline{c - b}) + c - b = 2c$$

Ex. A chain suspended from an upright post has a length of 2 feet lying on the ground, and on being drawn out to its full extent, so as just to touch the ground, the end is found to be 8 feet from the post; what is the length of the chain? Ans. 17 feet.

4. Another method.

$$\left\{\left(\frac{a}{2}\right)^2 \div \frac{c - b}{2}\right\} + \frac{c - b}{2} = c$$

Ex. There is a circular beam of wood in a wall, the dimensions of which are not known, but on inserting a saw, it is found that when it has entered an inch in depth, the saw-cut is a foot in length; what is the diameter of the piece of wood? Ans. 2 feet 6 inches.

5. Given, the sum of the altitude and hypothenuse, and the base, to find the altitude.

$$\frac{b + c - \{a^2 \div \overline{b + c}\}}{2} = b$$

Ex. There is a bamboo 10 feet high, the upper end of which being broken down, on reaching the ground, the tip is just 3 feet from the stem; what is the height to the break? Ans. $4\frac{11}{20}$ feet. This problem is said to be contained also in the Indian mathematical work Brahmegupta (see Journal Asiatique, Mars 1839).

6. Given, the base and altitude, to find the sum and difference of these and the hypothenuse.

$$2ab \div \overline{c + a + b}$$

Ex. What is the diameter of the largest circle that can be inscribed within a right-angled triangle, the two short sides of which are respectively 8 & 15. Ans. 6.

Such is a very superficial outline of this venerable memento of a bygone age. A curious characteristic deserving of notice, is the fact that every section and subdivision commences with a stanza of rhyme, embodying on a general way the rule in question; the meaning is not very apparent on the surface, but the quaintness of the phraseology is calculated to fix them on the memory; and on a minute inspection, it will be seen that they contain in a concise form, the leading ideas which they are intended to convey, very accurately expressed.

In examining the literary productions of the Chinese, one finds considerable difficulty in assigning the precise date for the origin of any mathematical process; for on almost every point, where we consult a native author, we find references to some still earlier work on the subject. The high veneration with which it has been customary for them to look upon the labours of the ancients, has made them more desirous of elucidating the works of their predecessors, than of seeking fame in an untrodden path; so that some of their most important formulæ have reached the state in which we now find them by an almost innumerable series of increments. One of the most remarkable of these is the (85) *Ta-yen,* "Great Extension," a rule for the resolution of indeterminate problems. This rule is met with in embryo in Sun

Tsze's Arithmetical Classic* under the name of (86) *Wuh puh che soo*, "Unknown numerical quantities," where after a general statement in four lines of rhyme, the following question is proposed:—

Given an unknown number, which when divided by 3, leaves a remainder of 2; when divided by 5, it leaves 3; and when divided by 7, it leaves 2; what is the number? Ans. 23.

This is followed by a special rule for working out the problem in terms sufficiently concise and elliptical, to elude the comprehension of the casual reader;—

Dividing by 3 with a remainder of 2, set down 140; dividing by 5 with a remainder of 3, set down 63; dividing by 7 with a remainder of 2, set down 30; adding these sums together gives 233, from which subtract 210, and the remainder is the number required.

A more general note succeeds:—

For 1 obtained by 3, set down 70; for 1 obtained by 5 set down 21; for 1 obtained by 7, set down 15; when the sum is 106 or above, subtract 105 from it, and the remainder is the number required.

In tracing the course of this process, we find it gradually becoming clearer, till towards the end of the Sung dynasty, when the writings of Tsin Keu-chaou put us in full possession of the principle, and enable us to unravel the meaning of the above mysterious assemblage of numerals. Applying the principles of the *Ta-yen* as there laid down:—Multiplying together the three divisors 3, 5 and 7, gives 105 for the (87) *Yen-moo*, "Extension parent." Divide this by the (88) *Ting-moo*, "Fixed parent" 7, the quotient 15 is the (89) *Yen-soo*, "Extension number." Divide this again by 7, and there is an overplus of 1, which is the (90) *Ching-suh*, "Multiplying term;" by which multiply the Extension number 15, and the product 15 is the (91) *Yung-soo*, "Use number," or as is given above,—for 1 obtained by 7 set down 15. Divide the Extension parent 105 by the Fixed parent 5, and the quotient 21 is the Extension number. Divide this again by 5, and the overplus 1 is the multiplying term. Multiply the Extension number 21 by this, and the product 21 is the Use number; which is given above,—for 1 obtained by 5 set down 21. Divide the Extension parent 105 by the Fixed parent 3, and the quotient 35 is the Extension number. Divide this again by 3, and there is a (92) *Ke*, "Remainder" of 2. This Remainder, being more than unity, is then submitted to a subsidiary process, termed (93) *Keu Yih*, "Finding unity" which is the alternate division of the Extension parent and Remainder by each other, till the remainder is reduced to one; the result in the present instance is 2 which is the Multplying term; by which multiply the Extension number, and the product 70 is the Use number; which is the meaning of the sentence,—for 1 obtained by 3 set down 70.

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* Native writers are divided in opinion as to the time when Sun Tsze lived; some consider him the same as Sun Woo-tsze, a military officer during the Heptarchy about B.C. 220. The more probable opinion however is that he lived towards the end of the Han or during the Wei dynasty in the third century of the Christian era.

Having thus obtained the several Use numbers, multiply the corresponding original remainders by these :—

$$70 \times 2 = 140;\ 21 \times 3 = 63;\ 15 \times 2 = 30,$$

add these three numbers together as stated in the rule and the sum is 233; from which subtract as many times the parent number 105 as it will admit, which making 210, the remainder is 23, the number required.

The general principles of the *Ta-yen* are probably given in their simplest form in the above rudimentary problem of Sun Tsze. Subsequent authors, enlarging on the idea, applied it with much effect to that complex system of cycles and epicycles which form such a prominent feature in the middle-age astronomy of the Chinese. The reputed originator of this theory as applied to astronomy is the priest Yih Hing who had scarcely finished the rough draft of his work (94) *Ta-yen leih shoo*, when he died A.D. 717.

But it is in the " Nine sections of the art of numbers " by Tsin Keu chaou that we have the most full and explicit details on this subject. Here we have the various applications of this theory worked out at great length; the first problem being to find a solution of a passage in the *Yih King* treating of the origin of the divining numbers :—

QU. In the *Yih King* it is said,—'The Great Extension number is 50, and the Use number is 49.' Again it is said,—'It is divided into 2, to represent the 2 spheres; 1 is supposed to represent the 3 powers; they are drawn out by 4, to represent the 4 seasons; three changes complete a symbol, and eighteen changes perfect a diagram.' What is the rule for the Extension and what are the several numbers?

The 4 cardinal numbers, 1 the senior male, 2 senior female, 3 the junior male, and 4 the junior female, are used to start with as the Origin numbers. In a second row, opposite each number respectively, is placed the product of the other three.

Origin numbers, ...	1	2	3	4.
Extension numbers, . . .	24	12	8	6.

The sum of these lower numbers gives 50, the Great Extension number alluded to in the text of the *Yih King*: 50 being an even number, and consequently unsuitable for the Use number, the Origin numbers are severally taken in pairs and common divisions sought; where this is obtained, the number which gives an odd quotient is divided by it, but the other is not. The result of this last operation gives the Fixed parent numbers, and the Extension numbers are obtained in the same manner as before.

Fixed parents, ...	1	1	3	4.
Extension numbers, . . .	12	12	4	3.

Subtract from the respective Extension numbers, as many of the corresponding Fixed parents as they will admit, so as always to leave some remainder.

Fixed parents, ...	1	1	3	4.
Remainders, ...	1	1	1	3.

The three Remainders 1 become the respective multiplying terms, and submitting the last remainder 3 to the process for Finding unity, the multplying term 3 is obtained. By these four terms multiply the respective Extension numbers, and the Expansion use numbers are obtained.

Fixed parents, ...	1	1	3	4.
Expansion use, ...	12	12	4	9.

The second Origin number 2 having been reduced to 1 at the beginning, once the Extension parent 12 is consequently added to the second Expansion number, and the Expansion numbers then become the Fixed use numbers, and stand as follows :—

Origin numbers,	1	2	3	4.
Fixed use numbers,	12	24	4	9.

The four upper numbers here are those employed in drawing the divining straws. The sum of the four lower numbers is 49, which is denominated the Use number in the text of the *Yih King*, being the number of the straws being divided at random between the two hands, one hand containing an even number, the other must contain odd. In the present instance, the left hand is supposed to hold the odd number 33, though unknown by the enquirer. In drawing them out by one at a time, as it is known there must be a remainder of 1, the number 1 is suspended at first, as it is said in the text, 1 is suspended to represent the 3 powers. The straws are then drawn in so many successive operations by 2, 3, and 4 at a time, which give respectively the remainders 1, 3 and 1, and these are put down in three parcels, making the suspended 1, the first parcel, we have then the following numbers :—

Use numbers,	12	24	4	9.
Parcels,	1	1	3	1.

Multiply these parcels by the respective Use numbers and the Full numbers are obtained.

Full numbers,	12	24	12	9.

Adding together these 4 numbers gives 57, from which subtract the Extension parent 12 as many times as it will admit and there is a remainder of 9, which make the dividend, and divide it by 3 the Extension measure, when the quotient 3 is the junior male, and represents the single stroke symbol.

In this manner by changing the odd number in the left hand different symbols might be obtained. 1, representing two whole strokes ; 2, representing one broken stroke ; 3, one whole stroke ; 4, one whole and one broken stroke. Such was the means employed in former times for diving into futurity ; the various diagrams of *Fo-he* being thus obtained were supposed to set forth the destiny of enquirers. Some knowledge of the *Yih King* is necessary to enable one to understand these diagrams which are evidently the relics of a very ancient system of divination.

The 2nd problem is to find the epoch of conjunction of the terms of concurrent cycles of time. The question is given as follows :—

Let the solar year be equal to 365$\frac{1}{4}$ days, the moon's revolution, 29$\frac{499}{940}$ days, and the *Kea-tsze* 60 days. Suppose in the year A.D. 1246, the 53rd day of the *Kea-tsze* or sexagenary cycle of days is the 1st of the 11th month ; the 57th day of the *Kea-tsze* is the Winter solstice or 1st day of the solar year ; and the 1st day of the *Kea-tsze* is the 9th day of the month. Required the time between two conjunctions of the commencement of these three cycles ; also, the time that has already elapsed and how much has yet to run. Ans. The time between two conjunctions, 18,240 years : 225,600 months : 6,662,160 days : number of years already past 9,163 ; number of years unexpired 9,077.

The 3rd problem relates to the calculation of labour ; 4 companies of men varying in number being engaged in raising an embankment ; each company has an equal portion of work allotted to it ; the amount of the work is not stated ; but the force of each company and the quantity left unfinished by each company after the last full day's work is given, from which it is required to find the gross amount of the labour performed.

The 4th problem supposes 7 treasuries, each possessing an equal sum of money originally, but being continuously reduced by daily drafts differing in amount, on the respective funds; the original stock and the number of days which the exhausting process has gone in each, are both unknown; from the known daily drafts and the remainders, it is required to find the original sums.

The 5th problem gives 3 agriculturists, having each an equal quantity of grain to dispose of; which being sold at 3 different places according to different measures, the respective surplus quantities over the full measures being given, it is required to find the whole amount.

The 6th problem supposes 3 companies of troops proceeding to the capital, the daily rate of each, and the hours of the day at which they arrive being given, to find the original distance.

The 7th problem refers to couriers of different speed, the daily rate of travelling being given, to find what would be the first place on the road, where both would halt for the night.

The 8th problem is on erecting the basement for a building, with one of 4 different kinds of bricks, which the builder is at liberty to select at will; the dimensions of the several bricks are given, with the surplus or deficiency in the gross dimensions, from which it is required to find the dimensions of the erection.

The 9th problem is as follows :—

> A report being raised that 3 rice bins, each containing the same amount, have been robbed, the original quantity is not known, but it is found that in the left hand one, there is 1 *ho* left; in the middle one there is 1 *shing* 4 *ho* left; in the right hand one, there is 1 *ho* remaining; the thieves being caught, A confesses that he took a horse-ladle at night and filled it several times out of the left hand bin, putting the contents in a bag; B confesses having hastily taken a wooden shoe several times full out of the middle bin; C says he took a bowl and filled it successively out of the right hand bin. Examining the three vessels, the horse-ladle is found to contain 1 *shing* 9 *ho*; the wooden shoe 1 *shing* 7 *ho*, and the bowl 1 *shing* 2 *ho*. What is the amount of rice lost, and how much did each take? Ans. Lost 9 *shih* 5 *tow* 6 *shing* 3 *ho*. Stolen by A, 3 *shih* 1 *tow* 9 *shing* 2 *ho*; B, 3 *shih* 1 *tow* 7 *shing* 9 *ho*; C, 3 *shih* 1 *tow* 9 *shing* 2 *ho*.

The second division of Tsin's work is on the calculation of astronomical terms which are also worked out by the *Ta-yen*. The 1st problem is how to calculate the (95) *Ke*, "time from the Winter Solstice to the end of the *Kea-tsze*," for any given year. The 2nd is to calculate the (96) *Jun*, "time to the Winter Solstice from any previous point in the 11th month," for any year. The 3rd is on the computation of the various terms (23 in number) requisite for adjusting the inequalities of the annual, lunar, and diurnal revolutions, reduced from a period of 7,848,183 years. The 4th is on calculating the inequalities of the apparent motions of the planets. The 5th is on the comparative length of the shadow on the sun-dial. The remaining four questions of this division, on measuring the fall of rain, and the fall of snow. do not properly belong to the *Ta-yen*.

At a time when astronomical instruments were far from having attained the precision of the present day, and observations were at best but comparatively rude, it will be seen that the above calculus was one of considerable importance, as enabling the astronomers, by a kind of dead reckoning to fix the length of the year and the various astronomical periods.

This appears to be the formula, or something very like it, which was known to the Hindoos under the name of the *Cuttaca,* or as it is translated "Pulveriser,"* implying unlimited multiplication, which is not far from the meaning of the *Ta-yen* or "Great Extension." The process with its results, as found in the mathematical works of India, has engaged much attention in Europe, and given rise to some singular conjectures as to the antiquity of the Science.

Considering the long period that the study of mathematics had been cultivated among the Chinese, one cannot feel surprised at their having attained a respectable proficiency in the use of numbers, while yet the science had scarcely obtained a footing in England; nor would it be unreasonable to expect a much higher stage of development than actual evidence goes to prove. From the days of Chow King downwards, every age had furnished its quota of professors, each for the time looked up to by their respective disciples as master of the science *par excellence;* but the extent of the various improvements on which they severally established their fame, merely amounted to matters of detail, while all paid homage to the *Kew chang,* in endeavouring to throw new light on its time-honoured formulæ, or as they would say, to restore it in some degree to its pristine lustre.

The close of the thirteenth century, however, brings to our notice an innovation on the long-prevalent monotony. At this period, we find a grand step made toward the theory of analysis, in the publication of the (97) *Leih tien yuen yih,* "Setting up the celestial monad," a new branch of Arithmetic, which may, with propriety, be termed the Chinese Algebra; as indeed the identity has been clearly seen and stated by native authors several centuries later, when the European Algebra was introduced by the Roman Catholic missionaries. It is somewhat remarkable that we find this branch of Science introduced almost contemporaneously by three different authors, living far apart, under different monarchies, and apparently unknown to each other. In their works there is not the slightest reference to each other, and it is to be regretted that we have not some explicit detail of the circumstances which gave rise to their several productions. Each has treated the subject in a different manner, and seems to have concentrated his force on a part which is less dwelt on by the others. The first which appeared before the public was the work of Tsin Kew-chaou, alluded to above, in which the Monad, as a symbol of an unknown quantity, is employed in the *Ta-yen;* but the illustration of its advantages there is so feeble, that were the name omitted, one might easily pass it by without observing a new germ in the calculus. The capabilities of the element however expand as he approaches the subject of

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* See Edinburgh Review, November 1817: Art. Arithmetic and Algebra of the Hindoos.

Evolution, and we find him resolving equations of the 6th, 7th, 8th degrees and upwards, with the greatest freedom and simplicity imaginable. Passing over that for the present, we shall find in the work of the next author, Leyay Jin-King, some elementary details better calculated to elucidate the theory. In the preface to the "Mirror of the mensuration of circles" he tells us that from his youth, he had given himself to the study of the (98) *Tung yuen Kew yung;* but as modern native authors seem quite at a loss to know what that can be, the information is of little practical value.

In the *Tien yuen Yih,* unity is employed as the representative of an unknown number; this being combined with an extension of the theory of local value, in order to represent the successive powers of the Monad or unknown number. The Monad has the character (99) *Yuen,* written by the side of it, to distinguish its place in the column. Immediately below the Monad is the place for the natural number, which is marked by the character (100) *Tae,* implying that it is the place of the (101) *Tae Keih,* or extreme limit. Immediately above the Monad is the square; next above is the cube; again above, the fourth power, and so on upwards ad infinitum. Thus the equation $x^3 + 15x^2 + 66x - 360 = 0$ would be written—

| 1 cube of Monad.
| 𝍭 15 square of Monad.
⊤ ⊥ (99) 66 Monad.
||| ⊤ 0 (100) — 360 natural number.

In the above it is seen that the cube, square, &c., are all indicated by their relative position with respect to *Tae,* the characters being used only to express the co-efficients. In practice, the character *Yuen* is omitted when *Tae* is used; or if *Yuen* be used, *Tae* is omitted. It is not a little remarkable, that while it has been gravely asserted by most respectable authorities in Europe, that the Chinese are ignorant of the meaning of local value, we find here on the contrary, that they have pushed the principle to a degree of refinement unpractised in the West. It may be noticed too that instead of the old form of writing the equation used in Europe, $x^3 + 15x^2 + 66x = 360$, the method invented by Hariot, of placing all the significant terms on one side, is precisely that used by the Chinese some five centuries earlier, and although in itself but a variation in algebraical language, yet it is said by De Morgan to have been the foundation of most important branches of the science.* In Tsin's original work, positive and negative numbers are distinguished by the former being in red ink, and the latter in black; and this custom seems to have been in use long before his time, for we find Lew Hwuy referring to it in the sixth century. It is said to represent the bamboo tally numerals used in ancient times. Le-yay was the first to introduce the diagonal stroke drawn through the right-hand figure, as above, which has been used ever since. The principle of the *Tien yuen* as given by these writers consists in expressing a compound numerical quantity called the (102) *Ke tso*, containing the unknown number involved to any degree, which term shall be equal to a given natural number, called the (103) *Tung soo* or (104) *Yew soo.* These two quantities being amalgamated, the resolution of the equation brings out the value of the monad or unknown number. Multiplication by the monad is accomplished by moving all the terms up one stage in the column; if by the square of the monad, they are moved up two stages; for the cube three stages and so on. If any of the places from *Tae* upwards happen to be left blank, they are supplied by cyphers. Of course, division is the inverse operation to the above, every term in the column being moved down a stage, two stages, three stages, &c., according to the power of the divisor; the place next below *Tae* is the square root of the monad; next is the cube root; biquadrate root, &c., all follow in succession.* After the amalgamation of the two equivalents above mentioned, the designations which the various places in the column had retained during the operation, are all effaced, and the order of the series inverted, the lowest term in the column being moved to the top and vice versa; the upper term then becomes the (105) *Shih* or sum of all the others; if there is only one more term, it is called (106) *Fa*, the divisor; if there are 2 more, the second is called (107) *Fang* or (108) *Tsung*, and the lower (109) *Yu*; for any greater number of terms, the *Yu* always remains at the bottom, and the additional terms are placed between it and the *Fang*, with the name of (110) *Leen*, according to their number beginning from *Fang*, as 1st leen, 2nd leen, &c.

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* "On the study and difficulties of Mathematics" by Professor De Morgan, p. 12.

Having the various terms thus arranged, the unknown number is then released, by a simple process called by Tsin (111) *Ling lung Kae fang*, "Harmoniously alternating Evolution," the knowledge of which in Europe is of very recent date. One example taken from a great number in Tsin's work will best illustrate this. It is required to extract the root of $-x_4 + 1534464x^2 - 526727577600$, or to find the value of x.

720	(112) *Shang.* Value of x, the Monad.
– 526727577600	(113) *Shih.*
– 14940217600	
14940217600	
0	(114) *Fang.*
731124800	
776249600	
747010880	
1534464	(115) *Shang-leen.*
1044464	
64464	
– 1405536	
– 1461936	

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* For the sake of simplicity, 1 is assumed as the co-efficicnt of these several powers, but it is obvious the principle is the same, any other nnmbers be used.

0 (116) *Hea-leen.*

− 700

− 1400

− 2100

− 2800

− 2820

− 1 (117) *Yu.*

− 1

In the above example, the several terms are set down with the Chinese names *Shih, Fang,* &c., against them, the units in each being 2 places more to the left than in the term above it, because there is found to be 3 figures in the root. The first figure of the root being 7, is set down at the top. The *Yu* − 1 is then multiplied by this 7, and the product added, to the *Hea leen* 0, regard being had to the place in the column, which gives − 700; the *Hea leen* − 700 is then multiplied by 7, the product being added to the *Shang leen* 1534464, still preserving the same place in the column, gives 1044464; again this is multiplied by 7, and the product added to the *Fang* 0, gives 731124800; this multiplied by 7 and the product combined with the *Shih,* leaves − 14940217600, &c. The second figure in the root being found to be 2, the same process is repeated, moving all the terms to the right 1, 2, 3, &c., places.

In this operation, the reader will at once detect the famous theorem of Horner, "for solving Equations of all orders," first published in 1819; of which De Morgan says, that it is a "method which must place its author among those valuable inventors who find out simple adaptations which have been overlooked by their predecessors,"—from such a source, no mean commendation. It appears some have thought proper to dispute the right of Horner to the invention, and it will perhaps be an unexpected occurrence to our European friends to find a third competitor coming forward from the Celestial Empire, with a very fair chance of being able to establish his claim to priority. However this cannot detract in any degree from the merit of Horner, whose right is doubtless well grounded. In the above example, the numbers which are added to the several terms are not set down, but merely the successive results with a line between; in the original, where a negative number is combined with a positive, or a positive with a negative, these are both set down; but as the rationalé of the process will be quite clear to any one who is familiar with Horner's method, the extra numbers are omitted here, to avoid prolixity. The original is accompanied with a very minute letterpress description of every step in the process.

We give for the sake of comparison, the same sum worked out according to Horner's directions.

– 1	0	1534464	0	526727577600(720
	– 700	– 490000	731124800	511787360000
	– 700	———	45124800	———
	———	1044464	———	14940217600
	– 1400	– 980000	776249600	14940217600
	– 700	———	– 29238720	———
	———	64464	———	
	– 2100	– 1470000	747010880	
	– 700	———		
	———	– 1405536 ;		
	– 2800 ;	– 56400		
	– 20	———		
	———	– 1461936		
	– 2820			

Although Tsin is the first author in which we find this rule fully developed with regard to compound equations, yet it should be observed that Yang Hwuy, a contemporary writer, in his analysis of the *Kew chang,* gives precisely similar rules, though unaccompanied with examples, for the extraction of the square and cube roots called (118) *Tsang ching fang,* "Accumulating Involution," these are quoted from one (119) *Kea Heen.*

At the beginning of the work of (120) Choo She-kee, the third author above alluded to, the following table of the "Ratio of the *Leens* in the involution of numbers up to the 8th power," is given; not as anything new however, for it is denominated the "Ancient method;" but at what time it was first discovered we have no sufficient data to determine :—

1	Original sum.
1 1	Factors.
1 2 1	Square.
1 3 3 1	Cube.
1 4 6 4 1	Biquadrate.
1 5 10 10 5 1	5th power.
1 6 15 20 15 6 1	6th power.
1 7 21 35 35 21 7 1	7th power.
1 8 28 56 70 56 28 8 1	8th power.

A slight examination will enable any one to understand the mechanical structure of this table, which it is clear might be carried to an unlimited extent. Having already attained such a dexterity in dealing with the higher powers, the transition from this to Horner's method was an easy and almost natural consequence.

Choo She-kee published his (121) *Sze Yuen Yuh Keen,* "Precious mirror of the four regions," about the year 1303; in which he makes a decided step in advance of his contemporaries, by using the Monad for known as well as unknown quantities. For several Monads are used, which are termed respectively, Heaven, Earth, Man and Thing. The three first are generally (not necessarily) employed to designate known quantities, and the last (122) *Wuh*, "Thing,"* for the unknown quantity. Here again we must yield precedence to the Chinese, for until the time of Vieta, who lived in the 16th century, arbitrary representations of numbers were never used in Europe for anything but quantities sought. The *Tien Yuen* or Heaven is written below *Tae*, the powers increasing as they descend; Earth is written on the left side of *Tae*, the powers increasing as they recede from *Tae* in a lateral direction; Man is written on the right side, the powers increasing towards the right; and Thing is written above, the powers increasing as they ascend.

Thus the four simple Monads or $a + b + c + x$ are written in the following form:—

$$\begin{array}{ccc} & 1 & \\ 1 & (100) & 1 \\ & 1 & \end{array}$$

The square of the above, or $a^2 + 2ab + 2ac + 2ax + b^2 + 2bc + 2bx + c + 2cx + x^2$, is as follows:—

$$\begin{array}{ccccccc} & & & 1 & & & \\ & 2 & & 0 & & 2 & \\ & & & & 2 & & \\ 1 & 0 & & (100) & & 0 & 1 \\ & & 2 & & & & \\ & 2 & & 0 & & 2 & \\ & & & 1 & & & \end{array}$$

A very little practice is sufficient to render any one perfectly familiar with the several terms of this algorithm. The left being the side for the Earth Monad or b, the 1 removed two stages from Tae is b^2; in the next column, the upper 2 standing opposite the place for the Thing Monad, signifies twice the product of Earth and Thing or $2bx$; the lower two standing opposite the place for the Heaven Monad, signifies twice the product of Heaven and Earth, or $2ab$; the 2 standing diagonally against *Tae* indicates twice the product of Earth and Man, the two Monads in the same horizontal line, or $2bc$; the other diagonal 2 above *Tae* is twice the product of Heaven and Thing, or $2ax$, &c. The operations connected with this formula are as simple as in the *Tien Yuen*, except it be with regard to multiplication, and even in that, the apparent intricacy is more imaginary than real, the several columns being taken separately and the products added together afterwards. The process of amalgamation and reduction is carried on till all the columns are gradually reduced to one, when the root is extracted the same as before.

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* "The Persians and Arabs employed the word answering to "Thing" in their language for the unknown quantity, and the Italians adopted the word 'Cosa' for the same purpose: hence Algebra came to be called the Regola de la Cosa, in Italy, and the Cossike Art in England." Penny Cyclopædia, Vol. I., p. 325. The coincidence here is curious.

Notwithstanding the advantage presented by the *Tien Yuen,* it does not seem to have been cultivated to any great extent by subsequent authors during the Yuen and Ming dynasties. We find it employed by Ko Show-King in the construction of his new system of astronomy, the (123) *Show she leih,* which was used throughout the Yuen dynasty. In the middle of the 16th century (124) Tung shun-Che revised Le Yay's "Mirror of the Mensuration of Circles," and added an elucidation in detail of the various problems; but this afterwards fell into the hands of (125) Koo Yiu-tseang, a self-taught mathematician, who being ignorant of the principles of *Tien Yuen* cast aside Tung Shung-che's addenda and published the original text with a series of rules in which the true meaning of Le's work is very much lost sight of. Ko Show-king's work on "Arcs and versed-sines" was also republished by this author, the principles of *Tien Yuen* being alike misunderstood and disregarded. After this time we do not hear of any one during the Ming dynasty giving attention to the subject; and the knowledge of it is said to have been preserved by a "mere thread" to subsequent times.

In the reign of Kang-he, towards the end of the 17th century, a treatise on algebra (126) *Tseay kan fang,* as it was then known in Europe was composed and laid before the emperor by some of the foreigners engaged in the Astronomical Board. The name of the author has not been preserved in connection with his work, but it is clear he could have had no knowledge of the *Tien Yuen.* We find this document made use of in the *Leuh leih Yuen Yuen,* which was prepared by imperial authority during the same reign, but as it is there presented, it does not show any advantage superior to the *Tien Yuen,* and had the imperial counsellors been familiar with the latter, it is probable it would not have been omitted in the work. A strange mistake has arisen in connection with this branch of the science, probably arising from some faulty wording in the original memorial; for the books that take notice of it, say that "in Europe it is known by the name of (127) *A-urh-je-pa-la,* 'Algebra,' which word being translated means, 'Rule derived from the east,'" and eking out this misconception in true Chinese style, they say "the people of the West thus acknowledge that it is of Chinese origin." That the compilers of the algebra in the work just mentioned were ignorant of the *Tien Yuen,* there is a strong reason to believe from the fact that the technical terms employed are all different. Thus instead of Tien Yuen Yih, they say (128) *Yih kan;* instead of Ching and Fow, they say (129) *To* and (130) *Shaou;* for *Tung soo,* they use (131) *Teng soo,* &c. In extracting the roots of equations, the former has clearly the superiority.

No arbitrary symbols are used in the *Tseay kan fang,* but the designation of the several powers are set down against the respective co-efficients; this however is not absolutely necessary, for the terms may be indicated by position the same as in *Tien Yuen;* they are written horizontally instead of being in a perpendicular column.

In the beginning of the 17th century, the study of the *Tien Yuen* was revived by Mei Wuh-gan in a work called the (132) *Chih Shwuy e chin,* "Pearls dropped in the Red river,"* in which he institutes a comparison between that and the *Tseay kan fang* which had been introduced by the Europeans. Since that time the subject has engaged much of the attention of students, among whom the most famous who have written, may be mentioned (133) Le Juy and Chang Tun-jin, both of Soochow at the end of last century; the former of whom in his (134) *E shoo,* "Posthumous works," has left some valuable details on the science. Among other subjects he treats largely on the number and ratio of roots real and imaginary to any equation, and the method of finding the same; the latter has published a number of the most important ancient works on arithmetic, illustrating the various formulæ at great length by the *Tien Yuen.* A later writer (135) Chang Tso-nan in a mathematical compendium styled (136) *Tsuy wei shan fang swan heo,* "Mathematics of the Lagerstræmia hill institution," annexed to the division on Plane and Solid Mensuration, gives a supplementary section, on the art of using the *Tien Yuen,* but there is little original matter in his work. In the same work, 4th volume on the Mensuration of Solids treats of the *Tseay kan fang;* this last form however seems to have received but little favour from native mathematicans.

An edition of the "Precious mirror of the four regions" has been recently published at Yang-chow with minute illustration of the whole text, and three volumes of rules by (137) Lo Ming-heang, in which he enters into an elaborate and interesting detail of the fundamental principles of the science. This author has likewise published a number of smaller works, showing the application of *Tien Yuen* to the trigonometrical canon, astronomy, &c. These last and many other works of a kindred character have been published at the expense of Yuen Yuen, who is well known to foreigners by his formerly having been Governor of Canton, and to natives by his munificent patronage of the arts and sciences. The exact sciences had evidently been in a declining state from the commencement of the Ming dynasty, down to the time of the arrival of the Romish missionaries. The consequences of this neglect were beginning to be felt in the confusion that was taking place in the calculations of the Astronomical Board. Mathew Ricci, the first who arrived at Peking, soon turned his attention to this subject, and translated a treatise on European arithmetic as a preliminary step which was afterwards revised and published by (138) Le che-tsaou, a Christian Mandarin, under the name of (139) *Tung wan*† *swan che,* "Guide to Arithmetic in the common character;" this comprehended the common rules as they then stood in Europe beginning with notation and reaching to the Rule of Three. Schaal and coadjutors by their labours did much to introduce the knowledge of European mathematics, and published some works of great importance on the subject. The compilation of the *Leuh leih yuen yuen* in the reign and under the special supervision of Kang-he, was an undertaking worthy of the monarch who conceived the plan. In it, various desultory efforts of preceding writers were wrought into a harmonious whole, while their works were thus placed on such a standing as would ensure their most extensive usefulness. As the publication of this work may be looked upon as an era in the history of the science, it will not be amiss to give an outline of the contents of the 3rd division of it, called the (141) *Soo le tsing wan,* "Repository of the subtleties of the principle of numbers," which treats entirely of the abstract sciences and which is at the present day the authorized guide for the calculations used in the Astronomical Board. It is divided into two parts; the first comprehending 5 volumes treating on the theory of quantity. The 1st volume commences with an enquiry into the origin of the principle of numbers, in which, true to the Chinese philosophy it conducts us quietly up to the time when the ancient sage Fo-he saw the dragon horse emerge from the Yellow river with the scheme of decimal notation portrayed on his back. This chart is then given and also another held equally sacred, which the great Yu is said to have observed on the back of a tortoise which rose out of the river Lo, containing the elements of multiplication and division. The first section of the *Chow-pe* with a commentary, completes the volume. The next three volumes contain a treatise on geometry in 12 books, on a much more diffuse plan than that of Euclid, and consequently deficient in the demonstrations of that author; it embraces planes and solids of every form, and the last book being on proportion, gives various mechanical devices for the construction of figures, including the use of the wooden square, the parallel rule, the diagonal scale, the protractor, and the sector, with methods for reducing plans, maps and drawings. The last volume contains what may be called palpable arithmetic, entering scientifically on the theory of calculation, which it illustrates by means of counters, and also by geometrical figures. The 2nd part treats on the practical application of the various branches of arithmetic, in 40 volumes, and is divided into 5 sections. The first or introductory section in two volumes includes Tables of Weights and Measures, Notation, Addition, Subtraction, Multiplication, Division and Fractions. The second section in 8 volumes on Lines, embraces Proportion, Alligation, Progression, Fellowship, Surplus and Deficiency, Position, and Equations. The third section in 12 volumes on surfaces, contains square root, Trigonometry, ancient and modern, the use of the 8 lines of the Trigonometrical canon, method for finding the several dimensions of the triangle from each other, Geometrical mensuration, measurement of figures of various forms, rectilinear and curvilinear, Ratio of various figures contained in the circle, and in which the circle is contained, and Relative ratios of regular polygons. The fourth section in 8 volumes on solids, treats on Cube Root, measurement of bodies of various forms, rectilinear and curvilinear, Ratio of various bodies contained in a sphere, and in which a sphere is contained, Relative ratios of regular polyhedrons, Relative weight of various substances, animal, vegetable and mineral, and Piling. The concluding section in ten volumes treats on Algebra, miscellaneous problems not embraced in the preceding rules, Logarithms, and two volumes in explanation of the use of the Sector. Besides the above there are eight supplementary volumes of tables, two containing the sines, cosines, tangents, co-tangents, secants and cosecants, for every 10 seconds throughout 90 degrees. The two next give a table of the divisors of numbers from 1 up to 100,000 to facilitate the computation of logarithms. At the end of every 10th thousand, a list of the prime numbers is also given. The two next volumes contain a table of the logarithms of natural numbers from 1 to 100,000 calculated to 10 places of figures. This is apparently a transcript of the table that was published in Holland by Adrian Vlacq in 1628. At the end is a catalogue of the logarithms of the ratios of a great number of mathematical terms, and the relative gravity of various substances. The last two volumes have a table of the logarithms of sines, cosines, tangents, co-tangents, secants and cosecants for every 10 seconds from zero up to 90 degrees. The very familiar phraseology in which this book is written no doubt laid open the science to the capacity of many who had hitherto been deterred from the study, by reason of the peculiarly rigid and to some extent quaint character of the ancient books. While there is every possible guarantee for the purity of the style and the absence of barbarisms, it embodies the substance of all that was really useful in the previous writings of Europeans. The illustrations and explanations to every rule are profuse almost to a fault; the principle of every operation being laid down with great minuteness, at the end of the problem. Perhaps the chief objection that this exposes it to, is that it swells out the work to such an extent as to place it beyond the reach of any but a very limited number. Since the publication of the above, a great number of works on the same subject have been constantly issuing from the press, all more or less characterized by conformity to this.

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* This singular title bears allusion to a tradition respecting the ancient sage Hwang-te, who, it is said, while out on a pleasure excursion on the Red river, in the neighbourhood of the Kwan-lun mountains, accidentally dropped some valuable pearls in the water, where they lay for a long time, but were ultimately regained.

† This title has reference to the (140) *Chung Yung,* chap. 28, sect. 3.

About the year 1633, the theory of Napier's Roads* was introduced in a work presented to the emperor by James Rho, an Italian missionary, then in the Astronomical Board. This theory has been reprinted in several native works under the name of (142) *Show swan*, "Tally calculation," in distinction from (143) *Pih swan*, "Pencil calculation," while the use of the abacus is termed (144) *Choo swan*, "Pearl or Ball calculation." Some books also give (145) *Chih swan*, "Foot or sector calculation," and there is an ancient method called (146) *Chang kin swan*, "Finger calculation."

Although it has been a predominant feeling with most native writers to trace everything of a scientific character to a remote native origin, yet they seem never to have questioned the right of John Napier to the invention of Logarithms; nor have they on that account undervalued the discovery; and even at the present day we find some applying themselves with a great zeal to the subject, ignorant of the advances that have been made in the West, since the days that it was first brought before the Chinese. There is one (147) Le Shen-len, a relative of Le Juy above mentioned, now (1852) residing in Shanghae, who has recently published a small work called (148) *Tuy soo tan yuen*, " Discovery of the source of Logarithms," in which he details an entirely new method for their computation, based upon geometrical formulæ which he says in his introduction is " ten thousand times easier than the methods used by Europeans," and that "although they can just calculate the numbers, yet they are ignorant of the principle." This small indication of self satisfaction may be very well overlooked, as quite pardonable in one who has had no better aid than that afforded by the *Leuh leih yuen yuen*, and who has here given us, as the result of four years' thought, a theorem, which in the days of Briggs and Napier, would have been sufficient to raise him to distinction.

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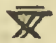
* First published in Napier's "Rabdologiæ, sive Numerationes per Virgulas," in 1617.

A mandarin at Hang-chow, named (149) Tae Hew has also been engaged for some time past on a new method of obtaining logarithms. It is well known that there are at present many of a similar turn of mind, in different parts of the empire; and it is surely interesting to find these signs of intellectual activity, striving against the difficulties with which they have to contend, and standing out in bold relief from the mass of mental torpor, so characteristically congenial to the institutions of this truly conservative empire. It is true the Celestials are disposed to look with a feeling akin to contempt on the mushroom antiquity of our Western lore; yet it is equally true that a spirit of inquiry still germinates among them, which if fostered by a greater freedom of intercourse, will doubtless tend much to smooth the asperities which now exist, and thus prove mutually advantageous. And may we not hope that the same spirit of inquiry which would lead them to seek after truth in science, would also, if rightly directed, teach and prepare them to investigate it in much higher and more important spheres of knowledge:—ever to understand themselves, and the relation they sustain to that great Being, whose name they have preserved,—whose knowledge they have lost.

CORRECTION.

In the footnote on p. 176 the sign  was accidentally omitted.

BEOBACHTUNGEN VON SONNENFLECKEN AM ASTROPHYSICALISCHEN OBSERVATORIUM ZU O'GYALLA.

Gruppe.	M. Z. O'Gyalla (bürg.)	p	ρ	l	L	b	
	1882.				Rotationsperiode 287.		
65	April 26·4857	294°·64	402″·4	247°·89	343°·69	+ 15°·50	Kleiner Fleck.
64	April 22·4888	54·51	820·0	169·23	321·05	+ 6·29	Kleiner sich auflösender Fleck.
	„ 23·4819	51·51	692·8	182·60	321·24	+ 6·37	
	April 22·9854				321·15	+ 6·33	
66	April 30·4852	313·54	408·6	244·99	283·74	+ 19·37	Schwacher Fleck.
	Mai 2·4885	278·20	678·4	277·06	287·42	+ 19·26	
	„ 3·4843	272·71	790·8	290·50	286·46	+ 19·35	
67	April 30·4852	41·82	721·1	189·31	228·06	+ 15·16	Behofter sich auflösender Fleck.
	Mai 2·4885	21·73	442·2	217·19	227·55	+ 15·43	
	„ 3·4843	355·11	331·4	231·63	227·59	+ 15·10	
	„ 4·4843	317·71	334·3	245·97	227·67	+ 15·82	
	Mai 2·7356				227·72	+ 15·38	
68	Mai 2·4885	40·37	903·4	168·95	179·31	+ 23·13	Grosser behofter Fleck.
	„ 3·4843	37·70	826·8	182·54	178·50	+ 22·65	
	„ 4·4843	32·35	727·1	195·93	177·63	+ 22·91	
	„ 7·4905	348·38	428·2	236·70	175·50	+ 22·92	
	Mai 4·4869				177·74	+ 22·90	$\xi = 13·5079$ $\xi_0 = 13·889$.
		Mittl. Abw. einer Beob.			± 0·073	± 0·196	
	Mai 4·4843	43·54	876·7	174·89	156·59	+ 19·69	Hauptfleck einer verwaschenen Schaar.
	„ 7·4905	24·42	547·3	215·41	154·21	+ 20·11	
	Mai 5·9824				155·40	+ 19·90	
71	Mai 7·4905	69·54	663·5	197·74	136·54	— 3·91	Mehrkerniger kleiner Fleck.
	„ 14·5224	246·85	789·5	305·18	143·67	— 3·21	Behofter Fleck.
73	Mai 7·4905	51·84	901·4	171·83	110·63	+ 13·57	Behofter Fleck.
	„ 14·5225	286·73	425·4	270·19	108·68	+ 13·52	
75	Mai 14·5224	114·71	569·1	220·66	59·15	— 27·57	Behofter Fleck im S.W. einer grossen Gruppe.
	„ 17·4769	177·99	425·8	260·68	57·01	— 27·39	
	„ 14·5224	107·98	684·0	209·09	47·58	— 28·87	Hauptfleck der Gruppe. Der Kern war am 14, sehr zerstückelt; später stets doppelt.
	„ 17·4792	153·96	422·6	248·64	44·94	— 28·45	
	„ 21·4835	214·65	762·1	303·64	42·81	— 29·70	
		214·92	746·3	302·02	41·19	— 28·89	
	„ 22·4916	218·94	839·0	315·37	40·16	— 29·25	
	Mai 18·9942				43·57	— 29·02	
76	Mai 14·5224	53·35	719·6	201·01	39·50	+ 10·13	Anfangs behofter, später sehr schwacher Fleck.
	„ 21·4835	267·05	694·7	301·26	40·43	+ 10·39	
	„ 22·4916	264·36	824·3	316·02	40·81	+ 10·41	
	Mai 19·4992				40·25	+ 10·31	

Gruppe.	M. Z. O'Gyalla (bürg.)	p	ρ	l	L	b	
77	Mai 14·5224	85°·35	891″·5	178°·85	17°·34	− 16°·20	Behofter Fleck; von Mai 21 ab war der Kern doppelt.
	„ 17·4769	96·50	526·2	220·60	16·93	− 16·25	
	„ 21·4835	217·75	416·8	277·88	17·05	− 15·58	
	„ 22·4916	228·05	577·2	291·84	16·63	− 15·31	
	„ 25·4911	236·95	912·2	333·78	15·78	− 15·28	
	Mai 20·2931				16·75	− 15·72	
	Mai 17·4792	89·57	562·8	216·68	12·98	− 13·30	Behofter Fleck mit doppeltem Kerne.
	„ 21·4835	216·76	322·4	271·82	10·99	− 12·48	
	„ 22·4916	229·52	490·0	285·85	10·66	− 12·52	
	„ 25·4911	238·77	876·8	326·91	8·91	− 13·12	
	„ 26·4870	239·57	928·6	339·41	7·26	− 13·15	
	Mai 22·6865				10·16	− 12·91	

Rotationsperiode 288.

Gruppe.	M. Z. O'Gyalla (bürg.)	p	ρ	l	L	b	
97	Mai 25·4911	57·93	904·6	187·20	229·20	+ 13·43	Behofter regelmässiger Fleck.
	„ 26·4870	57·12	830·6	200·07	227·92	+ 13·07	
	„ 27·4853	54·46	717·2	213·75	227·28	+ 13·29	
	„ 28·4923	49·56	565·5	228·08	227·27	+ 13·20	
	„ 29·4818	39·19	406·1	241·89	226·95	+ 13·30	
	„ 30·4738	13·56	269·2	257·57	226·74	+ 13·66	
	Juni 1·4837	294·26	375·2	274·27	226·50	+ 14·06	
	„ 2·4898	281·87	538·7	298·61	226·49	+ 14·45	
	„ 3·4807	276·20	686·7	312·39	226·13	+ 14·54	
	„ 4·5384	273·62	816·9	327·40	226·07	+ 14·98	
	Mai 30·9348				226·82 ± 0·139	+ 13·84 ± 0·143	$\xi = 14·0854$ $\xi_0 = 14·178$. $\Delta b = + 0·210$
80	Mai 29·4818	52·85	931·1	183·91	168·97	+ 20·49	Grosser behofter Fleck; der Kern spaltete sich Juni 3 in 3 Stücke (das mittlere kleinere ist nicht verzeichnet), deren vorangehendes sich Juni 7 ganz abgelost hatte.
	„ 30·4738	51·98	885·0	197·32	167·33	+ 20·43	
	Juni 1·4837	45·09	689·5	223·31	165·54	+ 20·83	
	„ 2·4898	37·24	555·9	237·28	165·16	+ 20·91	
	„ 3·4807	23·04	424·8	250·98	164·72	+ 20·63	
	„ 4·5384	354·81	345·2	265·47	164·14	+ 20·97	
	„ 7·4831	289·28	614·2	307·60	164·25	+ 20·30	
		290·20	597·1	306·04	162·69	+ 20·23	
	„ 8·5125	283·46	745·4	321·81	163·77	+ 20·26	
		283·87	730·9	320·28	162·24	+ 20·16	
	„ 9·5153	280·60	845·7	335·43	162·90	+ 20·26	
		280·95	834·3	333·77	161·24	+ 20·28	
	Juni 3·9399				164·67 ± 0·468	+ 20·55 ± 0·303	$\xi = 13·6648$ $\xi_0 = 13·977$.
87	Juni 8·5125	89·56	929·2	192·94	34·90	− 11·67	Hauptkern eines grossen vieltheiligen Fleckes.
	„ 9·5153	90·69	871·9	206·79	34·26	− 11·45	
	„ 13·4839	118·82	301·1	262·80	34·12	− 10·71	
	„ 18·5117	246·84	758·7	344·42	33·73	− 11·55	
	„ 19·4864	249·36	866·0	348·82	33·23	− 11·25	
	Juni 13·9019				34·05 ± 0·231	− 11·33 ± 0·377	$\xi = 14·1517$ $\xi_0 = 14·233$.

Gruppe.	M. Z. O'Gyalla (bürg.)	p	ρ	l	L	b	
88	Juni 9·5153	91°·35	902″·6	201°·16	28°·63	12°·53	Grosser in viele Stücke zerfallender Fleck.
	Juni 13·4839	102·23	570·1	242·58	13·90	— 12·70	Doppelfleck, der sich auflöste.

Rotationsperiode 289.

Gruppe.	M. Z. O'Gyalla (bürg.)	p	ρ	l	L	b	
90	Juni 18·5117	28·82	387·2	266·92	326·23	+ 20·54	Sehr veränderlicher Hauptfleck einer grossen Fleckenschaar. Nur Juni 23 einfacher runder Fleck.
	„ 19·4864	0·46	324·4	280·08	325·49	+ 21·42	
	„ 21·5090	307·70	468·9	307·45	324·00	+ 21·93	
	„ 23·4955	291·74	731·1	334·70	322·90	+ 22·38	
	Juni 20·7507				324·65 ± 0·107	+ 21·57 ± 0·244	ξ = 13·5949 ξ_0 = 13·940 Δb = + 0·331.
	Juni 23·4955	289·19	738·0	335·96	324·16	+ 20·66	Rundlicher Fleck.
91	Juni 18·5117	64·53	795·9	225·53	284·84	+ 15·38	Schwacher Doppelfleck.
		64·49	817·0	223·06	282·37	+ 15·77	
	„ 19·4864	61·47	640·7	242·04	287·45	+ 15·17	Rundlicher Fleck; stets von mehreren kleineren begleitet, Juni 24 getheilt.
	„ 21·5090	39·37	300·6	271·35	287·90	+ 14·48	
	„ 23·4955	308·98	276·4	299·17	287·37	+ 13·87	
	„ 24·4951	290·61	448·8	313·85	287·80	+ 13·94	
		302·59	436·0	312·65	286·60	+ 14·44	
	„ 25·4836	284·47	610·2	327·61	287·45	+ 14·15	
	„ 26·5150	281·52	756·6	342·27	287·40	+ 14·22	
	Juni 23·4974				287·56 ± 0·221	+ 14·31 ± 0·476	ξ = 14·2460 ξ_0 = 14·166
94	Juni 28·4841	227·48	549·9	320·23	337·27	— 19·15	Unregelmässiger behofter Fleck.
	„ 29·5475	237·14	680·8	334·00	235·86	— 18·90	
	Juni 29·0158				236·57	— 19·02	
92	Juni 23·4955	69·62	881·5	217·99	206·18	+ 14·27	Kleiner runder behofter Fleck.
	„ 24·4951	68·98	783·0	232·41	206·36	+ 14·08	
	„ 25·4836	66·64	652·0	246·27	206·61	+ 14·20	
	„ 26·5150	60·80	483·1	261·07	206·20	+ 14·17	
	„ 28·4841	7·08	193·4	289·35	206·39	+ 14·17	
	„ 29·5475	313·18	258·0	303·79	205·65	+ 14·01	
	Juni 26·3368				206·15 ± 0·264	+ 14·15 ± 0·092	ξ = 14·2626 ξ_0 = 14·170
95	Juni 26·5150	73·56	691·2	243·06	188·19	+ 10·25	Juni 26 einzelner Fleck; später Hauptfleck im W der Gruppe.
	„ 28·4841	62·11	297·6	274·47	191·51	+ 9·88	
	„ 29·5475	15·34	116·0	290·36	192·22	+ 9·34	
	Juli 5·4878	277·74	924·5	17·10	194·22	+ 8·69	
	Juni 30·0086				191·53 ± 0·939	+ 9·54 ± 0·035	ξ = 14·8498 ξ_0 = 14·266 Δb = — 0·169.
96	Juni 28·4841	108·13	711·0	245·43	162·47	— 14·42	Behofter Fleck im O der Gruppe; Juni 28 und 29 mit getheiltem Kerne, dessen grösseres n. Stück beobachtet wurde.
	„ 29·5475	115·63	564·0	259·94	161·80	— 14·43	
	Juli 5·4878	246·97	718·9	344·81	161·93	— 14·71	
	Juli 1·1731				162·07	— 14·52	

Gruppe.	M. Z. O'Gyalla (bürg.)	p	ρ	l	L	b	
97	Juni 29·5475	93°·79	804″·0	161°·80	136°·53	— 4°·65	Kleiner behofter Fleck.
	Juli 5·4878	244·19	316·1	315·87	132·99	— 5·07	
	„ 7·4810	259·28	663·0	343·84	132·52	— 5·24	
	„ 8·4902	262·20	802·2	358·59	132·88	— 5·44	
	Juli 5·2516				133·75 ± 0·535	— 5·10 ± 0·059	$\xi = 13{\cdot}8174$ $\xi_0 = 14{\cdot}323$ $\Delta b = - 0^\circ{\cdot}083$.
98	Juli 12·4852	297·32	417·7	329·32	46·62	+ 14·23	Hauptkern eines sich schnell ändernden Fleckes.
	„ 14·4806	288·96	736·7	357·73	46·55	+ 14·59	
	„ 15·5041	289·04	863·5	14·07	48·30	+ 15·48	
	„ 16·4788	289·84	920·9	26·88	47·20	+ 15·93	
	Juli 14·7622				47·17	+ 15·06	
	Juli 11·4865	338·97	225·1	309·55	41·09	+ 16·51	Einzelner Fleck.
	„ 12·4852	306·57	373·1	324·84	42·14	+ 16·45	
99	Juli 14·4806	310·41	129·1	313·23	2·05	+ 8·89	Verwaschener Fleck.

Rotationsperiode 290.

99	Juli 14·4806	320·91	79·4	310·07	358·89	+ 7·74	Verwaschener Fleck.
	„ 15·5041	287·32	280·5	324·59	358·82	+ 8·00	
	„ 16·4788	282·42	477·7	338·98	359·30	+ 7·78	
	Juli 15·4878				359·00	+ 7·84	
101	Juli 16·4788	358·84	104·6	309·31	329·63	+ 10·67	Verwaschener Fleck.
100	Juli 14·4806	80·52	888·5	236·09	284·91	+ 13·71	Grosser behofter Fleck, dessen Kern am 16, 17 und 19 in zwei grosse und ein kleines (nicht verzeichnetes) Stück getrennt war. Später schmolzen alle in einen einfachen langlichen Kern zusammen.
	„ 15·5041	80·68	789·5	250·95	285·18	+ 13·46	
	„ 16·4788	78·42	652·3	265·54	285·86	+ 14·19	
		79·54	671·8	263·75	284·07	+ 13·65	
	„ 17·4939	74·01	485·8	279·97	285·82	+ 14·48	
		75·61	502·5	277·56	283·41	+ 13·97	
	„ 19·4803	22·65	168·0	308·48	285·98	+ 14·45	
	„ 23·4855	291·08	724·4	5·36	285·74	+ 13·59	
	„ 24·4886	290·50	841·4	19·68	285·75	+ 13·48	
	„ 25·4816	291·02	911·6	32·88	284·75	+ 13·56	
	Juli 19·6117				285·24 ± 0·473	+ 13·80 ± 0·367	$\xi = 14{\cdot}3090$ $\xi_0 = 14{\cdot}179$
102	Juli 17·4939	109·92	818·4	251·27	257·12	— 10·72	Kleiner Fleck.
	„ 19·4803	121·24	569·0	277·93	255·43	— 11·43	
	„ 23·4845	218·62	451·6	330·60	250·98	— 19·31	Kleiner behofter Fleck.
	„ 24·4886	236·83	576·2	345·38	251·45	— 19·19	
	„ 25·4816	247·43	705·0	359·74	251·61	— 18·76	
	„ 26·4920	253·70	821·2	14·41	251·87	— 18·74	
	Juli 24·9869				251·48 ± 0·076	— 19·00 ± 0·099	$\xi = 14{\cdot}5495$ $\xi_0 = 14{\cdot}030$ $\Delta b = + 0{\cdot}214$
103	Juli 19·4803	116·44	869·1	247·33	224·83	— 17·01	Kleiner Fleck.

Gruppe.	M. Z. O'Gyalla (bürg.)		p	ρ	l	L	b	
104	Juli	23·4855	78°·15	577″·1	278°·69	199°·07	+ 15°·66	Kleiner Fleck.
	„	24·4886	71·67	397·8	293·32	199·39	+ 15·37	
	„	25·4816	49·78	227·3	307·81	199·68	+ 15·40	
	„	26·4920	347·53	183·5	322·32	199·78	+ 15·58	
	Juli	24·9869				199·48	+ 15·50	ξ = 14·5085 ξ_0 = 14·135
						± 0·067	± 0·140	
	Juli	23·4855	88·27	655·0	271·45	191·83	+ 9·97	Kleiner Fleck.
	„	24·4886	87·01	489·2	285·32	191·39	+ 10·06	
	„	25·4816	80·79	295·3	299·79	191·66	+ 10·29	
	„	26·4920	45·72	109·8	314·15	191·61	+ 10·52	
	Juli	24·9869				191·62	+ 10·21	ξ = 14·2275 ξ_0 = 14·254
						± 0·267	± 0·041	Δb = + 0·188
105	Juli	31·4833	151·68	339·5	309·73	115·99	— 10·70	Behofter Fleck mit doppeltem Kerne im W. der Gruppe.
107	August	10·4849	349·66	236·7	339·01	2·58	+ 19·23	Grosser behofter Fleck mit rundlichem Kerne.
	„	11·4801	319·12	369·6	353·44	2·81	+ 18·53	
	„	12·4811	308·10	533·5	7·55	2·64	+ 17·88	
	„	13·4876	304·03	687·5	21·72	2·45	+ 17·66	
	„	14·4796	303·07	805·1	34·99	1·56	+ 17·86	
	August	12·4827				2·41	+ 18·23	ξ = 14·0265 ξ_0 = 14·055
						± 0·312	± 0·311	Δb = — 0·324

Rotationsperiode 291.

107	August	10·4849	10·35	221·5	333·70	357·27	+ 19·65	Unregelmässiger behofter Fleck.
	„	11·4801	331·56	306·0	347·18	356·55	+ 19·63	
	„	12·4811	315·73	454·4	0·55	355·64	+ 19·72	
	„	13·4876	308·70	608·4	14·25	354·98	+ 19·53	
	„	14·4796	306·11	742·4	27·82	354·39	+ 19·69	
	August	12·4827				355·77	+ 19·64	ξ = 13·5375 ξ_0 = 14·009
			Mittl.	Abw. einer Beob.		± 0·087	± 0·073	
108	August	10·4819	90·47	873·9	264·49	288·06	+ 15·37	Kleiner behofter Fleck mit rundlichem Kerne.
	„	11·4801	90·52	774·1	278·50	287·87	+ 15·25	
	„	12·4811	89·25	636·8	292·58	287·67	+ 15·39	
	„	13·4876	85·74	469·4	306·72	287·45	+ 15·26	
	„	14·4796	75·25	292·6	320·55	287·12	+ 15·21	
	„	20·4912	301·09	828·0	43·69	284·32	+ 14·71	
	August	13·8169				287·08	+ 15·20	ξ = 13·8818 ξ_0 = 14·143
						± 0·239	± 0·078	Δb = — 0·067
111	August	20·4912	134·88	460·6	316·50	197·13	— 6·72	Kleiner Fleck.
112	August	20·4912	94·60	592·4	303·70	184·33	+ 13·49	Grosser behofter Fleck.
	„	26·4859	301·23	602·4	27·57	182·86	+ 12·55	
	„	20·4912	96·29	693·4	295·09	175·72	+ 13·02	Grosser behofter Fleck.
	„	26·4859	302·69	494·0	19·32	174·61	+ 12·54	
113	August	26·4859	91·93	258·2	322·87	128·16	+ 11·33	Verwaschener Doppelfleck.

Gruppe.	M. Z. O'Gyalla (bürg.)	p	ρ	l	L	b	
114	August 26·4859	144°·36	481″·7	323°·07	118°·36	— 10°·54	Behofter Fleck.
	Sept. 2·5097	271·49	793·4	48·25	103·41	— 12·53	Behofter Fleck mit doppeltem Kerne.
	,, 3·5434	275·35	885·5	62·05	102·40	— 12·55	
117	Sept. 2·5097	207·70	464·3	358·19	53·35	— 22·03	Kleiner Doppelfleck.
	,, 3·5434	223·20	520·4	13·71	54·06	— 21·16	
120	Sept. 4·4965	315·54	545·0	30·54	57·28	+ 19·00	Westlichster sehr veränderlicher Kern der Gruppe.
	,, 5·4976	312·06	705·5	45·41	57·86	+ 19·30	
	,, 6·4907	310·79	841·5	61·60	59·89	+ 19·42	
	,, 7·4805	311·53	905·3	72·79	56·95	+ 20·03	
	Sept. 4·4965	320·58	479·8	24·94	51·48	+ 20·13	Mitte des grossen Hauptflecks mit doppeltem Kerne.
	,, 5·4976	315·11	630·2	38·33	50·78	+ 20·23	
	,, 6·4907	313·04	760·3	51·73	50·02	+ 20·56	
	,, 7·4805	313·12	856·7	64·60	48·76	+ 21·38	Oestlicher Kern.
	Sept. 5·4949				50·76	+ 20·31	

Dr. H. Kobold.

O'Gyalla, 1882, September 14.

ON THE PARALLAX OF THE STAR BRADLEY 3077.

By O. BACKLUND.

As this star has a considerable proper motion its annual parallax might be supposed to be appreciable. In order to decide on this point M. Wagner, Vice Director of our Observatory, has observed with the great transit instrument the differences of Right Ascension between it and some neighbouring stars. The circumstances under which he was obliged to make the observations were very unfavourable, as the stars of comparison turned out to be too faint to be observed in the months of May, June and July, the season of the greatest negative influence of parallax. In December, when the influence of parallax is at its positive maximum, the images of stars are in our climate very bad. As the precision consequently could not equal the precision of observations made under ordinary conditions, M. Wagner gave up the proposed work before the number of observations was great enough to secure a definitive deduction of the value of the parallax. But when Professor Gyldén recently published his determination of the parallax of this star, it appeared to me a question of interest to see how far the results of M. Wagner's observations would agree with this determination. At my request M. Wagner kindly placed his observations at my disposal.

The position of Br. 3077 is given by Argelander in his "Untersuchungen über die Eigenbewegungen von 250 Sternen"; reduced to 1870·0 it becomes—

$$\alpha = 23^{h}\ 7^{m}\ 1^{s}{\cdot}93 \qquad \delta = +\ 56^{\circ}\ 27'\ 2''{\cdot}7$$
$$\text{PM} = +\ 0{\cdot}2499 \qquad +\ 0{\cdot}270$$

The approximate positions of the comparison-stars for the same epoch are[*]—

No. 1	$\alpha = 23^h\ 4^m\ 49^s{\cdot}3$	$\delta = +56^\circ\ 19'{\cdot}4$
2	$6\ \ 4$	$56\ \ 24{\cdot}7$
3	$9\ \ 8$	$56\ \ 1{\cdot}4$
4	$9\ \ 53$	$56\ \ 31{\cdot}0$
5	$10\ \ 5$	$55\ \ 58{\cdot}0$

For the reduction of the observations to 1870·0 M. Wagner has employed Struve's precession and Argelander's proper motion.

Br. 3077 was compared 35 times with No. 1, 16 times with No. 2, 22 times with No. 3, 4 times with No. 4, and 32 times with No. 5. The comparisons with the stars Nos. 2, 3 and 4 being made when the influence of the parallax was rather insignificant, I have only employed the comparisons with Nos. 1 and 5. These two stars were both compared with Br. 3077 on 28 nights, but having overlooked one comparison I have only employed 27.

From the Right Ascensions of these stars deduced by comparing them with the observations of Br. 3077 I formed the means. These means I denote by α', and by α I denote the true mean of the two stars; I further put $\pi = 15x$ and z equal to the correction to the proper motion in R.A. From the 27 equations of condition of the form—

$$\alpha' = \alpha + ax + by$$

I formed the mean, and by subtracting this from each of the original equations I eliminated α and obtained 27 new equations of the form—

$$n = a'x + b'y.$$

These equations treated by the method of least squares gave—

$$\pi = +\ 0''{\cdot}20 \pm 0''{\cdot}080.$$

I then treated the 32 observations of the star No. 5 separately in the same manner and obtained—

$$\pi = +\ 0''{\cdot}21 \pm 0''{\cdot}078.$$

The omission of one comparison in the former set does not alter the result by more than $0''{\cdot}01$.

These values of π agree within the limits of the probable errors with the value deduced by Gyldén from his observations made with the equatoreal of the Stockholm Observatory. This is—

$$\pi = +\ 0''{\cdot}28 \pm 0''{\cdot}045.$$

(See: "Bestämning af afständet tiu stjernon No. 3077 i Bradley's Catalog." Öfversigt af Kongl. Vetenskapsakademiens förhandlingar 1882, No. 2, Stockholm). The first observation of M. Wagner was made 1870, April 5, and the last 1873, April 4. All the observations except three made in 1873, will appear in Vol. XII. of the Observations de Poulkova.[†]

Pulkova, Sept. 20, 1882.

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[*] The magnitude of Br. 3077 is 5·9 according to Argelander. The magnitudes of the stars of comparison range between 8·7 and 9·5.

[†] [Professor Brünnow found the parallax of B. 3077 equal to $+\ 0''{\cdot}07 \pm 0''{\cdot}02$ by measuring the difference of declination from a star 9·5 mag. 4^s *f* and 83″ south. See Dunsink Observations, Part II.—Ed.]

PROFESSOR GYLDÉN'S NEUE UNTERSUCHUNGEN ÜBER DIE THEORIE DER BEWEGUNG DER HIMMELSKÖRPER.

Von Dr. O. BACKLUND.

I.

In der Wissenschaft giebt es nicht viele Hypothesen, denen ein so glänzender Erfolg zu Theil geworden ist, wie der Newtonschen Hypothese über die allgemeine Gravitation. Abgesehen von Bedenken, welche von Dilettanten auf dem Gebiete der Himmelsmechanik erhoben sind, ist eigentlich nur einmal ernsthafter Zweifel an der Richtigkeit dieser Hypothese ausgesprochen worden. Das Resultat, zu welchem Clairaut bei seinem ersten Versuche, die Bewegung des Mondapogäums theoretisch abzuleiten, gelangte, veranlasste ihn, wie bekannt ist, die Newtonsche Hypothese als ungenügend zu erklären. Da fast gleichzeitig solche Männer wie Euler und d'Alembert in derselben Frage dieselben Resultate erhielten, so könnte dieser Umstand nicht umhin den schon angeregten Zweifel zu stärken. Es war daher ein grosses Verdienst von Clairaut, als er selbst nachwies, dass seine theoretischen Resultate falsch waren, und dass eine richtige analytische Behandlung der Frage zu völliger Bestätigung der Newtonschen Hypothese führte. Schöneres Lob könnte ihm auch nicht gesagt werden, als das, was Euler über ihn äussert (Theoria Lunæ p. 120): "Gloria hujus insignis inventi cum industriae tum candori excellentissimi Clairaulti debetur, qui primus egregium hunc theoriæ concensum cum veritate detexit et publice est professus." Die glänzenden Untersuchungen von Lagrange und Laplace scheinen nachher die Newtonsche Hypothese so über allen Zweifel erhoben zu haben, dass in denjenigen Fällen, wo man nicht Theorie und Beobachtung in Einklang zu bringen vermochte, die Existens anderer Kräfte, welche die Bewegung nach den Newton'schen Gesetzen modificiren, neben der allgemeinen Attractionskraft angenommen wurde. Und es ist vielleicht leichter in kritischen Fällen Zuflucht zum wiederstehenden Mittel oder zum Weber'schen Gesetze zu nehmen, als zu untersuchen, ob die in der Störungstheorie üblichen Methoden ausnahmslos den Consequenzen jener Hypothese entsprechen.

Es gereicht Professor Gyldén zu grossem Ruhm diese Frage in allem Ernst aufgenommen und beantwortet zu haben. Er hat der Astronomie ohne Zweifel grosse Dienste von dauernder Bedeutung geleistet, indem er gezeigt (Vierteljahrsschrift, XVI, 4 Heft) dass die Annahme der Kepler'schen Ellipse als Primärcurve für die Planetenbewegung nicht nothwendig auf convergente Annäherungen führe, wenn diese ausserdem, wie gewöhnlich, nach den Potenzen der störenden Kräfte geordnet werden.

Um diesen principiellen Mangel der alten Theorie zu vermeiden baut Gyldén eine neue Theorie für die Bewegung der Himmelskörper auf. Von dem für diese

Theorie Characteristischen wollen wir beiläufig das folgende angeben. Als Primärcurve wird eine solche angenommen, die sich näher als die Kepler'sche Ellipse an die wirkliche Bewegung anschliesst. Gyldén nennt diese Curve "intermediäre Bahn." Weiter werden in der darauf folgenden ersten Annäherung die beträchtlichsten Glieder ebenso wie die Glieder langer Periode mit aufgenommen, aber dabei wird jede Entwickelung nach den Potenzen der störenden Kräfte vermieden, in welcher bei nachheriger Integration kleine Divisoren auftreten und die Convergenz illusorisch machen können.[*]

Von Gyldén's Arbeit: "Undersökningar af Theorien för Himlakropparnas rörelser," welche seine neue Theorie darstellen soll, sind bis jetzt zwei Theile erschienen. In diesen sind aber die Grundzüge der Theorie enthalten, und ich werde mich in den folgenden Zeilen bemühen, dieselben so kurz wie möglich darzustellen. Den bald erscheinenden dritten Theil werde ich vielleicht in einem späteren Aufsatze Gelegenheit haben zu besprechen.

Der erste Theil enthält die Theorie der intermediären Bahn und die Theorie für die Ermittelung der übrig bleibenden Abweichungen ("Störungen"). Der zweite Theil enthält wesentlich vorbereitende Untersuchungen über die "absolute Bahn" nebst numerischen Anwendungen.

In dem hier darzustellenden Theile der Gyldén'schen Theorie handelt es sich nur um die Bewegung in der Bahnebene, welche als Fundamentalebene angenommen wird. Die Differentialgleichungen der Bewegung sind also—

$$\frac{d^2x}{dt^2} + \frac{\mu_1 x}{r^2} = \frac{\delta(\Omega)}{\delta x}$$

$$\frac{d^2y}{dt^2} + \frac{\mu_1 y}{r^2} = \frac{\delta(\Omega)}{\delta y}.$$

Wenn aus der Störungsfunction (Ω) eine Function $f(r)$, die nur Function von r ist, ausgeschieden und—

$$\Omega = (\Omega) + f(r)$$

$$f'(r) = \frac{\mu_1}{r^2}\,\mathrm{F}$$

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[*] Schon die Newton'sche bewegliche Ellipse könnte als intermediäre Bahn dienen, denn durch gehörige Bestimmung der Coefficienten von $\frac{1}{r^3}$ kann sie die wirkliche Bewegung genauer als die Kepler'sche Ellipse darstellen. Dass Clairaut in seiner Mondtheorie Gebrauch von der intermediären Bahn gemacht, ist wohl bekannt. Dabei hat er aber nicht die Nothwendigkeit einer solchen eingesehen. Auch in seiner Planetentheorie (1747) legt Clairaut eine intermediäre Bahn seinen Untersuchungen zu Grunde. In der merkwürdigen Abhandlung "Investigatio perturbationum quibus planetarum Motus ob actionem eorum mutuam afficiuntur"—im Jahre 1756 der Pariser Akademie vorgelegt—finden wir, dass auch Euler die intermediäre Bahn als vortheilhaft betrachtet. Da die Astronomen aber an die Keplersche Ellipse zu sehr gewohnt waren, schien es ihm besser, diese als Primärcurve anzuwenden. Die wahre Bedeutung der intermediären Bahn war also Euler ebenso wie Clairaut vollständig entgangen. Es ist in dieser selben Abhandlung, wo, meines Wissens wenigstens, zum ersten Male die Idee der momentanen Ellipse gegeben wird.

gesetzt wird, so werden die Bewegungsgleichungen—

$$\frac{d^2x}{dt^2} + \frac{\mu_1}{r^2}(1 + \mathrm{F})\frac{x}{r} = \frac{\delta\Omega}{\delta x},$$

$$\frac{d^2y}{dt^2} + \frac{\mu_1}{r^2}(1 + \mathrm{F})\frac{y}{r} = \frac{\delta\Omega}{\delta y},$$

welche mit Einführung der Polarcoordinaten mittelst—

$$x = r\cos v, \quad y = r\sin v$$

in die folgenden transformirt werden können—

$$\left.\begin{aligned} \frac{d\left(r^2\frac{dv}{dt}\right)}{dt} &= \frac{\delta\Omega}{\delta v} \\ \frac{d^2r}{dt^2} - r\left(\frac{dv}{dt}\right)^2 + \frac{\mu_1}{r^2}(1 + \mathrm{F}) &= \frac{\delta\Omega}{dr} \end{aligned}\right\} \quad \ldots\ldots \quad (1)$$

oder, wenn man $r^2\frac{dv}{dt} = \sqrt{c}$ setzt—

$$\left.\begin{aligned} \frac{d\sqrt{c}}{dt} &= \frac{\delta\Omega}{\delta v} \\ \frac{d^2r}{dt^2} - \frac{c}{r^3} + \frac{\mu_1}{r^2}(1 + \mathrm{F}) &= \frac{\delta\Omega}{\delta r} \end{aligned}\right\} \quad \ldots\ldots \quad (2)$$

Die wahre Länge v wird in zwei Theile zerlegt, nemlich—

$$v = v_0 + \chi.$$

Bezeichnet c_0 einen constanten Mittelwerth von c, so soll diese Zerlegung näher angegeben werden durch—

$$\frac{dv_0}{dt} = \frac{\sqrt{c_0}}{r^2} \quad \ldots\ldots \quad (a)$$

Für $\Omega = 0$ mögen r und F mit r_0 und F_0 bezeichnet werden, und mit τ eine Function von t, welche folgendermassen definirt wird—

$$\frac{dv_0}{d\tau} = \frac{\sqrt{c_0}}{r^2} \quad \ldots\ldots \quad (b)$$

v_0 nennt Gyldén *intermediäre Länge,* χ *Variation,* r_0 *intermediären Radiusvector* und τ *reducirte Zeit.* Bei der Annahme $\Omega = 0$ können wir daher statt der zweiten der Gleichungen (2) schreiben—

$$\frac{d^2r}{d\tau^2} - \frac{c_0}{r_0^{\,3}} + \frac{\mu_1}{r_0^{\,2}}(1 + \mathrm{F}_0) = 0 \quad \ldots\ldots \quad (3)$$

welches die Differentialgleichung der intermediären Bewegung ist. Aus den Relationen (*a*) und (*b*) ergiebt sich—

$$\frac{d\tau}{dt} = \left(\frac{r_0}{r}\right)^2.$$

Wird die Differenz $\frac{1}{r_0} - \frac{1}{r}$ mit ρ bezeichnet, und—

$$r_0\rho = \frac{\nu}{1 + \nu}$$

gesetzt, so wird—

$$r = r_0 (1 + \nu).$$

$r_0\nu$ wird *Evection* genannt. Setzen wir ferner $t - \tau = \mathrm{T}$, so finden wir—

$$\mathrm{T} = \int (2\nu + \nu^2) d\tau = \int \frac{(2r_0\rho - r_0^2\rho^2)}{(1 - r_0\rho)^2}\, d\tau.$$

Da $\frac{dv}{dt} = \frac{\sqrt{c}}{r^2}$, so ist auch $\frac{dv_0}{dt} + \frac{d\chi}{dt} = \frac{\sqrt{c}}{r^2}$, ein Ausdruck, der wegen $\frac{dv_0}{dt} = \frac{\sqrt{c_0}}{r^2}$ in den folgenden verwandelt werden kann—

$$\frac{d\chi}{dv_0} = \sqrt{\frac{c}{c_0}} - 1.$$

Differentiiren wir diesen Ausdruck, so ergiebt sich—

$$\frac{d^2\chi}{dv_0^2} = \frac{1}{\sqrt{c_0}} \frac{d\sqrt{c}}{dv_0} = \frac{r^2}{c_0} \frac{d\sqrt{c}}{dt} = \frac{r^2}{c_0} \frac{\delta\Omega}{dv}, \qquad \ldots\ldots \quad (4)$$

welche Differentialgleichung zur Bestimmung der Variation dient. Um den Ort eines Planeten vollständig angeben zu können, muss noch der wahre Radius vector ermittelt werden. Zu dem Zwecke haben wir ν oder ρ zu bestimmen.

Differentiiren wir $\rho = \frac{1}{r_0} - \frac{1}{r}$ zweimal, so finden wir leicht mit Rücksicht auf (*a*) und (*b*)—

$$\frac{d^2\rho}{dv_0^2} = \frac{1}{c_0} \left\{ r^2 \frac{d^2r}{dt^2} - r_0^2 \frac{d^2r_0}{d\tau^2} \right\}.$$

Wird in der zweiten der Gleichungen (1) $\frac{dv}{dt}$ durch $\frac{\sqrt{c_0}}{r^2} \frac{d\chi}{dv_0} + \frac{\sqrt{c_0}}{r^2}$ ersetzt, so findet sich—

$$r^2 \frac{d^2r}{dv_0^2} = \frac{c_0}{r} + \frac{c_0}{r} \left\{ 2\frac{d\chi}{dv_0} + \left(\frac{d\chi}{dv_0}\right)^2 \right\} - \mu_1(1 + \mathrm{F}) + r^2 \frac{\delta\Omega}{\delta r}.$$

Die Differentialgleichung der intermediären Bewegung giebt—

$$r_0^2 \frac{d^2r_0}{d\tau^2} = \frac{c_0}{r_0} - \mu_1(1 + \mathrm{F}_0).$$

Werden diese beiden Ausdrücke in den Ausdruck für $\frac{d^2\rho}{dv_0^2}$ eingeführt, so finden wir als Differentialgleichung der Evection—

$$\frac{d^2\rho}{dv_0^2} + \rho = -\frac{\mu_1}{c_0} (\mathrm{F} - \mathrm{F}_0) + \frac{1}{r} \left[2\frac{d\chi}{dv_0} + \left(\frac{d\chi}{dv_0}\right)^2 \right] + \frac{r^2}{c_0} \frac{\delta\Omega}{\delta r} \qquad \ldots \quad (5)$$

Hiermit sind die Gyldén'schen Differentialgleichungen sowohl für die intermediäre wie für die wirkliche Bewegung gegeben.

Wie schon bemerkt, ist der Zweck der intermediären Bahn einen genaueren Ausgangspunkt für die weiteren Approximationen an die wirkliche Bewegung zu gewinnen, als die Kepler'sche Ellipse bietet. Wollte man nun, nachdem die intermediäre Bahn ermittelt worden, die folgenden Annäherungen, wie in den gewöhnlichen Methoden, nach den Potenzen der störenden Kräfte ordnen, so wäre es am Einfachsten $\chi = 0$ zu setzen. Dadurch würde man aber in die Schwierigkeiten der alten Methode zurückfallen, welche darin bestehen, dass die Integrationen kleine Divisoren erzeugen und dadurch die Convergenz verringern. Die Ausscheidung von χ aus der wahren Länge und ihre Bestimmung so wie die Bestimmung von ρ in der Weise, dass diese Schwierigkeiten vermieden werden, gehört daher zu den Cardinalpunkten der Gyldén'schen Theorie. Der Erfolg auf dem eingeschlagenen Wege hängt also wesentlich von der Integration der Differentialgleichungen (3), (4) und (5) ab. Die Schwierigkeiten, welche sich an die Integration von (4) und (5) knüpfen, sind bedeutend; die Art und Weise, wie Gyldén diese beseitigt hat, bezeichnet einen mathematischen Fortschritt, der gewiss für die Entwickelung der Astronomie in vielerlei Beziehung eine grosse Bedeutung haben wird.

Führen wir in (3) v_0 als unabhängige Variable ein, so wird die Differentialgleichung der intermediären Bahn—

$$\frac{d^2\mathrm{R}}{dv_0^2} + \mathrm{R} - \frac{\mu_1}{c_0} - \frac{\mu_1}{c_0}\mathrm{F}_0 = 0$$

wo $\mathrm{R} = \frac{1}{r_0}$ gesetzt ist. Wenn F_0 eine rationale Function von R ist, so führt diese Gleichung auf elliptische oder hyperelliptische Integrale. Es ist dies auch der Fall, wenn F_0 einfach einer ganzen Potenz von R gleich gesetzt wird, ausgenommen, wo $\mathrm{F}_0 = \kappa\mathrm{R}$, denn diese Substitution führt auf ein trigonometrisches Integral. Substituiren wir daher allgemein $\mathrm{F}_0 = \kappa\,\mathrm{R}^n$, so können wir—

$$\frac{d^2\mathrm{R}}{dv_0^2} + \mathrm{R} - \frac{\mu_1}{c_0} - \frac{\mu_1\kappa}{c_0}\mathrm{R}^n = 0$$

als die Differentialgleichung einer Bahn betrachten, in welcher ein Planet um die Sonne unter dem Einflusse einer Centralkraft von der Form—

$$-\frac{\mu_1}{r^2} + \frac{\mu_1\kappa}{c_0}\cdot\frac{1}{r^{n+2}}$$

sich bewegt. In einer solchen Bahn sind bekanntlich die Apsiden beweglich. Durch zweckmässige Wahl des zweiten Gliedes kann also schon in der intermediären Bahn ein wesentlicher Theil der Apsidenbewegung der wahren Bahn berücksichtigt werden, und zwar so, dass die Unterschiede der intermediären Orte von den wirklichen nicht die Ordnung der Excentricität der Bahn des "gestörten" Körpers überschreiten, während diese Unterschiede in der elliptischen Bewegung von der Ordnung der Summe der Bahnexcentricitäten des gestörten und der störenden Körper werden können. Für $n = 1$ kommt man auf den von Newton (Principia, sectio 9, Prop. 44) behandelten Fall, in welchem die obige Gleichung auf ein trigonometrisches Integral führt, und zwar wird die Gleichung der Bahn—

$$r = \frac{\frac{c_0}{\mu_1}}{1 + e\cos\sqrt{1 - \frac{\mu_1\kappa}{c_0}}\cdot v_0},$$

also eine Ellipse, die sich um das Bewegungscentrum mit gleichformiger Geschwindigkeit dreht.[*]

Die Newton'sche bewegliche Ellipse ist aber nicht diejenige intermediäre Bahn, welche sich unmittelbar und am natürlichsten darbietet. Betrachten wir nämlich die bekannten Entwickelungen der Störungsfunction—

$$\frac{1}{\mu_1}\,(\Omega) = \frac{1}{r'} + \tfrac{1}{4}\,\frac{r^2}{r_1^{\,3}} + \tfrac{3}{4}\,\frac{r^2}{r_1^{\,3}}\cos 2\mathrm{H} + \ldots$$

$$\frac{1}{\mu_1}\,(\Omega) = \frac{1}{r} + \left(\frac{r_1}{r^2} - \frac{r^2}{r_1^{\,2}}\right)\cos\mathrm{H} + \tfrac{1}{4}\,\frac{r_1^{\,2}}{r^3} + \tfrac{3}{4}\,\frac{r_1^{\,2}}{r^3}\cos 2\mathrm{H} + \ldots$$

so ist augenscheinlich am zweckmässigsten F_0 proportional r^3 anzunehmen, wenn der "störende" Körper stets ausserhalb, und proportional $\frac{1}{r^2}$ wenn der "störende" Körper stets innerhalb der Bahn des "gestörten" sich bewegt, da in diesem Falle die beiden ersten Glieder durch Transformation auf den Schwerpunkt des Systems weggeschaft werden können. Es ist auch diese Wahl, die Gyldén macht.[†]

Es wird daher angenommen—

$$\mu_1 F_0 = -\,\mu_2 r^3$$

für den ersten Fall, und—

$$\mu_1 F_0 = \frac{3\mu_2}{r^2}$$

für den zweiten, wodurch die Differentialgleichung der intermediären Bahn wird—

bezüglich—

$$\left.\begin{aligned} &\frac{d^2\mathrm{R}}{dv_0^{\,2}} + \mathrm{R} + \frac{\mu_2}{c_0}\cdot\frac{1}{\mathrm{R}^3} - \frac{\mu_1}{c_0} = 0 \ \ddagger \\ &\frac{d^2\mathrm{R}}{dv_0^{\,2}} + \mathrm{R} - \frac{3\mu_2}{c_0}\,\mathrm{R}^2 - \frac{\mu_1}{c_0} = 0 \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \ (6)$$

Die erste ist die Differentialgleichung der Bahn für den Fall, dass die Centralkraft durch den Ausdruck—

$$-\frac{\mu_1}{r^2} + \mu_2 r$$

gegeben ist, die zweite stellt dagegen die Bewegung dar, wenn die Centralkraft von der Form ist—

$$-\frac{\mu_1}{r^2} - \frac{\mu_2}{r^4}.$$

Die zweite Gleichung ist von Gyldén in der Abhandlung "Om banan för en punkt, som rör sig i en sferoids equatorsplan" integrirt worden. Da in der Arbeit, die wir jetzt besprechen, vorausgesetzt wird, der "störende" Körper befinde sich stets ausserhalb der Bahn des "gestörten" Körpers, so interessirt uns hauptsächlich die erste Gleichung, welche für die hier vorliegenden Zwecke vollständig integrirt wird. Dieser Aufsatz würde zu umfangreich werden, wenn wir Gyldén's Auseinandersetzung über die Integration im Detail verfolgen wollten; wir müssen uns daher mit der Angabe der Hauptzüge begnügen und können für die Einzelnheiten auf die interessante Schrift: "Über die Bahn eines materiellen Punktes, der sich unter dem Einflusse einer Centralkraft von der Form $\frac{\mu_1}{r^2} + \mu_2 r$ bewegt," Kongl. Swenska Wetenskapsakademiens Handlingar, Bandet 17 No. 1, verweisen.

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[*] Vergleich übrigens d'Alembert's ausführliche Untersuchungen für verschiedene Hypothesen über die Centralkraft, Recherches sur differents points importants du système du Monde, T. II, pp. 136 et seq.

[†] Die Newton'sche Bahn zu wählen ist hier durchaus kein Grund vorhanden. Es sei denn, dass gewisse Vorurtheile gegen die Theorie der Elliptischen Functionen und ein Bestreben, die Existens derselben zu ignoriren, entscheidend sind.

[‡] Die Art und Weise wie Clairaut die Bewegung des Mondapogeums das erste Mal ermittelte, war im wesentlichen nichts anders als eine Ableitung der Bewegung der Apsiden der durch diese Gleichung bestimmten intermediären Bahn. Dass er dabei nur den halben Betrag der wirklichen Bewegung erhielt, war also die Folge davon, dass er in der That die gegen dem Radius vector senkrechte Kraft unberücksichtigt liess. Ebenso kann die durch die intermediäre Bahn, so wie Gyldén sie definirt, ermittelte Apsidenbewegung nicht die vollständige sein.

Wird—

$$dr = \beta_0 r_0 du_0$$

oder, was dasselbe ist,

$$dv_0 = \beta_0 \frac{\sqrt{c_0}}{r_0} du_0$$

gesetzt, so erhalten wir nach einmaliger Integration aus der ersten der Differentialgleichungen (6)—

$$\frac{1}{\beta^2}\left(\frac{dr}{du}\right)^2 = -c + 2\mu_1 r - hr^2 + \mu_2 r^4$$

wo der Index $_0$ überall weggelassen ist. h bedeutet wie c eine Integrationsconstante. Die Wurzeln der Gleichung—

$$\mu_2 r^4 - hr^2 + 2\mu_1 r - c = 0$$

sind alle reell, darunter zwei positiv; es sollen diese mit r_1 und r_2 bezeichnet werden. Weiter wird gesetzt—

$$\rho = r\sqrt{\mu_2}; \; \rho_1 = r_1\sqrt{\mu_2}; \; \rho_2 = r_2\sqrt{\mu_2}$$

$$\mu = \tfrac{1}{2}(\rho_2 + \rho_1), \;\; \nu = \sqrt{\frac{2\mu_1}{r_2 + r_1} + \left(\frac{\rho_2 - \rho_1}{2}\right)^2} \, *$$

Es ergiebt sich somit—

$$\frac{1}{\beta^2}\left(\frac{dr}{du}\right)^2 = -(r - r_1)(r - r_2)(\nu - \mu - \rho)(\nu + \mu + \rho).$$

Um diesen Ausdruck auf die Normalform für die elliptischen Functionen zu reduciren wird für r eine andere Variable y eingeführt, und zwar durch die Gleichung—

$$r = \frac{r_1 + my}{1 + ny},$$

wo m und n folgendermassen bestimmt werden—

$$m = \frac{(r_2 - r_1)(\nu + \mu)}{\nu + \mu + \rho_2}$$

$$n = -\frac{\rho_2 - \rho_1}{\nu + \mu + \rho_2}.$$

Wird ausserdem—

$$k^2 = \frac{2(\rho_2 - \rho_1)\nu}{(\nu + \mu + \rho_2)(\nu - \mu - \rho_1)}$$

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* ρ und ν hier sind nicht zu verwechseln mit denselben Bezeichnungen in den Ausdrücken für die Evection.

gesetzt, und β durch die Gleichung—

$$\beta = \frac{2}{\sqrt{(\nu - \mu - \rho_1)(\nu + \mu + \rho_2)}}$$

bestimmt, so erhalten wir—

$$y = \operatorname{sn} u$$

wo die Integrationsconstante der Kürze wegen fortgelassen ist. Die gesuchten Grössen lassen sich vortheilhaft durch die θ-Function ausdrücken, wenn noch folgende Bezeichnungen eingeführt werden—

$$n = -k^2 \operatorname{sn} \omega^2, \quad \frac{m}{r_1} = -k^2 \operatorname{sn} i\,\sigma^2,$$

wo $i = \sqrt{-1}$. Nach diesen Vorbereitungen können die folgenden Ausdrücke für den intermediären Radius Vector, die intermediäre Länge und die reducirte Zeit abgeleitet werden—

$$r = r_1 \left[\frac{\theta(\omega)}{\theta(i\sigma)}\right]^2 \frac{\theta\left(\frac{K}{\pi}\varepsilon + i\sigma\right)\theta\left(\frac{K}{\pi}\varepsilon - i\sigma\right)}{\theta\left(\frac{K}{\pi}\varepsilon + \omega\right)\theta\left(\frac{K}{\pi}\varepsilon - \omega\right)}$$

$$v - v_0 = (1 + \varsigma)\,\varepsilon + i \log \frac{\theta\left(\frac{K}{\pi}\varepsilon - i\sigma\right)}{\theta\left(\frac{K}{\pi}\varepsilon + i\sigma\right)}$$

$$N(\tau - \tau_0) = \varepsilon + \frac{N}{\sqrt{\mu_2}} \log \frac{\theta\left(\frac{K}{\pi}\varepsilon - \omega\right)}{\theta\left(\frac{K}{\pi}\varepsilon + \omega\right)}$$

wo v_0 und τ_0 Integrationsconstanten sind. ϵ wird intermediäre Anomalie genannt und ist mit u durch die Gleichung—

$$\varepsilon = \frac{\pi}{K} u$$

verbunden. ς und N bedeuten mittlere Bewegungen und ergeben sich aus den Ausdrücken—

$$1 + \varsigma = -i\,\frac{K}{\pi}\left[\frac{\theta'(\omega - i\sigma - iK')}{\theta(\omega - i\sigma - iK')} - \frac{\theta'(\omega + i\sigma - iK')}{\theta(\omega + i\sigma - iK')}\right]$$

$$\frac{\sqrt{\mu_2}}{N} = \frac{K}{\pi}\left[\frac{\theta'(\omega + i\sigma - iK')}{\theta(\omega + i\sigma - iK')} + \frac{\theta'(\omega - i\sigma + iK')}{\theta(\omega - i\sigma + iK')}\right]$$

Nachdem so die intermediäre Bahn bestimmt ist, erübrigt es die Differentialgleichungen der Evection und der Variation zu integriren. Die beiden höchst interessanten Paragraphen, in welchen Gyldén diese Aufgabe erledigt, werden wir vorlaüfig nur leicht berühren, da sie keine nothwendige Voraussetzung für das Verständniss der zweiten Abhandlung enthalten. In einem zweiten Aufsatze wollen wir aber einen vollständigen Ueberblick der für die Gyldén'sche Theorie eigenthümlichen Differentialgleichungen geben.

Die Differentialgleichungen der Evection und der Variation können nur durch successive Annäherungen integrirt werden, aber es sollen schon in der ersten Annäherung die Glieder zweiter Ordnung und die von langer Periode mitgenommen werden. Wird die Grösse ρ kleiner als eins vorausgesetzt, so können wir uns die rechte Seite der Gleichung (5) nach den Potenzen von ρ entwickelt denken, wonach wir die Differentialgleichung der Evection folgendermassen schreiben können—

$$\frac{d^2\rho}{dv_0^2} + (1 + \beta_0^{(1)} + \psi_1)\,\rho = \psi_0 + \psi_2\rho^2 + \psi_3\rho^3 + \cdot\ \cdot\ \cdot$$

$\beta_0^{(1)}$ ist eine Constante, ψ_0 . . . periodische Functionen von der Ordnung der störenden Kraft. Die in der ersten Annäherung zu integrirende Gleichung ist folglich—

$$\frac{d^2\rho}{dv_0^2} + (1 + \beta_0^{(1)} + \psi_1)\,\rho = \psi_0 \cdot$$

Diese wird in mehrere andere zerlegt, deren Integrale leichter zu ermitteln sind. Zu dem Zwecke wird vorausgesetzt, dass ψ_1 und ψ_0 durch folgende convergente Entwickelungen dargestellt werden können—

$$\psi_1 = \beta_1^{(1)} \cos(\lambda_1^{(1)}v_0 + b_1^{(1)}) + \beta_2^{(1)} \cos(\lambda_2^{(1)}v_0 + b^{(1)}) + \ldots$$
$$\psi_0 = \beta_0^{(0)} + \beta_1^{(0)} \cos(\lambda_1^{(0)}v_0 + b_1^{(0)}) + \beta_2^{(0)} \cos(\lambda_2^{(0)}v_0 + b_2^{(0)}) + \ldots$$

wo die β, λ und b Constanten sind. Von den λ können einige schr klein und also die entsprechende Glieder von langer Periode sein. Es wird nun angenommen, die Zahl der in ψ_1 enthaltenen merkbaren Glieder sei s, und ρ dann folgendermassen zerlegt—

$$\rho = \mathrm{R}_0 + \mathrm{R}_1 + \mathrm{R}_2 + \ldots + \mathrm{R}_s + \mathrm{R}_{s+1}\,.$$

Die Art und Weise, wie die Functionen R bestimmt werden, führt auf Differentialgleichungen von der Form—

$$\frac{d^2\mathrm{R}}{dv_0^2} + (1 + \beta_0 + \beta \cos(\lambda v + b)\,)\mathrm{R} = \mathrm{U} \quad .\ .\ .\quad \text{(A)}$$

wo U von R unabhängig ist. Diese Gleichung wäre also integrirt, wenn man—

$$\frac{d^2\mathrm{R}}{dv_0^2} + (1 + \beta_0 + \beta \cos(\lambda v + b)\,)\mathrm{R} = 0$$

integriren könnte. Direct kann man auch diese nicht integriren, und die bisherigen Versuche sie durch successive Annäherungen zu integriren sind unbefriedigend ausgefallen (vergl. Heine, Handbuch der Kugelfunctionen, 2te Auflage, Th. I, pp. 404–405). Dagegen ist die von Gyldén gegebene Methode eine vollständig zum Ziele führende. Er setzt—

$$\lambda + b = \frac{\pi}{\mathrm{K}}\,x$$

und eliminirt $\cos 2\frac{\pi}{2\mathrm{K}}x$ durch die Reihe—

$$\gamma_0^{(2)} + 2\gamma_2^{(2)} \cos 2 \operatorname{am} x + 2\gamma_4^{(2)} \cos 4 \operatorname{am} x + \ldots$$

wo die Coefficienten gemäss den Vorschriften in "Studien auf dem Gebiete der Störungstheorie," Mém. de St. Pétersbourg 1871, zu bestimmen sind. Mit n bezeichnen wir eine ganze positive Zahl, bestimmen den Modul k aus der Gleichung—

$$n\,(n+1)\,k^2 = 16\gamma_4^{(2)}\,\frac{\beta}{\lambda^2}\left(\frac{\pi}{2\mathrm{K}}\right)^2$$

und setzen—

$$-\,h = 4\,\frac{1+\beta_0}{\lambda^2}\left(\frac{\pi}{2\mathrm{K}}\right) + \frac{n\,(n+1)}{2}\,k^2 + 4\gamma_0^{(2)}\,\frac{\beta}{\lambda^2}\left(\frac{\pi}{2\mathrm{K}}\right)^2.$$

Dadurch geht die Differentialgleichung (A) nach gehöriger Reduction in—

$$\frac{d^2\mathrm{R}}{dx^2} - [n(n+1)k^2\,\mathrm{sn}\,x^2 + h]\,\mathrm{R} = \frac{4}{\lambda^2}\left(\frac{\pi}{2\mathrm{K}}\right)^2\mathrm{U} - 8\,\frac{\beta}{\lambda^2}\left(\frac{\pi}{2\mathrm{K}}\right)^2[\gamma_4^{\,2}\cos 4\,\mathrm{am}\,x + \ldots]\,\mathrm{R}$$

über. Die Coefficienten von R auf der rechten Seite kann man so klein machen, wie man will, wenn n hinreichend grosse Werthe erhielt. Durch fortgesetzte Annäherungen wird man dann diese Gleichung integriren können. Die Integration der Differentialgleichung der Evection ist also auf die Integration der Lamé'schen Gleichung—

$$\frac{d^2\mathrm{R}}{dx^2} = [n(n+1)k^2\,\mathrm{sn}\,x^2 + h]\,\mathrm{R}$$

zurückgeführt. Aber durch Hermite's klassische Aufsätze "Sur quelques applications des fonctions elliptiques" in den Comptes rendus wissen wir, wie diese Differentialgleichung integrirt werden kann.

Ueber die Integration der Differentialgleichung der Variation, die auch auf die Lamé'sche Gleichung zurückgeführt wird, findet man in den Comptes Rendus XCV einen erschöpfenden Aufsatz.

II.

In Folge der Gyldén'schen Methode die Differentialgleichungen zu integriren treten in den Integralausdrücken Glieder auf, in welchen die störende Masse nicht in die Coefficienten sondern in das Argument multiplicirt ist. Solche Glieder, deren Perioden entweder sehr lang oder nahe gleich den Umlaufszeiten sind, verschwinden nicht mit der störenden Masse. Gyldén nennt sie elementare Glieder, im Gegensatz zu den coordinirten Gliedern, welche mit der störenden Masse verschwinden. In den elementaren Gliedern sind die in der gewöhnlichen Störungstheorie sogenannten secularen Störungen enthalten. In dieser Hinsicht besitzt also die neue Theorie einen wesentlichen Vorzug vor der alten, denn da Gyldén die Integration derart ausführt, dass Bögen nicht ausserhalb der Zeichen *cos* und *sin* auftreten, wo dies der Natur des Problems fremd ist, leuchtet es ein, dass die Gyldén'sche Theorie im Stande sein wird mehr als Interpolationsformeln zu liefern. Löst man dagegen das Dreikörperproblem durch Entwickelung nach den Potenzen der störenden Kräfte, so kommt man nicht über Interpolationsformeln mit beschränkter Geltung hinaus. Grössere Ansprüche stellt die gegenwärtige Störungstheorie freilich nicht, aber in gewissen Fällen werden auch diese nicht befriedigt.

Wenn nun in passender Weise die elementaren Glieder ausgeschieden und die schon gewonnenen Resultate der intermediären Bahn zweckmässig verwendet werden, so wird man eine neue intermediäre Bahn herstellen können, die sich nur um Grössen von der Ordnung der störenden Kräfte von der wirklichen Bahn unterscheidet. Es wird zu keinem Wiederspruch führen, wenn man sich die Elemente einer solchen Bahn als absolute Constanten denkt—wenn sonst das System stabil ist—, dagegen können die Elemente der im Vorübergehenden besprochenen intermediären Bahn Naturgemäss nicht unveränderlich sein. Eine Bahn, deren Elemente absolute Constanten sind, und welche sich an die wirkliche Bahn bis auf Grössen von der Ordnung der störenden Kräfte anschliesst, nennt Gyldén *absolute Bahn.*

Die Bestimmung der absoluten Bahn kann selbstverständlich nur durch successive Annäherungen geschehen, zu dem Zwecke wird es also nöthig sein zuerst die intermediäre Bahn zu kennen. Uebrigens versteht es sich aber von selbst, dass es für die Kenntniss der absoluten Bahn nicht mehr ausreichen wird nur eine Function von r aus der Störungsfunction auszuscheiden, vielmehr muss die auszuscheidende Function sowohl eine Function von r wie von v sein.

Da es zunächst wichtig ist die absoluten Bahnen der Planeten abzuleiten, und zwar in erster Linie diejenigen der grossen Planeten, so wird in den folgenden Untersuchungen angenommen, dass die Excentricitäten kleine Grössen sind, weil eine solche Annahme wesentliche Erleichterungen herbeiführen wird. Es sollen demgemäss die elementaren Glieder von der ersten Ordnung genannt werden, wenn ihre Coefficienten die Excentricitäten, von der zweiten Ordnung, wenn sie die Quadrate oder Producte der Excentricitäten als Factor enthalten, u.s.w.

Unter der Annahme kleiner Excentricität lassen sich Ausdrücke für die intermediären Coordinaten ableiten, welche für die folgenden Untersuchungen zweckmässiger sind als die Ausdrücke, welche in der ersten Abhandlung gegeben sind.*

Wenn wir in der Differentialgleichung—

$$\frac{d^2R}{dv_0^2} + R - \frac{\mu_1}{c_0} - \frac{\mu_1}{c_0} F_0 = 0$$

$$aR = \frac{a}{r_0} = \frac{\mu_1 a}{c_0}(1 + \rho_0)$$

setzen, so erhalten wir sofort—

$$\frac{d^2\rho_0}{dv_0^2} + \rho_0 = F_0.$$

---

* Diese Ableitung hat auch den Vorzug, dass keine andere Annahme über die Function F nöthig ist, als dass sie eine Function von r ist.

$F_\circ$ können wir uns in die Reihe $\beta_\circ + \beta_1 \rho_\circ + \beta_2 \rho^2_\circ + \ldots$ entwickelt denken; denn $F_\circ$ ist eine Function nur von $r_\circ$ und $\rho_\circ$, also offenbar von der Ordnung der Excentricität. Die β-Coefficienten enthalten ausserdem die störende Masse als Factor. Die Differentialgleichung können wir daher schreiben—

$$\frac{d^2\rho_0}{dv_0^2} + (1 - \beta_1)\rho_0 = \beta_0 + \beta_2\rho_0^2 + \beta_3\rho_0^3 + \ldots$$

Abgesehen davon, dass hier die Coefficienten Constanten sind, ist diese Gleichung von derselben Form wie die Differentialgleichung der Evection. Die Integration wird auch in analoger Weise ausgeführt.

ρ_0 wird in zwei Theile zerlegt, nämlich $\rho_0 = R_0 + S_0$, und R_0 mittelst der Gleichung—

$$\frac{d^2R_0}{dv_0^2} + (1 - \beta_1) R_0 = \beta_3 R_0^3 \quad . \quad . \quad . \quad . \quad . \quad . \quad (\alpha)$$

bestimmt. Für S_0 erhalten wir dann—

$$\frac{d^2S_0}{dv_0^2} + (1 - \beta_1 - 3\beta_3 R_0^2) S_0 = B_0 + B_1 S_0 + B_2 S_0^2 + \ldots$$

wo folgende Bezeichnungen eingeführt sind—

$$B_0 = \beta_0 + \beta_2 R_0^2 + \beta_4 R_0^4 + \beta_5 R_0^5 + \ldots$$
$$B_1 = 2\beta_2 R_0 + 4\beta_4 R_0^3 + 5\beta_5 R_0^4 + \ldots$$
$$B_2 = \beta_2 + 3\beta_3 R_0 + 6\beta_4 R_0^2 + \ldots$$
$$. \quad . \quad . \quad . \quad . \quad . \quad . \quad .$$

Zerlegen wir ferner S_0 durch $S_0 = R_1 + S_1$ und bestimmen R_1 mittelst—

$$\frac{d^2R_1}{dv_0^2} + (1 - \beta_1 - 3\beta_3 R_0^2) R_1 = B_0 \quad . \quad . \quad . \quad . \quad . \quad (\beta)$$

so wird sich S_1 aus—

$$\frac{d_2S_1}{dv_0^2} + (1 - \beta_1 - 3\beta_3 R_0^2) S_1 = C_0 + C_1 S_1 + C_2 S_1^2 + \ldots$$

ergeben, wenn wir in ähnlicher Weise wie oben setzen—

$$C_0 = B_1 R_1 + B_2 R_1^2 + {'B_3} R_1^3 + \ldots$$
$$C_1 = B_1 + 2B_2 R_1 + 3B_3 R_1^2 + \ldots$$
$$C_2 = B_2 + 3B_3 R_1 + 6B_4 R_1^2 + \ldots$$

Wird in dieser Weise die Zerlegung fortgesetzt, so erhält man ρ unter der Form—

$$\rho_0 = R_0 + R_1 + R + \ldots$$

wo die Reihe auf der rechten Seite als convergent varausgesetzt wird.

Von den hier zu integrirenden Gleichungen bietet (α) keine wesentliche Schwierigkeiten. Sie giebt nach einmaliger Integration—

$$\left(\frac{dR_0}{dv_0}\right)^2 = g^2 - (1 - \beta_1) R_0^2 + \tfrac{1}{2}\beta_3 R_0^4$$

wo g eine Integrationsconstante bedeutet. Setzen wir—

$$\frac{g^2}{\kappa^2}(1 + k^2) = 1 - \beta_1$$
$$\frac{g^2}{\kappa^4}k^2 = \tfrac{1}{2}\beta_3$$

so können wir diese Gleichung wie folgt schreiben—

$$\left(\frac{d\mathrm{R}_0}{dv_0}\right)^2 = g^2\left(1 - \frac{\mathrm{R}_0^2}{\kappa^2}\right)\left(1 - k^2\frac{\mathrm{R}_0^2}{\kappa^2}\right)$$

woraus wir—

$$\mathrm{R}_0 = \kappa \operatorname{sn}\frac{g}{\kappa}(v_0 - v_0^0), \bmod k$$

erhalten. Dieser Ausdruck wird einfacher, wenn—

$$\frac{g}{\kappa}v_0 = x$$
$$\frac{g}{\kappa}v_0^0 = x_0$$
$$\kappa = \frac{k}{\sqrt{1+k}}\sqrt{\frac{2(1-\beta_1)}{\beta_3}}$$

gesetzt wird. Es ergiebt sich nämlich dann, wenn x_0 fortgelassen wird—

$$\mathrm{R}_0 = \kappa \operatorname{sn} x.$$

Die Integration der Gleichung (β), welcher die übrigen Differentialgleichungen zur Bestimmung von R_2, R_3 . . . vollkommen ähnlich sind, wird folgendermassen ausgeführt. Mit Rücksicht auf den gewonnenen Ausdruck für R_0 und nach Einführung von x als unabhängige Variable, nimmt (β) die Gestalt—

$$\frac{d^2\mathrm{R}_1}{dx^2} + \frac{\kappa^2}{g^2}[1 - \beta_1 - 3\beta_3\kappa^2 \operatorname{sn} x^2]\,\mathrm{R}_1 = \frac{\kappa^2}{g^2}\mathrm{B}_0 \quad . \quad . \quad . \quad . \qquad (\gamma)$$

an. Aus der Theorie der Lamé'schen Differentialgleichung wissen wir, dass diese Gleichung (die rechte Seite gleich Nul gesetzt) die beiden particulären Integrale—

$$\mathrm{R}_{1,1} = \mathrm{C}_1 \operatorname{cn} x \operatorname{dn} x$$
$$\mathrm{R}_{1,2} = \mathrm{C}_2 \operatorname{cn} x \operatorname{dn} x \int \frac{dx}{\operatorname{cn} x^2 \operatorname{dn} x^2}$$

hat. Wie man mit Hülfe dieser particulären Integrale zu dem folgenden Ausdruck des vollständigen Integrals der Gleichung (γ) gelangt—

$$\begin{aligned}\mathrm{R}_1 = \mathrm{C}_1 \operatorname{cn} x \operatorname{dn} x + \mathrm{C}_2\left\{\frac{\operatorname{sn} x}{k_1^2} - \frac{1+k^2}{k_1^4}\frac{\theta_1^1(x)}{\theta_1(x)}\operatorname{cn} x \operatorname{dn} x - \frac{l}{k_1^4}x \operatorname{cn} x \operatorname{dn} x\right\}\\ - \frac{\kappa^2}{g^2}\operatorname{cn} x \operatorname{dn} x \int \mathrm{B}_0\left[\frac{\operatorname{sn} x}{k_1^2} - \frac{1+k^2}{k_1^4}\frac{\theta_1^1(x)}{\theta_1(x)}\operatorname{cn} x \operatorname{dn} x\right]dx\\ + \frac{\kappa^2}{g^2}\left[\frac{\operatorname{sn} x}{k_1^2} - \frac{1+k^2}{k_1^4}\frac{\theta_1^1(x)}{\theta_1(x)}\operatorname{cn} x \operatorname{dn} x\right]\int \mathrm{B}_0 \operatorname{cn} x \operatorname{dn} x\,dx - \frac{\kappa^2}{g^2}\frac{l}{k_1^4}\operatorname{cn} x \operatorname{dn} x \int dx \int \mathrm{B}_0 \operatorname{cn} x \operatorname{dn} x\,dx\,.\end{aligned}$$

ist in der Vierteljahrsschrift d. a. G., *l. c.*, auseinandergesetzt. Eben daselbst ist die bemerkenswerthe und für die weiteren Untersuchungen wichtige Eigenschaft nachgewiesen, welche darin besteht, dass R_1 (und aus denselben Gründen auch R_2, R_3 etc.) rein periodisch ist, wenn $C_1 = 0$ und C_2 so bestimmt wird, dass die mit x multiplicirten Glieder verschwinden.*

Da die intermediäre Bahn als Mittel zum Uebergang zu der absoluten benutzt werden soll (und darinn liegt die grosse Bedeutung der intermediären Bahn) so ist es vorläufig nur von Interesse die elementaren Glieder in der Entwickelung von ρ_0 zu berücksichtigen; von den übrigen Gliedern wird es jedoch von Nutzen sein das constante Glied κ_0 von $R_1 + R_2 + \ldots$ zu behalten. Offenbar kann nur R_0 elementare Glieder liefern. Entwickeln wir die rechte Seite von—

$$R_0 = \kappa \sin \frac{g}{\kappa}(v_0 - v_0^0)$$

nach den Vielfachen des Argumentes, so ersieht man leicht, dass nur das erste Glied dieser Entwickelung elementaren Characters ist. Dieses Glied ist—

$$\kappa \frac{4}{k} \cdot \frac{\pi}{2K} \cdot \frac{\sqrt{q}}{1-q} \sin \sqrt{\frac{1-\beta_1}{1+k^2}} \cdot \frac{\pi}{2K}(v_0 - v_0^\circ).$$

Setzen wir dann—

$$\sqrt{\frac{1-\beta_1}{1+k^2}} \cdot \frac{\pi}{2K} = 1 - \varsigma\,; \quad \pi = (1-\varsigma)\,v_0^\circ - qv^\circ,$$

so erhalten wir den folgenden Ausdruck für ρ_0—

$$\rho_0 = \kappa_0 + \kappa_1 \cos\left[(1-\varsigma)\,v_0 - \pi\right].$$

ςv_0. kann hier aus verschiedenen Ursachen nicht der vollständige Ausdruck für die mittlere Bewegung der Apsiden sein; π bedeutet die Länge des Perihels. Der grösste Werth von ρ_0 ist $\kappa_0 + \kappa_1$, der kleinste $\kappa_0 - \kappa_1$, bezeichnen $\frac{1}{1-e}$ und $\frac{1}{1+e}$ die entsprechenden Werthe von $\frac{a}{r_0}$, so findet sich—

$$\frac{\mu_1 a}{c_0} = \frac{1}{(1+\kappa_0)(1-e^2)}$$

$$\frac{\kappa_1}{1+\kappa_0} = e$$

und—

$$\frac{a}{r_0} = \frac{1 + e\cos\left[(1-\varsigma)\,v_0 - \pi\right]}{1-e^2}$$

Diese Ausdrücke für ρ_0 und $\frac{a}{r_0}$ werden in der Folge Werwendung finden.

Ebenso wie bei der intermediären Bahn soll bei der absoluten Bahn v zerlegt werden, indem gesetzt wird—

$$v = v_0 + \chi$$

---

* Diese Formlen können, wie sich später zeigen wird, nahezu unverändert angewandt warden, wenn β_0 eine bekannte Function von der Form—

$$\Sigma\, a_i \cos(\sigma_i v_0 - A_i) + \Sigma\, b_i \cos((1-\sigma_i)\,v_0 - B_i)$$

wo σ_i von der Ordnung der störenden Kräfte ist.

Führen wir diesen Ausdruck in die Gleichung—

$$\frac{dr^2 \frac{}{dt}}{dt} = \frac{\delta(\Omega)}{\delta v}$$

ein und bestimmen v_0 durch—

so wird—

$$\left.\begin{array}{l} \dfrac{dr^2 \frac{dv_0}{dt}}{dt} = \dfrac{c_0}{r^2}[Q_{0\cdot 0}], \\[2em] \dfrac{dr^2 \frac{d\chi}{dt}}{dt} = \dfrac{\delta(\Omega)}{dv} - \dfrac{c_0}{r^2}[Q_{0\cdot 0}] \end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

$[Q_{0\cdot 0}]$ ist freilich eine Function sowohl von r und ρ wie von v; aber da ρ schliesslich durch v ausgedrückt wird, so kann $[Q_{0\cdot 0}]$ als eine Function nur von v angesehen werden. Das Differential der Zeit ergiebt sich nun folgenderweise; setzt man nämlich—

$$r^2 \frac{dv_0}{dt} = \sqrt{c_1}\ ,$$

so liefert die vorletzte Gleichung—

$$\sqrt{c_1}\, d\sqrt{c_1} = c_0\,[Q_{0\cdot 0}]\,dv_0$$

woraus, indem die Integrationsconstante $= c_0$ angenommen wird, leicht hervorgeht—

$$dt = \frac{r^2 dv_0}{\sqrt{c_0\,(1 + 2\int[Q_{0\cdot 0}]\,dv_0)}}\,.$$

In der absoluten Bahn braucht man offenbar nicht irgend einen Theil der Länge als Variationsglieder anzugeben. Dagegen wird es nöthig sein, den Ausdruck für die reducirte Zeit zu modificiren. Bezeichnet ζ die reducirte Zeit, Z eine noch zu bestimmende Function von v_0 und (r) den Radius vector der absoluten Bahn, so können wir folgenden Ausdruck aufstellen—

$$\frac{dt}{d\zeta} = \frac{r^2}{(r)^2}(1 + Z)$$

Ersetzen wir hier dt durch dv_0, so wird der Ausdruck für die reducirte Zeit—

$$d\zeta = \frac{(r)^2}{(1 + Z)\sqrt{c_1}}\,dv_0\,.$$

Eliminiren wir ebenso dt aus der zweiten der Gl. (7), so ergiebt sich—

$$\frac{\sqrt{c_1}}{r^2}\,\frac{d\sqrt{c_1}\,\frac{d\chi}{dv_0}}{dt} = \frac{\delta\,(\Omega)}{\delta v} - \frac{c_0}{r^2}[Q_{0\cdot 0}].$$

Die Differentialgleichung der Variation können wir also schreiben—

$$\frac{d^2\chi}{dv_0^2} + \frac{1}{2}\frac{d \log c_1}{dv_0}\cdot\frac{d\chi}{dv_0} = \frac{c_0}{c_1}\left\{\frac{r^2}{c_0}\,\frac{\delta\,(\Omega)}{\delta v} - [Q_{0\cdot 0}]\right\} \quad . \quad . \quad . \quad (8)$$

Eine ähnliche Gleichung soll nun auch für den Radius vector abgeleitet werden. Die zweite der Gleichungen (1) giebt nach Einführung von $v_0 + \chi$ statt v_0—

$$\frac{d^2r}{dt^2} - r\left\{\left(\frac{d\chi}{dt}\right)^2 + 2\frac{dv_0}{dt}\frac{d\chi}{dt} + \left(\frac{dv_0}{dt}\right)^2\right\} + \frac{\mu_1}{r^2} = \frac{\delta\,(\Omega)}{\delta r}\,.$$

Berücktichtigen wir die Relation—

$$\frac{dv_0}{dt} = \frac{\sqrt{c_1}}{r^2}$$

so finden wir—

$$\frac{d^2r}{dt^2} = -\frac{\sqrt{c_1}}{r^2}\,\frac{d\,\sqrt{c_1}\,\frac{d\frac{1}{r}}{dv_0}}{dv_0}$$

oder—

$$\frac{d^2r}{dt^2} = -\frac{c_1}{r^2}\,\frac{d^2\frac{1}{r}}{dv_0^2} - \frac{\sqrt{c_1}}{r^2}\,\frac{d\,\sqrt{c_1}}{dv_0}\,\frac{d\frac{1}{r}}{dv_0}.$$

Führen wir diese Ausdrücke in die Differentialgleichung ein, so können wir sie, nachdem wir mit a multiplicirt und $\frac{r^2}{c_0}\,a\,\frac{\delta(\Omega)}{\delta r}$ mit P bezeichnet haben, folgendermassen schreiben—

$$\frac{d^2\,\frac{a}{r}}{dv_0^2} + \frac{1}{2}\frac{d\log c_1}{dv_0}\,\frac{d\,\frac{a}{r}}{dv_0} + \frac{a}{r} - \frac{\mu_1 a}{c_1} + \frac{a}{r}\left[2\frac{d\chi}{dv_0} + \left(\frac{d\chi}{dv_0}\right)^2\right] = -\frac{c_0}{c_1}\,\mathrm{P}\;.\quad .\quad .\ (9)$$

Statt r wird es zweckmässig sein hier ρ mittelst—

$$\frac{a}{r} = \frac{1+\rho}{p_0}$$

einzuführen, wo—

$$\frac{1}{p_0} = \frac{\mu_1 a}{c_0}$$

ist; in gleicher Weise setzen wir auch—

$$\frac{1}{p_1} = \frac{\mu_1 a}{c_1}.$$

Dadurch wird die Differentialgleichung—

$$\frac{d^2\rho}{dv_0^2} + \frac{1}{2}\,\frac{d\log c_1}{dv_0}\,\frac{d\rho}{dv_0} + \rho = -\left(1 - \frac{p_0}{p_1}\right) - (1+\rho)\left\{2\frac{d\chi}{dv_0} + \left(\frac{d\chi}{dv_0}\right)^2\right\} - p_0\,\frac{p_0}{p_1}\,\mathrm{P}\ \ .\,(10,$$

Setzen wir nun in der absoluten Bahn—

$$\frac{1}{(p)} = \frac{\mu_1 a}{(c)}$$

$$\frac{a}{(r)} = \frac{1+(\rho)}{(p)}$$

$$\frac{a}{r} - \frac{a}{(r)} = \frac{(\mathrm{R})}{(p)}$$

so wird—

$$\frac{a}{r} = \frac{1+(\rho)+(\mathrm{R})}{(p)}$$

oder, wenn $(\rho) + (\mathrm{R}) = [\rho]$ gesetzt wird—

$$\frac{a}{r} = \frac{1+[\rho]}{(p)}.$$

Führen wir diesen Ausdruck in (9) ein, so finden wir für (ρ) eine der Gleichung (10) ähnliche Gleichung, die aber nicht ganz so einfach ist, denn (p) ist veränderlich und zwar eine Function von denselben Variabelen wir ρ. Der veränderliche Theil von (p) ist aber von höherer Ordnung als ρ. In die Diffrentialgleichung für $[\rho]$ soll dann $(\rho) + (\mathrm{R})$ eingeführt werden, die resultirende Gleichung zerfällt in Uebereinstimmung mit den Principien der absoluten Bahn in zwei andere, von welchen die eine für (ρ), die andere für (R) ist. Für die vorläufige Untersuchung der absoluten Bahn wird es aber bequemer sein eine Function $\bar{\rho}$ von (ρ) statt ρ selbst zuerst zu bestimmen. Setzt man nämlich—

$$\frac{a}{r} = \frac{1 + \bar{\rho} + \bar{\mathrm{R}}}{p_0}$$

wo—

$$\bar{\mathrm{R}} = \frac{p_0}{(p)}\,(\mathrm{R})$$

so wird—

$$\bar{\rho} = -\frac{(p) - p_0}{(p)} + \frac{p_0}{(p)}\,(\rho)$$

Hierdurch werden zwei Vortheile gewonnen: erstens wird die Differentialgleichung für $\bar{\rho}$ einfacher als diejenige für (ρ), und zweitens wird es nach der Bestimmung von $\bar{\rho}$ möglich (p) so zu wählen, dass (ρ) die einfachste Form erhält.

Um $\bar{\rho}$ zu bestimmen muss also der Ausdruck—

$$\rho = \bar{\rho} + \bar{\mathrm{R}}$$

in (10) substituirt werden. Diese Zerlegung von ρ ist noch wilkürlich. Erwägen wir aber, dass es vor allen Dingen nöthig ist die elementaren Glieder zu ermitteln, und dass der Ausdruck für den Radius vector keine Variationsglieder enthalten soll, so können wir die Zerlegung derart ausführen, dass $\bar{\rho}$ aus—

$$\frac{d^2\bar{\rho}}{dv_0^2} + \tfrac{1}{2}\,\frac{d \log c_1}{dv_0}\,\frac{d\bar{\rho}}{dv_0} + \bar{\rho} = \left(1 - \frac{p_0}{p_1}\right) - p_0\frac{p_0}{p_1}\,[\mathrm{P}],$$

bestimmt wird, oder da—

$$\frac{p_1}{p_0} = \frac{c_1}{c_0}$$

ist, durch—

$$\frac{c_1}{c_0}\,\frac{d^2\bar{\rho}}{dv_0^2} + \tfrac{1}{2}\,\frac{d\dfrac{c_1}{c_0}}{dv_0}\,\frac{d\bar{\rho}}{dv_0} + \frac{c_1}{c_0}\,\bar{\rho} = 1 - \frac{c_1}{c_0} + p_0\,[\mathrm{P}] \quad . \quad . \quad . \quad . \quad . \quad (11)$$

wo [P] die Summe der in P zu berücksichtigenden Glieder bedeutet, und ausserdem eventuell das constante Glied in $\left(\frac{d\chi}{dt}\right)^2$ enthalten kann. Die Differentialgleichung für $\bar{\mathrm{R}}$, die sich jetzt von selbst ergiebt, werden wir hier nicht anführen, da sie bei der vorbereitenden Untersuchung über die absolute Bahn keine Verwendung findet.

Die Gleichung (11) integrirt Gyldén zunächst durch die Methode der unbestimmten Coefficienten. Um diese Methode mit Erfolg anwenden zu können muss eine vorläufige Untersuchung über die Form von $\bar{\rho}$ angestellt werden. Dazu, und überhaupt um weiter gehen zu können, wird eine zweckmässige Entwickelung der

Störungsfunction erforderlich sein. Die im §VI der zweiten Abhandlung gegebene Methode die Störungsfunction zu entwickeln hat Gyldén schon in den Astr. Nachr. LXX auseinandergesetzt; hier werden wir uns damit beschränken müssen die Resultate, welche wir brauchen, anzuführen. Diese sind—

$$\frac{r^2}{c_0} a \frac{\delta(\Omega)}{\delta r} = \Sigma\,\Sigma\,\mathfrak{B}^{(0)}_{i,i'}\,\rho^i\rho'^{i'} + \left[\Sigma\,\Sigma\,\mathfrak{B}^{(1)}_{i,i'}\,\rho^i\rho'^{i'}\right] \cos \mathrm{H} + \ldots$$

$$\frac{r^2}{c_0} \frac{\delta(\Omega)}{\delta v} = \left[\Sigma\,\Sigma\,\mathfrak{A}^{(1)}_{i,i'}\,\rho^i\rho'^{i'}\right] \frac{\delta \cos \mathrm{H}}{\delta v} + \left[\Sigma\,\Sigma\,\mathfrak{A}^{(2)}_{i,i'}\,\rho^i\rho'^{i'}\right] \frac{\delta \cos 2\mathrm{H}}{\delta v} + \ldots$$

wo die ρ, wie früher bemerkt, mit den Radien vectoren durch—

$$\frac{a}{r} = \frac{1+\rho}{p_0},\ \frac{a_1}{r_1} = \frac{1+\rho^1}{p_0^{\ 1}}$$

verbunden sind, und H der Winkel zwischen r und r' ist. Die accentuirten Grössen beziehen sich auf den störenden Körper. Die Co-efficienten $\mathfrak{B}_{i,i'}$ und $\mathfrak{A}_{i,i'}$ sind rationale Functionen von p_0 und p_0'. Setzt man nun—

$$(\mathrm{R}) = (\rho_1) + (\rho_2) + \ldots$$

$$\frac{p_0}{(p)} (\mathrm{R}) = \overline{\mathrm{R}} = \bar{\rho}_1 + \bar{\rho}_2 + \ldots$$

woraus—

$$\frac{a}{r} = \frac{1 + (\rho) + (\rho_1) + \ldots}{(p)}$$

oder—

$$\frac{a}{r} = \frac{1 + \bar{\rho} + \bar{\rho}_1 + \ldots}{p_0}$$

so führt weder die Substitution von (ρ), noch diejenige von $\bar{\rho}$ auf vollständige Ausdrücke für die Differentialquotienten der Störungsfunction; es müssen also durch successive Annäherungen die (ρ_1), (ρ_2) etc. oder $(\bar{\rho}_1)$, $(\bar{\rho}_2)$ berücksichtigt werden.

Ebenso wie $\frac{r^2}{c_0} a \frac{\delta(\Omega)}{\delta r}$ durch P, soll $\frac{r^2}{c_0} a \frac{\delta(\Omega)}{\delta v}$ durch Q bezeichnet werden, und dann sollen $\mathrm{P}_{i,i'}$ und $\mathrm{Q}_{i,i'}$ die Werthe dieser Functionen bedeuten, welche die Grössen (ρ), $(\rho_1) \ldots (\rho_i)$ und $(\rho') \ldots (\rho_i')$ enthalten. Demgemäss bezeichnen also $\mathrm{P}_{0\cdot 0}$ und $\mathrm{Q}_{0\cdot 0}$ diejenige Werthe von P und Q, welche nur (ρ) und (ρ') enthalten.

Statt H soll v eingeführt werden. Es ist bekanntlich—

$$\cos \mathrm{H} = \cos(v + \Pi)\cos(v' + \Pi') + \sin(v + \Pi)\sin(v' + \Pi')\sin \mathrm{J}$$

wo Π der Bogen zwischen dem aufsteigenden Knoten der Bahn des gestörten Körpers auf der Bahn des störenden und dem Anfangspunkt von v ist, und Π der Bogen zwischen demselben Knoten und dem Anfangspunkt von v'. J ist die gegenseitige Neigung der beiden Bahnen. Diese Grössen sind veränderlich, aber die Differenz $\Pi - \Pi'$ ist nahe constant wenn J klein ist, was angenommen wird. Setzen wir—

$$\Pi - \Pi' = \Delta,\ \cos^2\tfrac{1}{2}\mathrm{J} = \mu,\ \sin^2\tfrac{1}{2}\mathrm{J} = \nu,$$

so wird—

$$\cos \mathrm{H} = \mu \cos(v - v' + \Delta) + \nu \cos(v + v' + \Pi + \Pi').$$

Durch die von Herrn Tisserand in den Pariser Annalen Bd XV. dargestellte Methode lässt sich dann H durch v und v' in den Differentialgleichungen der Störungsfunction ersetzen.

Bei der Integration der Gl. (11) beschränkt sich Gyldén auf die Ermittelung der elementaren Glieder erster, zweiter, dritter und vierter Ordnung. Man braucht daher in den Ausdrücken von P und Q nicht über die Glieder in cos H und $\frac{\delta \cos \mathrm{H}}{\delta v}$ hinauszugehen, da die übrigen Glieder auf Grössen höherer Ordnung führen.

Da der Ausdruck von ρ' in der absoluten Bahn noch nicht als bekannt angenommen werden darf, so muss der angeführte intermediäre Ausdruck angewandt werden; dabei kann aber das constante Glied weggelassen werden, weil dasselbe keine elementaren Glieder verursacht. Es sei also—

$$\rho' = \kappa' \cos \left[(1 - \varsigma') v_0' - \Gamma' \right].$$

Bedenkt man nun, dass die Argumente der elementaren Glieder im Allgemeinen nicht die mittlere Bewegung oder die mittlere Länge des störenden Körpers, sondern nur die mittlere Bewegung der Apsiden desselben enthalten, so ist ersichtlich, dass solche Glieder nur von den Producten $\rho' \cos v'$, $\rho' \sin v'$, $\rho'^2 \cos 2v'$, $\rho'^2 \sin 2v'$ etc. herrühren können; und zwar sind die, welche aus $\rho' \cos v'$ und $\rho' \sin v'$ entstehen—

$$\tfrac{1}{2}\kappa' \cos (\varsigma' v_0' + \Gamma') \text{ und } \tfrac{1}{2}\kappa' \sin (\varsigma' v_0' + \Gamma')$$

Die übrigen Producte geben Glieder höherer Ordnung als die hier mitgenommen werden soll. Mit Rücksicht auf die hier beabsichtigte Genauigkeit kann man $v_0' = \mu v_0$ setzen, wo μ das Verhältniss $\frac{n'}{n}$ der mittleren Bewegungen bedeutet. Die Ausdrücke, welche für $\rho' \cos v'$ und $\rho' \sin v'$ in P und Q eingeführt werden sollen, sind also—

$$\tfrac{1}{2}\kappa' \cos (\mu\varsigma' v_0 + \Gamma'), \;\; \tfrac{1}{2}\kappa' \sin (\mu\varsigma' v_0' + \Gamma').$$

Dadurch wird—

$$\rho' \cos \mathrm{H} = \tfrac{1}{2}\kappa' \cos^2 \tfrac{1}{2}\mathrm{J} \cos (v_0 - \mu\varsigma' v_0 + \Delta - \Gamma')$$
$$\rho' \frac{\delta \cos \mathrm{H}}{\delta v} = \tfrac{1}{2}\kappa' \cos^2 \tfrac{1}{2}\mathrm{J} \sin (v_0 - \mu\varsigma' v_0 + \Delta - \Gamma')$$

und—

$$\mathrm{P} = \Sigma\, \mathfrak{B}^{(0)}_{s,0}\, \bar{\rho}^{s} + \tfrac{1}{2}\kappa' \cos^2 \tfrac{1}{2}\mathrm{J} \cos (v_0 - \mu\varsigma' v_0 + \Delta - \Gamma')\; \Sigma\, \mathfrak{B}^{(1)}_{s,1}\, \bar{\rho}^{s}$$
$$\mathrm{Q} = -\tfrac{1}{2}\kappa' \cos^2 \tfrac{1}{2}\mathrm{J} \sin (v_0 - \mu\varsigma' v_0 + \Delta - \Gamma')\; \Sigma\, \mathfrak{A}^{(1)}_{s,1}\, \rho_0^{s}$$

indem in P $\bar{\rho}$ statt ρ gesetzt ist. Substituiren wir nun in Q den Ausdruck für ρ_0, den wir von der intermediären Bahn kennen, nämlich—

$$\rho_0 = \kappa_0 + \kappa_1 \left[(1 - \varsigma) v_0 - \Gamma \right)$$

und setzen—

$$\mathrm{V}_0 = (1 - \varsigma) v_0 - \Gamma$$
$$\mathrm{V}_1 = (\varsigma - \mu\varsigma') v_0 + \Delta + \Gamma - \Gamma'$$

so soll folgender Ausdruck angenommen werden—

$$[\mathrm{Q}_{0 \cdot 0}] = -a_2 \sin (\mathrm{V}_0 + \mathrm{V}_1) + a_3 \sin \mathrm{V}_1$$

wo a_2 und a_3 leicht zu ermitteln sind. Aus der Gleichung—

$$\sqrt{c_1}\, d\sqrt{c_1} = c_0\, [\mathrm{Q}_{0\cdot 0}]\, dv_0$$

findet man dann—

$$\frac{c_1}{c_0} = \frac{p_1}{p_0} = 1 + \mathfrak{A}_2 \cos(\mathrm{V}_0 + \mathrm{V}_1) - \mathfrak{A}_3 \cos \mathrm{V}_1$$

wenn—

$$\mathfrak{A}_2 = \frac{2a_2}{1 - \mu\varsigma'}$$
$$\mathfrak{A}_3 = \frac{2a_3}{\varsigma - \mu\varsigma'},$$

ist. Aus der Form, welche der vorstehende Ausdruck von P annimmt, wenn der intermediäre Ausdruck für ρ_0 eingeführt wird, schliesst Gyldén, dass die bedeutendsten Glieder im Ausdrucke für $\bar{\rho}$ die Argumente V_0, $\mathrm{V}_0 + \mathrm{V}_1$ und V_1 enthalten müssen; er setzt also—

$$\bar{\rho} = \kappa_0 + \kappa_1 \cos \mathrm{V}_0 + \kappa_2 \cos(\mathrm{V}_0 + \mathrm{V}_1) + \kappa_3 \cos \mathrm{V}_1 + \bar{\bar{\rho}}.$$

$\bar{\bar{\rho}}$ bedeutet die Summe der nicht angesetzten Glieder. Wenn hier κ_1 und κ_2 als Grössen der ersten Ordnung angesehen werden, so wird es sich zeigen, dass κ_3 von der dritten und κ_0 von der vierten Ordnung ist. Bildet man nun $\bar{\rho}^2$ und $\bar{\rho}^3$ (weiter braucht man der gesteckten Genauigkeitsgrenze gemäss nicht zu gehen) und führt sie in P ein, so findet sich—

$$[\mathrm{P}_{0\cdot 0}] = b_0 + b_1 \cos \mathrm{V}_0 + b_2 \cos(\mathrm{V}_0 + \mathrm{V}_1) + b_3 \cos \mathrm{V}_1 + [[\mathrm{P}_{1\cdot 0}]]$$

wo die Ausdrücke für b_0, b_1, b_2 und b_3 ohne Schwierigkeit zu finden sind. $[[\mathrm{P}_{1\cdot 0}]]$ ist die Summe der übrigen in P enthaltenen Glieder. Setzt man die so gewonnenen Ausdrücke in die Gl. (11), so findet man ausser einer Differentialgleichung in $\bar{\bar{\rho}}$ vier Bedingungsgleichungen, aus denen die Grössen κ_0, κ_1, κ_2 und κ_3 zu bestimmen wären; indessen fällt κ_1 als gemeinschaftlicher Factor gerade aus der Gleichung fort, durch welche diese Grösse bestimmt werden sollte, aber die übrig bleibende Bedingung kann zur Bestimmung von ς angewandt werden. κ_1 muss also anderweitig bestimmt werden. Dies kann folgenderweise geschehen. Der intermediäre Ausdruck für $\frac{a}{r_0}$ kann man schreiben—

$$\frac{1 + e \cos(\mathrm{V}_0 + \Gamma - \pi)}{1 - e^2}$$

woraus sich ergiebt—

$$\bar{\rho} = \frac{p_0}{1 - e^2} - 1 + \frac{p_0}{1 - e^2}\,[e \cos(\pi - \Gamma) \cos \mathrm{V}_0 + e \sin(\pi - \Gamma) \sin \mathrm{V}_0].$$

Vergleicht man diesen Ausdruck mit dem obigen, κ_0, κ_1 etc. enthaltenden, so findet man Bedingungsgleichungen, aus welchen κ_1 und Γ durch successive Annäherungen bestimmt werden können, indem man beachtet, dass $2\kappa_1\kappa_2$ ein angenäherter Werth von κ_3 ist.

Die Differentialgleichung in $\bar{\bar{\rho}}$ wird dann dazu benutzt die Glieder dritter und vierter Ordnung zu berechnen; die für diesen Zweck nöthige Bedingungsgleichungen erhält man leicht mit Rücksicht darauf, dass $\overset{=}{\rho}$ keine Glieder erster und zweiter Ordnung enthalten darf.

Die Function Z bestimmt Gyldén der Bedingung gemäss, dass die reducirte Zeit keine Glieder langer Periode enthalten.

Die zweite Abhandlung wird durch eine numerische Anwendung der hier auseinandergesetzten Theorie beschlossen.

Die d'Alembert'sche Methode die Differentialgleichung (11) zu integriren leistet freilich für die vorbereitenden Untersuchungen gute Dienste; wollte man aber in derselben Weise fortsetzen, so würden die Bedingungsgleichungen so complicirter Natur werden, dass man schliesslich nicht mit gehöriger Sicherheit die Unbekannten ermitteln könnte. Nachdem aber angenäherte Resultate schon gewonnen sind, kann man Methoden anwenden, welche erlauben die Annäherungen so weit zu treiben wie man wünscht, ohne dass die Rechungsschwierigkeiten sich anhäufen. Die mathematischen Schwierigkeiten sind dazu wesentlich überwunden, indem die Differentialgleichungen von den Formen—

$$\frac{d^2\chi}{dv_0^2} = W$$

$$\frac{d^2\rho}{dv_0^2} + (1 - \beta_1 + \psi_1)\,\rho = \psi_0$$

$$\frac{d^2\rho}{dv_0^2} + (1 - \beta_1 + \psi_1)\,\rho = \psi_0 + \psi_2\,\rho^2 + \psi_3\,\rho^3$$

integrirt sind; denn auf diese lassen sich die übrigen für die Gyldén'sche Theorie eigenthümlichen Differentialgleichungen reduciren, Gyldén nennt daher diese Formen "canonische" Formen.

Von besonderer Wichtigkeit in der Störungstheorie im Allgemeinen, aber für die Kenntniss der absoluten Bahn unumgänglich nothwendig, ist das Wegschaffen der in den Integralausdrücken explicite erscheinenden Bögen, wenn dies der Natur des Problems zuwieder ist; in unerwartet einfacher Weise lässt sich dies erreichen durch die Anwendung der Lamé'schen Gleichung. Nur durch successive Bestimmungen von überzähligen Integrationsconstanten kann man nämlich mit beliebiger Vollständigkeit die nach und nach auftretenden Glieder besagter Form aus dem Resultate eliminiren.

Pulkowa, Sept. 20, 1882.

NOTES.

The American Commission on the Transit of Venus has equipped eight parties for observing the coming Transit, the same number as in 1874. The principal instruments are, as then, the horizontal photoheliographs and the 5-inch Equatoreals. The stations and the chiefs of the respective parties are: Cape Town, Prof. S. Newcomb; Santa Cruz, Patagonia, Lieutenant S. W. Very; Santiago de Chili, Prof. Lewis Boss; New Zealand, Mr. Edwin Smith; San Antonio, Texas, Prof. A. Hall; Washington, D.C., Prof. W. Harkness; Cedar Keys, Florida, Prof. J. R. Eastman; Fort Thorn, New Mexico, Mr. George Davidson. The Commission has issued a very valuable pamphlet, "Instructions for observing the Transit of Venus," which chiefly deals with the photographic work and enters very fully into the adjustments of the instruments and the photographic processes to be employed (collodio-bromide emulsion process with alkaline development); dry plates are to be used.

The French stations are: Port-au-Prince (M. d'Abbadie), Mexico (M. Bouquet de la Grye), Martinique (M. Tisserand), Florida (Col. Perrier), Santa Cruz (Capt. Fleuriais), Chili (M. de Bernardieres), Chubut (M. Hatt), Rio Negro (M. Perrotin).

The Brazilian stations are: St. Thomas, Magellan, Pernambuco and Rio de Janeiro (*Comptes Rendus*, Oct. 2.)

The German stations are: (Punta Arenas (Prof. Auwers), Bahia Blanca (Dr. Hartwig), Aiken (Dr. Franz) and Hartford (Dr. Müller). *Astr. Nachr.* No. 2469.

We have received *The American Ephemeris and Nautical Almanac* for 1885. The contents of the volume are similar to those of the volumes for 1882–84 (see this journal Vol. I, p. 58). For the additional stars, of which only the mean places were given in previous volumes, the apparent Right Ascensions are now given. The diagrams of the apparent orbits of the satellites of the outer planets, as well as many minor details (such as the star-constants f, G and H given both in arc and in time, with the Sidereal Hour of the corresponding Midnight) make this ephemeris extremely convenient and useful.

A National Observatory is being founded at Chapultepec, Mexico, which is to be under the direction of Senhor Anguiano. The principal instrument will be a 15-inch Refractor by Grubb.

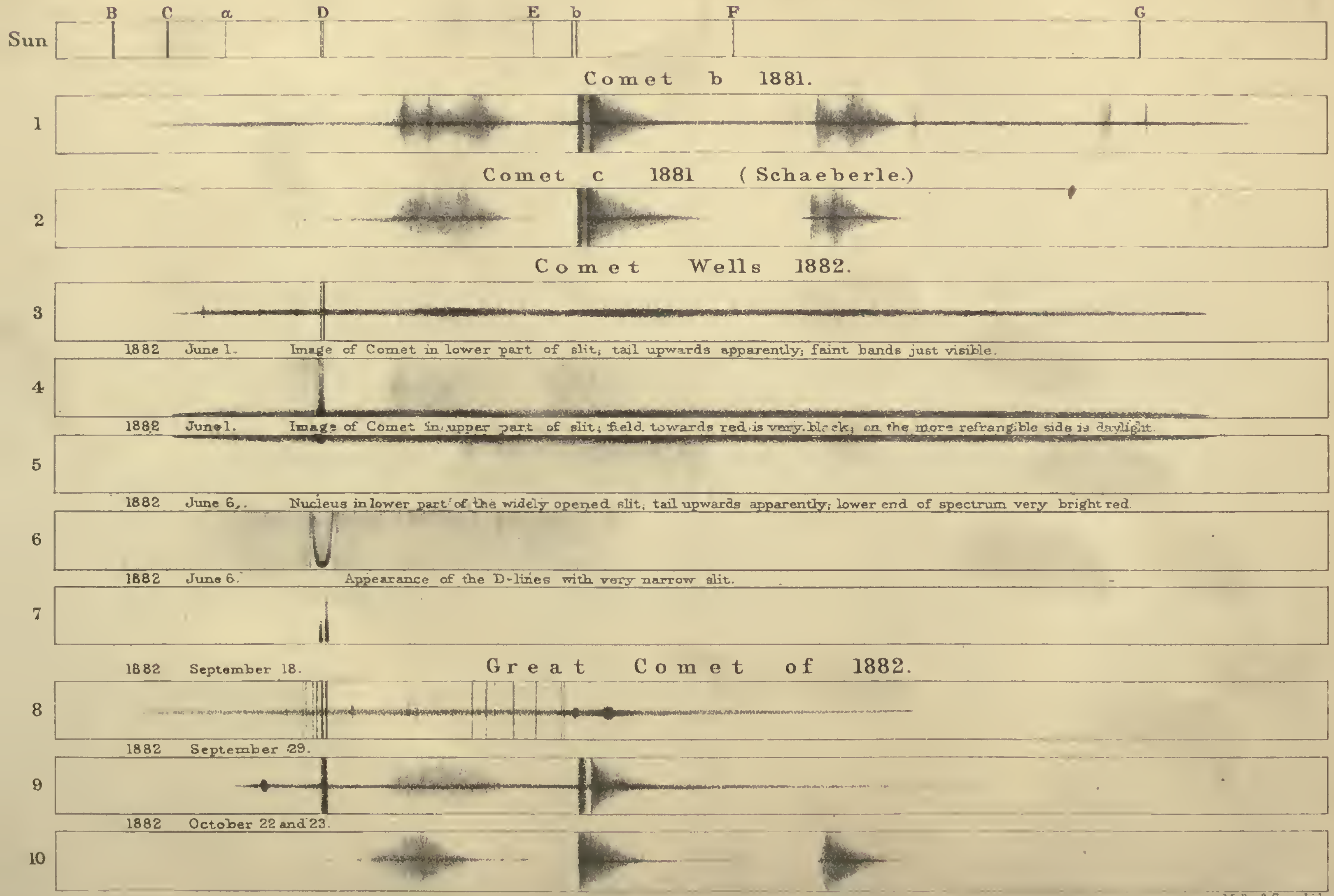
The annual Report of Prof. Hough, Director of the Dearborn Observatory, Chicago, gives a full account of the physical observations of Jupiter which have been carried on with the 18-inch Refractor during the last three oppositions. Prof. Hough, who maintains that micrometrical observations are greatly superior to any other method of estimation in determining the positions of markings on the surface, thinks that the phenomena seen on the latter will be found to be periodical in a manner analogous to that exhibited on the Sun. He believes the markings to be comparatively permanent in locality and that the surface of the planet is liquid or in a plastic condition. The observations of 1879 and 1880 showed that the red spot was retrograding with increasing velocity. This drifting has continued up to the present time with great uniformity. The report contains ten drawings of Jupiter and two wood cuts representing the Observatory building and the interior of the dome.

At the annual meeting of the Board of Governors of the Armagh Observatory (October 14) a committee was formed to raise a fund for the purpose of erecting in the Observatory an Equatoreal Refractor in memory of the late Director, Dr. T. R. Robinson. The committee has just issued a circular, making an appeal to the friends and colleagues of Dr. Robinson to contribute to the fund. It is hoped that sufficient means may be obtained to erect an 8-inch refractor in one of the existing domes at Armagh.

We have been obliged to defer to our December number an account of the Dun Echt Observations of the Great Comet.

Spectra of Comets

observed at Lord Crawford's Observatory Dun Echt, 1881-2.



Comet b 1881.

Comet c 1881 (Schaeberle.)

Comet Wells 1882.

1882 June 1. Image of Comet in lower part of slit; tail upwards apparently; faint bands just visible.

1882 June 1. Image of Comet in upper part of slit; field towards red is very black; on the more refrangible side is daylight.

1882 June 6. Nucleus in lower part of the widely opened slit; tail upwards apparently; lower end of spectrum very bright red.

1882 June 6. Appearance of the D-lines with very narrow slit.

Great Comet of 1882.

1882 September 18.

1882 September 29.

1882 October 22 and 23.

Malby & Sons Lith.

SPECTROSCOPIC OBSERVATIONS OF COMETS III. AND IV., 1881, COMET I., 1882, AND THE GREAT COMET OF 1882.

BY RALPH COPELAND AND J. G. LOHSE.

(Communicated by the Earl of CRAWFORD and BALCARRES.)

Although on former occasions several comets have been spectroscopically examined at Dun Echt, yet none of them could compare in brightness with any of the four comets, the spectroscopic observations of which we are now about to give. And it is only natural that under the more favourable conditions the spectroscope should better disclose the real nature of such phenomena as the several maxima noticed in the first or least refrangible band of Winnecke's Comet (*b* 1877) and of Hartwig's Comet (*d* 1880), and the well-defined sharp edge of the brightest or second band. That, however, the near approach to the Sun of Comet Wells, and of the Great Comet of 1882, and the vast increase of temperature connected therewith, should develop in their spectra bright lines, and the brightest of them in regions outside that of the three usual bands, and of a brilliancy far surpassing that of the bands, could not well be foreseen, although in the case of Comet Wells we expected, long before any trace of the D-lines became visible, that something quite unusual would appear in the spectrum near the time of perihelion passage. For the very bright and extended continuous spectrum indicated a dense, compact nucleus, which could not but undergo violent changes, while under the influence of the intense radiation of solar heat. But even if no displacement of the lines had been observed, the very circumstance that such bright lines existed in the spectra of these two comets, arising almost certainly from substances also existing on our Earth, would be sufficient to make the two comets rank as the most important ones yet observed with the spectroscope. On the other hand it is worthy of note, that, in spite of their great brightness, Comet Schaeberle (1881 IV.), and the Great Comet of 1882, when last seen on October 22 and 23, showed nothing except the three bands and only very faint traces of a continuous spectrum, both comets at the same time exhibiting very bright and long tails.

Comet III., 1881.

Although the earliest information respecting this comet, received in Europe, was telegraphed by Dr. Gould from Buenos Ayres, on June the first, it had already been perceived by Mr. J. Tebbutt, at Windsor, N.S.W., as early as May 22. The great resemblance of its elements to those of the comet of 1807, announced by Dr.

Gould, together with rough elements, telegraphed by H.M. the Emperor of Brazil, aided greatly in preparing northern observers for its sudden apparition on June 22. It was not, however, until June 25 that it could be examined at Dun Echt. The diameter of the bright nucleus was twice on that day found to be 6″·8 and 5″·6. It threw out a number of most remarkable jets, which varied from night to night, but here we must restrict ourselves to observations of its spectrum. That of the nucleus was intensely bright and in general strictly continuous, while that of the coma consisted of luminous bands.

We shall now give the observations in the order and, as nearly as possible, in the exact words, in which they were recorded, in general, however, omitting the actual measures and referring the reader to the tabular statement of the resulting wave-lengths.

Observations.

1881.

June 25.—The spectrum of the nucleus is continuous; that of the coma shows the usual bands. With a narrow slit there are indications of many lines just beyond the verge of distinct visibility. A dark band was noticed at wave-length $567^{\text{mmm}}\cdot 9$. The sky was then very bright, too bright for making further observations. A little before sunrise the comet became invisible in the telescope. Sodium used as a comparison spectrum.

June 28.—The first band almost extends to the second band, it does not appear black until just before the great line. With a very narrow slit a very bright line ($517^{\text{mmm}}\cdot 9\underline{+}$) stands completely separate and distinct at the beginning of the green band; there is a second, but fainter line. The blue band, although fairly sharp at the redward edge ($474^{\text{mmm}}\cdot 9\underline{+}$), is not bounded by any distinct line like that in the green band.

June 29.—The brightest line passes across both the nucleus and coma; this is also the case with the other line, which however is fainter. There are several lines in the third or blue band. Sodium used as comparison spectrum.

July 27.—Three bands easily visible, but the least refrangible one presents the usual difficulty of measurement, viz., that it seems to have two or more maxima. The green band has again a distinct detached bright line at the redward end. There was also a trace of a fourth band in the violet. The continuous spectrum of the nucleus is far fainter than at the end of June, but in moments of good definition it seemed to be crossed by many *very* short bright bands. The spectrum seemed to change in intensity from moment to moment, like a dancing aurora borealis, but this may have been due to passing clouds.—R. C.

On August 20th, both comets, III. and IV., were seen with the naked eye.—R. C.

August 22.—The comet is still visible to the naked eye. The spectrum of the nucleus is much more continuous than that of Comet IV.; it ends at wave-length 447^{mmm}.

The observations were made with the 15·06-in. refractor, and the Grubb compound prism in the Grubb spectroscope.

In the tabular arrangement of the wave-lengths some of the repeatedly measured objects are described by different words, as *limit, maximum, line,* because they were not always seen alike and with the same distinctness, and every time they were measured, their appearance was described by the most suitable term.

Observed Wave-lengths for the different parts of the Spectrum of the Great Comet 1881, III.

	Object.	June 25. Wave-length.	June 28. Wave-length.	June 29. Wave-length.	July 27. Wave-length.
		mmm.	mmm.	mmm.	mmm.
	Red limit of spectrum of nucleus,	664·0	670·5		
	Middle of a *dark* band,	567·9			
1st Band.	Red limit of first band; first maximum of the band; a line,	564·1 563·0	563·0	563·0 563·0 563·1	
	Middle; second maximum of broad band; brightest part of band,	555·5	553·0	555·9 556·2	546·8 ? 556·6
	Still bright at,				539·7
	The band extends to,		534·5		
	But does not appear black until just before the great line.				
2nd Band.	Bright line, well defined, standing alone at commencement of green band,	517·5 516·8 516·0	517·9 516·4 517·3 516·9 515·9 515·5	516·3 516·0	517·0 517·0
	A second line, fainter,		514·9	513·3 512·7 512·8	
	Declines in brightness up to,		498·1		
3rd Band.	Fairly sharp edge of band,	471·7	474·9		
	Maximum of band,		468·4 465·3	468·3 468·1	
	End of band,		461·0		
	A trace of light,			457·7	
4th Band.	A faint group of bright lines,	434·0	434·6 434·0	437·1	
	A bright line, common to spirit lamp and comet,		430·2		
	Violet end of spectrum of nucleus,		419·0		

The spectroscope is so constructed that the spectrum of the object under examination can be seen together with the comparison spectrum which appears above and below it. We were thus enabled to compare the spectra of this and of some of the other comets directly with the hydro-carbon spectrum of a spirit lamp and found that they always agreed in wave-length and outward appearance.

Comet 1881, *IV.* (SCHAEBERLE).

This comet (also known as Comet *c* 1881) was discovered by Mr. J. M. Schaeberle at Ann Arbor on the 14th of July (civil reckoning). It was studied with the spectroscope at Dun Echt on one occasion only.

Aug. 22, 1881.—In comparison to Comet III. this comet showed but a feeble continuous spectrum which barely sufficed to connect the bands. In fact it may be considered as a specially bright representative of the numerous class of faint comets which exhibit spectra, composed of three luminous bands. Particularly interesting is the measurement of the excessively faint line at 599^{mmm}, which in spite of the very considerable deviation *may* have been an extremely faint trace of the D-line. It should be remembered that this line was so faint as almost to defy any attempt at measurement, so the resulting wave-length can but count for little beyond a guarantee, that a faint line was seen near the place of the Sodium lines. If, however, the observed wave-length is considered fairly correct, it corresponds to a luminosity, almost amounting to a line, which was repeatedly seen and measured in Comet Wells. This view is to a certain extent supported by the total disappearance of the D-lines in the spectrum of the Great Comet of 1882, when last seen on October 22 and 23, while the three bands were still very bright.

The observations were made with the Grubb compound prism in the Grubb spectroscope, and Sodium was used as comparison spectrum.

Observed Wave-lengths for the different parts of the Spectrum of Comet 1881, *IV.*, (*Schaeberle*).

	Object.	August 22. Wave-length.
		mmm.
	Faint traces of a band; the spectrum here decidedly red,	599·0
	Beginning of the continuous spectrum,	593·9
1st Band.	Beginning of first band,	566·8
	First maximum in band 1,	561·2
	” ”	557·3
	Traces of 4 or 5 bright lines on band 1, beginning at,	561·0
	Almost a uniform brightness up to,	546·1
	End of bright maximum,	545·3
	The continuous spectrum just goes through, it almost fades off immediately before the great bright line.	
	Second maximum in band 1,	544·4
	End of first band,	534·2
2nd Band.	Great bright line, standing alone at the commencement of the second band. There is a stellar nucleus in the continuous spectrum at the intersection of the bright line,	516·3 516·7 516·8 516·4 516·8 516·2
	A bright line,	515·7 515·6
	A bright point in the spectrum of the nucleus; this is perhaps a third one, but may be identical with the one in the second bright line, for they were very small and difficult to fix.	513·1
	End of second band,	497·7
3rd Band.	Beginning of third band, rather sharp,	473·9 474·1
	Maximum of last band,	470·0 470·1 470·1
	End of spectrum,	461·3

Comet Wells, 1882, *I*.

This comet was discovered by Mr. C. S. Wells, Assistant at the Dudley Observatory, Albany, on March 17. The spectrum was first examined here on April 21, and found to be almost altogether continuous like a star spectrum. It differed so entirely from the usual band-spectrum, that from the appearance of the spectrum alone the real nature of the object in view would hardly have been recognised at once. At the same time it proved that the amount of light, emitted by the small coma, visible in the telescope, was quite insignificant, compared with that of the highly condensed nucleus. This peculiar conduct of the comet invited conjecture and led us to expect something unusual, as already mentioned in the introduction. From time to time it was viewed through different direct vision prisms, held before the eye-piece, but on May 27 the Grubb spectroscope was again resorted to. To obtain a brighter spectrum the compound prism was replaced by a Steinheil 60° prism, for which the reducing curve has already been given in "Copernicus," Vol. II., p. 109. Soon a short bright line was noticed, but only on favourable moments; the increasing twilight, however, prevented measuring it. On May 28 the line was already considerably brighter, and coincided with the D-line of a spirit-lamp. The position of the comet was so unfavourable, and the phenomena it presented so interesting and wholly unexpected, that on Monday, May 29, we considered it safest to publish in a Dun Echt Circular (No. 52), the following notice: "The spectrum of the nucleus of Comet Wells deserves the closest attention, as it shows a sharp bright line coincident with D, as well as strong traces of other bright lines, resembling in appearance those seen in the spectra of γ Cassiopeiæ and allied stars."

In the sequel it will be seen, that none of the suspected lines ever attained the sharpness of D, although this was very nearly the case with 620^{mmm} + and 615^{mmm} ±.

For some unknown reason the D-line of the comet, compared with that of the spirit lamp, always appeared a little reddish.

The observations are given in full; for the reducing curve see "Copernicus," Vol. II., p. 109.

Observations.

1882.

April 21.—Spectrum with Grubb compound prism almost altogether continuous, with a feeble indication of bright bands. Sky not very favourable at time of spectroscopic observations.—R. C. & J. G. L.

May. 27.—Grubb spectroscope, Steinheil 60° prism. Noticed for the first time a bright line in the less refrangible end of the spectrum, but it was still so faint as to require some attention to see it, and during moments of bad definition it became invisible.—R. C. and J. G. L. (Date and heading "Comet 1882 *a* (Wells)" only, in note-book.)

1882.

May 28.—Grubb spectroscope, Steinheil 60° prism; lowest power eye-piece.

	Micrometer Reading.		Reduction to Curve.
A bright line,	62^{r}	$17^{p}\cdot 8$	$+ 24^{p}\cdot 9$
Beginning of spectrum,	63	56·5	+ 24·9
A bright line (a feeble cylindrical lens before the eye-piece),	62	11·6	+ 24·9
	62	17·5	+ 24·9
	62	6·0	+ 24·9
D, spirit lamp,	62	15·9	+ 24·9
	62	9·6	+ 24·9
	62	7·8	+ 24·9
Maximum of a bright part, rather brighter than neighbouring parts,	56	60·5	+ 24·9
Same,	55	94·8	+ 24·9
A line, seen as a point,	51	23·5	+ 24·9
A line; bright point,	53	13	+ 24·9

R. C.

A bright line,	62	12·5	+ 21·55
A point almost at violet end. There is merely a trace of light extending slightly beyond.	47	80·0	+ 21·55
Towards redward end in the brightest part of a green band,	57	23·4	+ 21·55
	57	45·0	+ 21·55
Fine continuous spectrum with a fine bright line (D) in the yellow.			
A line in the red,	63	62·0	+ 21·55
		73·7	+ 21·55

J. G. L.

Bright part of spectrum,	60	49·2	+ 21·55
D-line in comet,	62	23·7	+ 21·55
		17·1	+ 21·55
D, spirit lamp,	62	15·9	+ 21·55
		13·0	+ 21·55
Edges of slit in D-light,	61	94·6	+ 21·55
" "	62	32·9	+ 21·55

May 29.—Grubb spectroscope, long collimator, Steinheil 60° prism, on 15·06-in. telescope, in the same position as at the close of last night's work. Daylight spectrum with narrow slit; lowest power eye-piece.

	Micrometer Reading.		Reduction to Curve.
B	65^{r}	$84^{p}\cdot 8$	$+ 32^{p}\cdot 8$
		84·2	+ 32·8
C	64	80·1	+ 32·8
		80·0	+ 32·8
D	62	3·2	+ 32·8
		3·2	+ 32·8
E	58	34·0	+ 32·8
		33·8	+ 32·8
b_1	57	72·2	+ 32·8
		72·9	+ 32·8

	Micrometer Reading.		Reduction to Curve.
b_{2-4}	57^{r}	$62^{p}\cdot 6$	$+ 32^{p}\cdot 8$
		60·6	+ 32·8
F	54	98·8	+ 32·8
	55	1·2	+ 32·8
	55	1·7	+ 32·8
G	48	40·0	+ 32·8
difficult to see		38·2	+ 32·8
		39·8	+ 32·8

The deviations from the curve not being large, the unaltered curve readings were used in the reductions.

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Spectrum of Comet; same date.

	Micrometer Reading.		Reduction to Curve.
Bright reddish yellow line,	62^{r}	9^{p}·0	+ 23^{p}·0
		11·0	+ 23·0
		6·0	+ 23·0
		13·2	+ 23·0
Another line,	62	67·8	+ 23·0
		59·3	+ 23·1
Bright D-line,	62	10·2	+ 23·0
Brightest part in the green,	57	27·3	+ 23·0
Same,	57	55·8	+ 23·0
D-line of spirit lamp,	62	14·5	+ 23·0
		13·4	+ 23·0
		12·0	+ 23·0
		12·0	+ 23·0
Limits of slit in D-light,	62	26·5	+ 23·0
	61	98·0	+ 23·0

Observer R. C.

May 31.—Spectrum very fine with 60° Steinheil prism.

	Micrometer Reading.		Reduction to Curve.
Bright D-line,	62^{r}	17^{p}·0	+ 19^{p}·7
		12·7	+ 19·7
		16·7	+ 19·7
A slightly brighter place, not decidedly a line,	62	62·3	+ 19·7
	59	12·8	+ 19·7
Red end; narrow slit,	64	27·6	+ 19·7
Brightest in this part of the green,	59	63·5	+ 19·7
Slightly brighter than neighbourhood,	61	38·0	+ 19·7
Last light in violet,	49	46·8	+ 19·7
Cometary D-line,	62	20·2	+ 19·7
		14·4	+ 19·7
D-line, spirit lamp,	62	15·3	+ 19·7
		16·8	+ 19·7
		14·1	+ 19·7
		18·6	+ 19·7
		16·9	+ 19·7

A bright part,	63	26·1	+ 18·6
Slightly brighter than the neighbourhood,	63	79·3	+ 18·6
Beginning of spectrum,	64	49·0	+ 18·6
A nebulous luminosity, rather long in the direction of the slit; longer than the D-line,	64	27·9	+ 18·6
A bright part, line?	63	32·6	+ 18·6
A similar bright part,	62	63·6	+ 18·6
D-line, very striking,	62	17·0	+ 18·6
A sharp bright part,	61	73·0	+ 18·6
A maximum,	59	58·1	+ 18·6
Another maximum,	56	55·9	+ 18·6
A marked maximum,	57	45·7	+ 18·6
D-line of spirit lamp; it coincides exactly with the splendid bright line in the spectrum of the comet.	62	17·9	+ 18·6
		17·8	+ 18·6
		16·7	+ 18·6
		17·1	+ 18·6

Observers: R. C. and J. G. L.

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June 1.—Grubb spectroscope, long collimator, Steinheil prism of 60°. Comet's nucleus just at end of slit. The D-line extends into the coma [tail] by at least the whole length of the slit. Faint bands just visible in the green (Sketch, see Fig. 4); the slit on axis of tail; tail upwards apparently. When the nucleus was placed apparently on the upper end of the slit, the D-line was reduced to a bright point, and the daylight spectrum could be traced from 60^{r} to 56^{r}, or from $549^{\mathrm{mmm}}\cdot5$ to $495^{\mathrm{mmm}}\cdot3$; while to the redward of D the field appeared very black. (Sketch, see Fig. 5.) With slit at right angles to tail, and just behind the nucleus, the D-line was seen right across, but broadened out and soft on the edges.

	Micrometer Reading.	Reduction to Curve.
Bright part in green,	57^{r} $55^{\mathrm{p}}\cdot3$	$+ 20^{\mathrm{p}}\cdot0$
Another bright part,	59 47·3	+ 20·0
,, ,,	59 83·2	+ 20·0
Bright part in orange,	62 85·1	+ 20·0
A line or nearly so,	63 58·2	+ 20·0
Beginning of spectrum,	64 75·0	+ 20·0
Cometary D-line,	62 16·0	+ 20·0
Brighter part (brighter than neighbourhood),	60 11·9	+ 20·0

Altered main focus.

Brighter part,	53 0·0	+ 16·1
Extreme end of spectrum,	48 27·2	+ 16·1
A line, the brighter one,	63 42·0	+ 16·1
A line, a little fainter,	62 65·0	+ 16·1
In accordance with what is sketched a couple of pages back (see Fig. 4), the comet is seen en miniature in the D-light.		
D-line of the comet,	62 19·9	
Bright part,	59 95·0	+ 16·1

June 4.—Comet seen for a minute or so between clouds. D-line very much brighter. The comet was also seen through fairly dense cloud.

June 5.—Very much cloud.

June 6.—Grubb spectroscope, long collimator, Grubb compound prism in minimum deviation for F., 13^{h} 45^{m} G.M.T. We see the D-line of the comet beautifully double. D_2, or the one to the violet, is about twice as bright as D_1. The lines are visible in the whole length of the slit, which is nearly parallel to the tail of the comet, so that this D-light is present in the nucleus, and the beginning of the tail as well. (Dark line, most probably in the daylight 44^{r} $89^{\mathrm{p}}\cdot2$.) Daylight, so that writing was plainly legible. With a wide slit, the D-light of comet's tail visible for 6′ and 7′ from nucleus by two trials.

With the nucleus in the lower part apparently of the widely opened slit, the sight was one of great magnificence. (Sketch, see Fig 6.) The beginning of the spectrum was then very bright red.

Comet well seen with naked eye, and also through dark yellow glass. Comet seen splendidly in finder with a small direct vision M'Lean prism.

Displacement, if any, of the D-lines one-tenth of interval towards the red.

Just at sunrise, with a very narrow slit, the D-line towards the violet is fully twice as long as the other; both lines appear much thicker in the nucleus. (Sketch, see Fig. 7.)

(The following further notes were written on June 11.)

Dr. Copeland estimated the brightness of the comet equal to 1·5th magnitude, and I found for it quite independently as limits, between which it must be classified, the first and second magnitude.

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With a wide slit the sight was really magnificent: exactly like a prominence, the perfect image of the comet, head and tail, being visible in the light of the D-line, very bright and clear, and the comet being very near the horizon, the atmospheric disturbances of the image were sometimes distinctly recognised by a waving motion of that part of the limb of the head, which was furthest from the tail, at once reminding me of Jupiter when seen at a time of bad definition; the definition to-night, however, was very good, only the comet at first was very low down. The head appeared elongated transversely to the tail, the diameter being about three to four times the interval between the D-lines.—J. G. L.

June 10.—Tried in vain to find the comet in day time.—R. C. & J. G. L.

June 12.—Searched for the comet from 1^h to 3^h G.M.T., but the daylight was too powerful.—J. G. L.

July 15.—About the place, assigned to comet Wells by the ephemeris, the sky was so bright that not a single star could be recognised, although specially looked for with the 15·06-in. refractor. The comet must therefore be considered as lost in these latitudes.—J. G. L.

Observed Wave-lengths for the different parts of the Spectrum of Comet Wells, 1882.

	Object.	May 28. Wave-length.	May 29. Wave-length.	May 31. Wave-length.	June 1. Wave-length.
		mmm.	mmm.	mmm.	mmm.
	Beginning of spectrum,	621·3		644·1	651·7
	„ „			638·4	
	A brightness rather long in the direction of the slit,			638·2	
	A line or nearly so,	621·9		625·5	620·5
	„ „	624·8			
	A bright part; line (?)			614·1	615·7
	„ „			612·5	
	A bright part; line (?)		600·6	598·7	603·7
	„ „		598·8	598·8	598·6
	Cometary Sodium (D)-line,	590·6	588·4	589·3	589·2
		589·3	588·8	588·4	589·2
		590·5	587·8	589·2	
		588·1	589·2	590·0	
		588·8	588·6	588·8	
		591·1			
		589·7		589·1	
	A sharp bright part,			580·3	
	Slightly brighter than neighbourhood,			573·8	
1st Band.	A bright part,	558·0			
	A bright part; a maximum,			543·6	548·1
	„ „			542·6	551·5
	„ „			535·8	546·8
	„ „				541·1
2nd Band.	Brightest part in the green; maximum,	510·2	510·9	512·7	514·1
	„ „	513·0	514·6		
	Another maximum,	503·0		501·7	
	„ „	495·3			
3rd Band.	A line, bright point; bright part,	466·8			464·8
	A line, seen as a point,	450·7			
	A point almost at violet end,	426·1			
	Last light in violet,			437·1	428·8

The Great Comet of 1882.

On September 11,* Monsieur Cruls, in Rio de Janeiro, discovered a large comet, visible to the naked eye in the morning sky before sunrise. The telegraphic announcement of this discovery was received in Dun Echt on the evening of September 13, and communicated in a Dun Echt Circular on the following day, as well as telegraphed to the Science Observer, Boston, U.S.

On two occasions, but especially on September 16, the sky was examined with a small comet-seeker, and the finder of the 15-in. Equatoreal, but the atmosphere not being perfectly clear, the approaching daylight soon became so powerful as to render even the bright stars invisible in the finder, so that a further continuation of the search seemed useless.

On Monday morning, the 18th, a telegram, kindly forwarded by A. A. Common, Esq., was received, announcing that a bright comet had been discovered by him at ten hours forty-five minutes on Sunday morning, five minutes preceding the Sun, approaching fast, and being most likely identical with Comet Cruls. As it happened to be clear, Dr. Copeland at once went outside to look for the comet. The first glance at the sky, with the Sun just concealed behind the western side of the dome, showed the comet splendidly visible with its head towards the Sun, and a tail about 15′ long, pointing westward.

As the telescope revealed no jets or any other striking phenomena not visible to the naked eye beyond the same majestic symmetrical form, it was at once decided to examine it spectroscopically, and if possible, observe it with the meridian circle.

The Grubb spectroscope in its original form with the Grubb compound prism, approximately adjusted to minimum deviation for F, was, therefore, mounted on the 15-in. Refractor, and the image of the comet at once brought on the slit. The expected bright Sodium lines exhibited themselves to the eye of the observer with a brilliancy and neatness quite comparable to the C-line in prominences, so well defined and clear did they stand out on the bright daylight spectrum.

Widely separated as they were, by the powerful prism, it was easily and with the greatest certainty seen that the more refrangible D_2 was slightly but decidedly brighter, and a little broader than D_1 and just perceptibly soft on the redward side, but this difference between the two D-lines was far less obvious than in the spectrum of Comet Wells, where it was impossible to overlook it.

The presence of the dark Frauenhofer lines of the daylight-spectrum did not permit a decision as to whether the violet edge of the lines was soft or well defined, nor in the short time at our disposal was it possible to determine the displacement of the bright lines with regard to the dark ones otherwise than by estimation. We may safely say that the displacement of the bright lines towards the red was not greater than one-fourth, and not less than one-twelfth of the interval between the D-lines, and that any quantity between one-sixth and one-tenth of this interval may be considered as a probable value. The actual few and rather discordant measurements, however, to which only little time was devoted, make the displacement of the lines as large as one-fifth or one-fourth of the interval between the D-lines. corresponding to an increase of distance between the Earth and the comet of about 37 to 46 miles (Engl.) in a second of time.

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* It was already seen as early as September 7 by Mr. Ellery in Melbourne, and on the 8th by Mr. Finlay at the Cape of Good Hope.

By trial the D-lines were found extending fully twice the length of the slit, the latter equalling about $1\frac{1}{2}'$ when on the 15-in. Refractor.

No similar trials were made for any of the other bright lines.

Next to the sodium-lines in brilliancy came several lines in the green, notably E and some other prominent iron lines, and in every case the bright line was just to the redward side of a black Frauenhofer line. Among the observations two exceptions will be found, but these exceptions are only apparent. In the one case the bright objects observed, with sharp dark lines to the redward side, were not lines but bands, and, therefore, most probably not at all physically related to the black lines. In the other case two bright lines, both very soft, were observed very close together with a black line between them, which evidently proves nothing. Besides in both cases the objects observed were of the faintest and most ill-defined kind, and only measured once. Every time, however, when a well-defined bright line and a dark one were seen, so to say, in contact, the bright one was invariably on the redward side, the amount of displacement being in each case, so far as could be judged, the same.

The spectrum thus seen was singularly beautiful. There were however five well-defined although not very prominent bright lines on the red side of D with no corresponding visible Frauenhofer lines. Four of these lines were measured, the fifth however was plainly seen in favourable moments by both observers and was about as bright as the two fainter ones, but the time at our disposal was so pressing that no setting could be effected.

All these lines as well as the green ones were about as long as the slit. All the bright lines crossed the nucleus showing an increased intensity there, but not sufficiently so to give them a knotty appearance, as was the case with the D-lines in Comet Wells, thus indicating according to Prof. Zöllner's investigations* a less density of the sodium vapours in this comet than in Comet Wells. Judging from the continuous ribband-like spectrum of the nucleus, which was roughly $\frac{1}{10}$ of the length of the slit in width, the diameter of the nucleus amounted to about 10″, which partly accounts for the difficulty experienced in keeping the image on the slit; this difficulty was greatly increased by the rapid motion in Right Ascension.

That the two brightest lines were caused by incandescent Sodium vapours, there can be little doubt. But to what origin do the other bright lines point? E and one or two of the bands might indicate Calcium, but E is a very inferior Calcium-line and if it owed its great brilliancy to incandescent Calcium vapour, the more prominent Calcium lines ought to have been giants. Quite similar is the case with Titanium, some of the lines point to it, but the main Titanium lines were not noticed. But if we turn to Iron, the case is very different. The spectrum being rich in lines like those of Calcium and Titanium, it would not be very wonderful, if a single one of the observed lines could be matched with an Iron-line. But not merely an odd one, nay, all the six brighter lines observed in the green and yellow correspond to the most prominent Iron lines in that part of the spectrum and even the two broad bands, noticed there, fall into a group of numerous Iron lines. And further, Monsieur Lecoq de Boisbaudran in his "Spectres Lumineux," Paris 1874, puts down his group κ and a few not very faint lines, in almost the very places where the five faint lines in the orange were observed. We do not attach great weight to the fact that to these five bright lines corresponding Iron lines can be found, but it remains nevertheless remarkable.

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* Berichte der königl. sächs. Gesellschaft d. W. 31 October, 1870. Poggendorff's Annalen d. Physik u. Chemie. Bd. 218, pp. 88–111.

It is necessary specially to mention a luminosity between b_1 and b_2 and also a bright soft band, the wave-lengths of which coincide with the brightest band, generally observed in the spectra of comets, for that band must have been very bright to be noticed on a bright daylight spectrum. And this was partly proved by the brilliancy with which that band shone on later occasions.

Observations.

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September 18.—Grubb spectroscope in its original state; Grubb compound prism.

D-line bright and double.—R. C. & J. G. L. An approximate position of the comet was received by bringing it on the centre of the slit and reading the circles, the comet then being exactly one hour east of the meridian. The resulting place, corrected for refraction and instrumental errors, is:—

Date.	Dun Echt M. T.	App. α Comet.	App. δ Comet.	Observer.
Sept. 17,	22^h 42^m 49^s	11^h 31^m $43^s.9$	$+1°$ $27'.8$	R. C.

	Micrometer Reading.	Redn. to Curve.	Wave-length. mmm.	Observer.
At 11^h A.M., G.M.T.—Many bright lines in the green just to the redward side of solar lines.				R. C.
Bright line in spectrum of the comet, . .	49^r $48^p{\cdot}0$	$+$ $7^p{\cdot}6$	526·9	"
Frauenhofer line of daylight, ...	49 49·4	$+$ 7·6	527·0	"
Bright line in spectrum of the comet, . .	50 16·3	$+$ 7·6	533·0	"
Frauenhofer line of daylight, ...	50 13·2	$+$ 7·6	532·7	"
Bright line in spectrum of the comet, . .	51 21·8	$+$ 7·6	542·9	"

At this time no more observations could be secured, the comet being visible only during short intervals between the clouds, of which there were two layers, moving in different directions. Fortunately there was an opening between the clouds at the time of meridian passage, and the comet was observed on six wires, which agree well, the setting in declination being made at the

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middle wire. The comet entered the field very high; and in setting the telescope the first wire was lost, and apparently also the count of the seconds, for this was found to be one second ahead of the clock at the close of the observation. All the wires were therefore lessened one second, and there is hardly any possibility that this correction should not be right. The meridian circle was then in succession set on several stars, but clouds always obstructed the view. At last, some wires of Polaris, and a good observation of α Bootis were obtained.

The meridian observation, corrected for refraction and instrumental errors, gives the following position for the comet:—

Date, 1882.	Dun Echt M. T.	App. α Comet.	App. δ Comet.	Observer.
Sept. 17,	23^{h} 41^{m} 58^{s}	11^{h} 31^{m} 8^{s}·98	+ 1° 23′ 26″·2	J. G. L.

At about 3^{h} 50^{m} P.M., G. M. T.—The following Frauenhofer lines of daylight were measured by J. G. Lohse:

	Micrometer Reading.			Micrometer Reading.	
B	61^{r}	8^{p}·9	D_2	55^{r}	32^{p}·5
C	59	56·1	E	49	55·6
a	57	95·7	b_1	48	54·0
D_1	55	36·9	F	44	10·9

Immediately afterwards it became clear in the west, and the comet was beautifully seen for fully half an hour. Having been tantalized so long by the constant interference of the clouds, it was determined, in view of the still threatening aspect of the sky, to measure quickly every visible bright line and band in the spectrum of the comet, and not to try a tardy identification of some of the lines in Ångström's map.

	Micrometer Reading.	Reduction to Curve.	Wave-length.	Observer.
			mmm.	
Bright D_1 { The bright lines are on the redward side of the black ones.	55^{r} 37^{p}·8	— 2^{p}·5	589·3	J. G. L.
Bright D_2 { D_2 is somewhat thicker than D_1, . .	55 34·8	— 2·5	588·9	,,
Bright line, ...	56 35·3	— 2·5	602·8	,,
2 faint bright lines before D, the first is the brighter one.	55 67·3	— 2·5	593·3	,,
	55 58·6	— 2·5	592·1	,,
Fairly bright line, with a black line to the violet, .	51 30·8	— 2·5	542·8	,,
E bright, also dark, ...	49 58·1	— 2·5	526·9	,,
Fairly bright line, also dark, ...	50 26·3	— 2·5	533·0	,,
Fairly bright line, also dark, ...	51 33·0	— 2·5	543·0	,,
Broad band } Both as broad as interval of D, with	53 0·0	— 2·5	560·1	,,
Broad band } sharp dark lines on redward side.	52 75·0	— 2·5	557·4	,,
Bright line } Fainter than the 2 lines nearer to D; both	55 89·0	— 2·5	596·3	,,
Bright line } visible through the whole length of the slit.	55 82·1	— 2·5	595·3	,,
Bright line } The 2 lines nearer to D; also both visible	55 68·2	— 2·5	593·4	,,
Bright line } through the whole length of the slit.	55 59·1	— 2·5	592·2	,,
A short bright line, ...	54 64·3	— 2·5	579·7	,,
A faint soft brightness, ...	55 42·8	— 2·5	590·0	,,
At this time the comet was seen through yellow glass with the naked eye fully half a degree long.				R. C.
Bright part } The second luminosity is a little brighter;	48 85·2	— 2·5	520·7	J. G. L.
Bright part } there is a black line between them.	48 80·5	— 2·5	520·3	,,
Unfortunately the spectroscope was at this time subjected to an accidental strain.				
A brightness to redward side of a dark line between b_1 and b_2.	48 67·1	— 21·8	517·6	,,
A soft band, ...	47 91·1	— 21·8	511·5	,,

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		Micrometer Reading.	Reduction to Curve.	Wave-length. mmm.	Observer
Bright line	The bright lines are about 1^p of the screw to the redward side of the black lines.	51^r $97^p{\cdot}3$	$-21^p{\cdot}8$	547·4	R. C.
Bright line		51 48·8	— 21·8	542·6	”
Bright line		51 16·0	— 21·8	539·5	”
Bright line		50 88·3	— 21·8	536·9	”
Bright line		50 41·0	— 21·8	532·6	”
Bright line		49 77·4	— 21·8	526·9	”
E, the dark Frauenhofer line of daylight, ...		49 76·5	— 21·8	526·8	”

The images being too bad for any further spectroscopic observations, the comet was again brought on the middle of the slit and the circles read. The resulting place, corrected for refraction and instrumental errors, is—

Date, 1882.	Dun Echt Mean Time.	App. α Comet.	App. δ Comet.	Observer
Sept. 18	4^h 14^m 40^s	11^h 29^m $6^s{\cdot}5$	+ 1° 13′·0	”

At 4^h 35^m P.M., G. M. T.

	Micrometer Reading.	Reduction to Curve.	Wave-length.	Observer
The continuous spectrum extends from, . . .	61 32·4	— 21·8	685·5	”
to, . . .	39 31·4	— 21·8	457·5	”
The nucleus is not visible in an open slit as was the case with comet Wells.				R. C. & J. G. L.
A bright band (with wide slit), ...	47 78·8	— 21·8	510·5	?

There is nothing surprising in the disappearance of the image of the nucleus, shining in the yellow Sodium light, on opening the slit wide, for prominences also disappear at last under similar circumstances. The reason is that the apparent brightness of the monochromatic image of an object is independent of the breadth of the slit, while that of the continuous daylight increases proportionally to the breadth of the slit.

With the aid of the eight Frauenhofer lines, measured in the spectrum of daylight, a new reducing curve was constructed on the basis of another curve, formerly prepared for this prism. The readings for a sufficiently extended part of the curve are given here.

Reducing curve for transforming into wave-lengths the corrected Micrometer Readings of the Grubb Spectroscope, the Grubb compound prism being approximately adjusted to minimum deviation for F :

Wave-length. mmm.	Micrometer.	Wave-length. mmm.	Micrometer.	Wave-length. mmm.	Micrometer.
480	43^r $9^p{\cdot}4$	540	50^r $99^p{\cdot}3$	600	56^r $13^p{\cdot}2$
485	43 94·0	545	51 51·2	605	56 48·2
490	44 73·0	550	52 1·5	610	56 82·2
495	45 47·2	555	52 50·0	615	57 15·4
500	46 17·4	560	52 96·5	620	57 47·8
505	46 85·0	565	53 41·0	625	57 79·6
510	47 50·3	570	53 83·7	630	58 10·7
515	48 13·3	575	54 24·5	635	58 40·8
520	48 74·4	580	54 64·1	640	58 69·7
525	49 33·6	585	55 2·8	645	58 97·6
530	49 90·7	590	55 40·6	650	59 24·6
535	50 45·8	595	55 77·4	655	59 50·4
540	50 99·3	600	56 13·2	660	59 75·4

For the sake of uniformity the Reduction to Curve was always deduced from the readings for the bright E-line, although this proceeding is not strictly correct; for it assumes the displacement of all the bright lines to be always the same number of parts of the screw, and moreover the resulting wave-lengths, thus obtained, are all too small by a corresponding quantity. But, besides that the instrument does not measure to such a nicety, these errors are insignificant compared with the neglected and not easily determinable variations of the zero correction, caused by the flexure in the spectroscope.

To facilitate comparisons the observations are now given in the order of the wave-lengths, together with the corresponding dark Frauenhofer-lines, for which the wave-lengths are taken either from Ångström's or from M. Lecoq de Boisbaudran's tables, and marked accordingly A or B. It is hardly necessary to point out, that the wave-lengths given for the lines, seen both bright and dark, only serve to identify in the solar spectrum these dark lines, which were nearly all conspicuous ones, and that it is altogether inadmissible to draw conclusions from them as to the displacement of the lines.

	Wave-length. mmm.	Lines in the Solar Spectrum or in the Spectrum of Iron (B). mmm.	Authorities.
Seen beginning of spectrum before B or about, . . .	700		
Beginning of spectrum, measured, ...	685·5		
Bright line, ...	602·8	? 602·30 Fe ? 601·8	A. B.
Bright line, not measured, but seen by both observers.			
Bright line — Fainter than the two lines nearer to D; both visible through the whole length of the slit.	596·3		
Bright line	595·3	? 594·74 Fe	A.
Bright line — The first line is just perceptibly brighter than the second one; both visible through the whole length of the slit.	593·3	? 593·6 — Group κ	B.
Same	593·4		
Bright line	592·1	? 591·6	B.
Same	592·2		
Faint soft brightness, ...	590·0		
Bright D_1 — The bright lines are on the redward side of the black ones; D_2 is somewhat thicker than D_1.	589·3	589·50 Na	A.
Bright D_2	588·9	588·90 Na	A.
Short bright line, ...	579·7	? 579·72 Fe	A.
		? 560·17 Fe — Group: θ, Boisbaudran.	A.
		? 559·72 Fe	A.
		? 559·12 Fe	A.
Broad band — Both as broad as interval of D; they have sharp dark lines on the redward side.	560·1	? 558·56 Fe	A.
Broad band	557·4	? 558·38 Fe	A.
		? 557·76 Fe	A.
		? 557·49 Fe	A.
		? 557·17 Fe	A.
		? 556·85 Fe	A.
Bright line, to the redward side of a dark line, . . .	547·4	547·33 Fe, Ti	A.
Bright line, to the redward side of a dark line, . .	542·9	542·88 Fe, Ti	A.
Same, . .	542·8		
Same, . .	543·0		
Same, . .	542·6		
		? 539·60 Fe	A.
Bright line, to the redward side of a dark line, . . .	539·5	? 540·31 Fe	A.
		? 540·48 Fe	A.
Bright line, to the redward side of a dark line, . . .	536·9	537·05 Fe	A.

	Wave-length. mmm.	Lines in the Solar Spectrum or in the Spectrum of Iron (B). mmm.	Authorities.
Bright line, to the redward side of a dark line,	533·0		
Same,	533·0	532·73 Fe	A.
Same,	532·6		
Bright E-line, to the redward side of the dark E-line,	526·9		
Same,	526·9	526·90 Fe, Ca	A.
Same,	526·9		
Bright part The second luminosity is a little brighter; there	520·7		
Bright part is a black line between them.	520·3		
A brightness to redward side of a dark line, between b_1 and b_2,	517·6	Great band of the comet.	
Soft band,	511·5		
Bright band (with open slit),	510·5		
End of continuous spectrum,	457·5		

Although the observations were begun at about 10^h 50^m A.M., the clouds were so troublesome that we probably had not more than an hour's unobstructed view of the comet from that time until 5^h P.M., about which time rising clouds to the westward completely cut off the comet from our view.

At Noon we telegraphed to America and in the evening prepared a telegram to the Academy at Vienna and a Dun Echt Circular No. 56, ready for dispatch to Aberdeen by the early post (8 A.M.) on the following morning. The next day, Tuesday, September 19th, was very cloudy and wet and the weather remained so for the rest of the week, until all hope of renewing the daylight observations was at an end.

The first opportunity of seeing the comet in the early morning twilight we had on September 29th. It was at first visible to the naked eye but was soon overpowered by the approaching daylight, even in the field of the telescope.

In the spectroscope the brightest band of the ordinary comet spectrum was wonderfully distinct, the initial green line being fully separated from the rest of the band. The D-lines, merged into one, were also very bright so that on opening the slit gradually the comet was seen in two colours, yellow and green. It was not possible to get a full view of the nucleus and the rest of the tail in this manner, as the continuous spectrum of the increasing daylight overpowered the comet's spectrum, before the slit was sufficiently wide open. The great group of bands in the citron, which we have seen in so many comets with extreme difficulty, was visible as a complicated group of bright bands. It is very much to be hoped that spectroscopic observations may have been made in the Southern hemisphere at this critical time, when we believe that the increasing distance of the comet from the Sun was gradually changing its spectrum from the type shown by Comet Wells, to the usual type of cometary spectra. By the time that a few measures had been taken the spectrum of the daylight extinguished that of the comet and the observer found himself measuring the familiar Frauenhofer lines.

Observations.

1882.

September 29.—Grubb compound prism in the Grubb spectroscope. Lowest power eye-piece.

Comet visible to the naked eye at 17^{h} 6^{m} G.M.T.

Object.	Micrometer Reading.	Reduction to Curve.	Wave-length.	Observer.
			mmm.	
Redward edge of bright band, ...	48^{r} 65^{p}·7	— 40^{p}·9	515·9	R. C
Almost a line, ...	57 25·0	— 40·9	610·3	”
Frauenhofer line E of daylight, ...	49 96·5	— 40·9	526·9	”
Frauenhofer lines *b*, seen as one line, . . .	48 77·2	— 40·9	516·9	”

With a rather wide slit the nucleus and neighbouring part of the tail were seen by the light of D and also of the great green line. The latter was seen quite separate and distinct like D in an open slit. On opening the slit wider the daylight overcame the light of the comet. There was a broad region of light on the green side of D, where we have often measured faint bands in hydrocarbon comets.

The comet is of a rich golden colour and some 30° in length.

The spectrum was also seen by Mr. J. G. Lohse, but not measured.

October 22.—Grubb compound prism in the Grubb spectroscope. Lowest power eye-piece.

In the finder the nucleus appears elongated in the direction of the tail. The sight with the naked eye is very splendid; the tail is exceedingly bright, a little curved, the convex side to the south, which is also the brightest and sharply bounded, while the north side fades off. Gradually broadening, it attains at the end a breadth equal to about one-sixth of its apparent length, and as the brightness is almost uniform it presents the appearance of being cut off. But on closer examination the southern third of the tail is seen to have a very faint conical continuation. The tail is fully 20° long.

Looked in the finder for Professor Schmidt's companion, but did not see it.

In the spectroscope not a trace of the D-lines is visible, although diligently searched for.

The continuous spectrum of the nucleus is faint and only just sufficient to connect the bands, almost like in Comet IV., 1881 (Schaeberle); it begins a little before the first band and ends not very far beyond the third one. The first band is the faintest, it has only one maximum and fades off on both sides. The second band is by far the brightest, it begins with a sharp line, which is however not any more separated from the rest of the band, most probably on account of the wide slit; with a narrow slit the spectrum became too faint.

At 18^{h} G.M.T. the comet was still tolerably bright in the spectroscope, but soon disappeared.

	Micrometer Reading.	Reduction to Curve.	Wave-length.	Observer.
			mmm.	
1st band: maximum, ...	53^{r} 61^{p}·9	— 57^{p}·65	560·9	J. G. L.
2nd band: sharp edge, ...	49 12·5	— 57·65	518·4	”
3rd band: maximum; the edge is a little soft, .	42 ·22·2	— 57·65	471·8	”
D of spirit lamp, ...	55 91·8	— 57·65		”
”	92·5	— 57·65		”
Limits of slit in D-light, ...	56 40·1	— 57·65		”
”	55 46·9	— 57·65		”
1st band: maximum, ...	53 38·5	— 49·45	559·2	”

1882.	Micrometer Reading.	Reduction to Curve.	Wave-length. mmm.	Observer.
2nd band: bright edge,	48^{r} 85^{p}·2	— 49^{p}·45	516·8	J. G. L.
3rd band: maximum,	41 90·0	— 49·45	470·4	"
Same,	41 99·0	— 49·45	470·9	"
2nd band: bright sharp edge,	48 75·1	— 49·45	516·0	"
1st band: maximum,	52 98·8	— 49·45	554·9	"
Same,	52 82·9	— 49·45	553·3	"
D of spirit lamp,	55 82·1	— 49·45		"
Same,	86·2	— 49·45		"
Limits of slit in D-light,	56 17·3	— 49·45		"
"	55 46·0	— 49·45		"

D of spirit lamp,	55 80·2	— 45·5		"
1st band: maximum,	52 97·5	— 45·5	555·2	"
2nd band: sharp edge,	48 86·8	— 45·5	517·3	"
3rd band: maximum,	42 7·0	— 45·5	471·6	"

Systematically arranged, the wave-lengths for the three bands are—

	Wave-length. mmm.		Wave-length. mmm.
1st band: maximum,	560·9	2nd band: sharp edge,	516·0
Same,	559·2	Same,	517·3
Same,	554·9	3rd band: maximum,	471·8
Same,	553·3	Same,	470·4
Same,	555·2	Same,	470·9
2nd band: sharp edge,	518·4	Same,	471·6
Same,	516·8		

October 23.—Grubb spectroscope in its original form, with Grubb compound prism approximately in minimum deviation for F. The telescope was, as usual, the 15·06-in. Equatoreal.

Viewed the spectrum of the comet at about 18^{h} G.M.T.; only the three bands are visible. In the second band a bright point was noticed at the intersection of the sharp edge and the spectrum of the nucleus; a similar, but fainter point appeared at the brightest part of the third band. The spectrum of the nucleus is brightest, where the bands are brightest, and fades away exactly as the bands do, extending only just sufficiently to connect the latter. The length of the tail is about equal the distance between β and γ Leonis.—J. G. L.

General Remarks.

Judging from the totality of the appearance presented by these four large comets, we are strongly inclined to think, that the difference of spectra of nucleus, coma and tail is only one of intensity, however that may be brought about. The maxima of brightness in the spectrum of the nucleus always coincide with the brightest parts of the bands of the coma and tail. With increased brilliancy the bands dilate in breadth, and show a tendency to unite and become continuous, as was very nearly the case in Comet III., 1881. In fact, on the first days there was any quantity of continuous light between the bands, but whether it was entirely owing to twilight could not be decided. On the other hand in Comet IV., 1881, and in the great comet of 1882, when last examined, the spectrum of the nucleus was reduced to that of a bright, shining coma, the connexions between the bands being only faint, nebulous lines. In the bright spectrum of Comet Wells, where—not counting for the moment the sodium lines—the coma and tail practically showed no spectrum, the bands nevertheless were strongly indicated by increased radiance in these regions, and it will always be difficult to decide how much of the light is identical with the banded coma light; and whether the reflected sunlight may not be sufficient to account for the rest of the light.

We have positive proof, that, when a comet is very near to the Sun, the changes in the light, caused by the Sun's vast heating power, are not restricted to one part, but are the same in the nucleus, coma and tail. On June 6 the whole Comet Wells was seen brilliantly in the light of sodium in an open slit, the tail as well as the head, and in the spectrum of the great comet of 1882 all the brighter lines extended through the whole length of the slit, being, however, fuller of light in the nucleus, thus indicating their presence in the nucleus as well as in the commencement of the tail.

A conjecture, perhaps not new, of the origin of the sodium and iron lines may here not be altogether out of place. Although probable, it is not absolutely necessary, that the substances sodium and iron originally existed in the comet; the phenomena would have been essentially the same, had these metals been present in those regions of interplanetary space, and the comet only been the means of making them luminous.

But if they form an essential part of a comet's substance, the appearance and disappearance of their spectra might, perhaps, be utilized as indicating the amount of heat; under all circumstances, however, they will be very valuable.

We cannot omit to say a few words about the first, yellowish green band. It is generally described as similar to the two other bands, beginning brightest towards the red, and fading gradually away towards the violet. It is true, the dispersive power of the instrument greatly modifies the appearance, but we must say, that under high dispersion we have never seen the first band like the others: it always faded away on both sides, and had seldom a very marked maximum, sometimes it had two, and, perhaps, more, and it seems to be the only band, which shows an essentially different appearance in different comets, and, therefore, deserves always a special examination. Unfortunately, it is nearly always the faintest band, and difficult to deal with, and only in Comet III., 1881, traces of what may be bright lines were recognisable; that the iron lines have any connexion with it is very doubtful, since E falls outside of it.

With respect to the measures of fainter lines and bands, we cannot too strongly insist on the fact that the spectroscope used is so constructed that the observer can never know what the reading, and still less the wave-length, will turn out to be; each measure is, therefore, quite independent of personal bias, and any feature, however difficult to perceive, if referred by measurement again and again to the same part of the spectrum, must actually exist.

The drawings, which accompany this paper, are intended to represent, as nearly as possible, the actual appearance of the spectra of the comets in the spectroscope, but, being negatives, can only do so insufficiently; the difficulties of representing the bands and lines bright, are, however, extreme. As scale the micrometer readings for the compound prism were taken, to which the observations made with other prisms have been reduced for the sake of uniformity. In the case of Comet III., 1881, all the details are shown in one single drawing, although observed partly on different nights; the same is the case with the first drawing of the spectrum of Comet Wells; the D-lines are represented there as seen on the last day; at first they were only a faint point, but became brighter and longer from day to day.

In Fig. 8 all the bright lines and bands, that were seen, are given, but to convey a real idea of the splendid appearance, an artist would have to represent them on a coloured solar spectrum side by side with the dark lines.

Fig. 9 is incomplete for want of observations.

Nearly all the observations were made conjointly by both observers: either the one, who observed, noted also the observations, while the other kept the image of the comet on the slit, or if the latter caused little difficulty, the observer would do it, and dictate the readings to the other, who also, from time to time, showed the comparison-spectrum. To avoid fatiguing the eyes, the observers changed once, twice or oftener, according to circumstances, and as a rule, both always examined the spectrum, that nothing might be overlooked, and, if possible, measured it both. The omission of the initials R. C. and J. G. L., therefore, indicates that the observations were made by both observers promiscuously.

RALPH COPELAND.
J. GERH. LOHSE.

CORRECTIONS AND ADDITIONS.

Vol. I., p. 237. Comet 1881 *e* (BARNARD).

	$\Delta\alpha$	$\Delta\delta$	α Comet.	δ Comet.
For Oct. 11,	$+\,0^{m}\;41^{s}{\cdot}81$	$-\,5'\;4''{\cdot}8$	$13^{h}\,46^{m}\,5^{s}{\cdot}56$	$+\,20^{\circ}\;26'\;9''{\cdot}5$
Read Oct. 11,	$+\,0^{m}\;41^{s}{\cdot}83$	$-\,5'\;4''{\cdot}8$	$13^{h}\,46^{m}\,5^{s}{\cdot}84$	$+\,20^{\circ}\;30'\;14''{\cdot}0$

and

	α	Redn.	δ	Redn.	
For Oct. 11,	$13^{h}\,45^{m}\;22^{s}{\cdot}03$	$+\,1^{s}{\cdot}72$	$+\,20^{\circ}\,31'\,24''{\cdot}9$	$-\,10''{\cdot}6$	
Read Oct. 11,	$13^{h}\,45^{m}\;22^{s}{\cdot}30$	$+\,1^{s}{\cdot}71$	$+\,20^{\circ}\,35'\,29''{\cdot}4$	$-\,10''{\cdot}6$	D. M. + 20°, 2884, connected with 1 Boötis on 1881, Oct. 11, and 1882, March 10.

The mistake in the deduction of the star-place was pointed out by Prof. E. Millosevich, at Rome.

Vol, II., p. 118. *North following* of star No. 60 is a faint star of the 15th magnitude, the place of which is about—$\alpha = 21^{h}\,37^{m}\,0^{s}{\cdot}5$, $\delta = +\,42^{\circ}\,17'\,36''$.

It was plainly seen by both observers on 1882, August 19, when also 2 stars were seen near No. 25; tho brightness of Nova Cygni was on that night estimated of the 14th magnitude.

„ p. 119. In the List of Memoirs on Nova Cygni was accidentally omitted

Astronomical Register—Vol. 15, p. 46, Isaac Ward, position and sketch.

Page 129, line 18, *for* trepidatis *read* trepidatio.
„ „ „ 20, *for* universe *read* universæ.
„ 130, „ 20, *for* $\frac{2k}{r'}$ *read* $-\frac{2k}{r'}$.
„ „ „ 22, *for* $=$ *read* $=-$
„ „ „ 24, *for* $v' = p'$ *read* $r' = p'$.
„ 167, „ 9, *for* and *read* et.

STORAGE